

# Computer algebra independent integration tests

2-Exponentials/2.1-u-F<sup>c</sup>-a+b-x<sup>n</sup>

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3.56	$\int e^{-a-bx}x^3(a+bx)^3 dx$	278
3.57	$\int e^{-a-bx}x^2(a+bx)^3 dx$	282
3.58	$\int e^{-a-bx}x(a+bx)^3 dx$	286
3.59	$\int e^{-a-bx}(a+bx)^3 dx$	290
3.60	$\int \frac{e^{-a-bx}(a+bx)^3}{x} dx$	294
3.61	$\int \frac{e^{-a-bx}(a+bx)^3}{x^2} dx$	298
3.62	$\int \frac{e^{-a-bx}(a+bx)^3}{x^3} dx$	302
3.63	$\int \frac{e^{-a-bx}(a+bx)^3}{x^4} dx$	306
3.64	$\int F^{a+b(c+dx)}x^m(e+fx)^2 dx$	310
3.65	$\int F^{a+b(c+dx)}x^3(e+fx)^2 dx$	314
3.66	$\int F^{a+b(c+dx)}x^2(e+fx)^2 dx$	325
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3.69	$\int \frac{F^{a+b(c+dx)}(e+fx)^2}{x} dx$	347
3.70	$\int \frac{F^{a+b(c+dx)}(e+fx)^2}{x^2} dx$	351
3.71	$\int \frac{F^{a+b(c+dx)}(e+fx)^2}{x^3} dx$	355
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3.80	$\int \frac{e^{-a-bx}(a+bx)^4}{(c+dx)^3} dx$	399
3.81	$\int \frac{e^{-a-bx}(a+bx)^4}{(c+dx)^4} dx$	405
3.82	$\int \frac{e^{-a-bx}(a+bx)^4}{(c+dx)^5} dx$	412
3.83	$\int F^{c(a+bx)} x^m \log^n(dx) (e+en+e(1+m+bcx \log(F)) \log(dx)) dx$	420
3.84	$\int F^{c(a+bx)} x^2 \log^n(dx) (e+en+e(3+bcx \log(F)) \log(dx)) dx$	423
3.85	$\int F^{c(a+bx)} x \log^n(dx) (e+en+e(2+bcx \log(F)) \log(dx)) dx$	426
3.86	$\int F^{c(a+bx)} \log^n(dx) (e+en+e(1+bcx \log(F)) \log(dx)) dx$	429
3.87	$\int \frac{F^{c(a+bx)} \log^n(dx) (e+en+bcx \log(F) \log(dx))}{x} dx$	432
3.88	$\int \frac{F^{c(a+bx)} \log^n(dx) (e+en+e(-1+bcx \log(F)) \log(dx))}{x^2} dx$	435
3.89	$\int \frac{F^{c(a+bx)} \log^n(dx) (e+en+e(-2+bcx \log(F)) \log(dx))}{x^3} dx$	438
3.90	$\int \sqrt{e^{a+bx}} x^4 dx$	441
3.91	$\int \sqrt{e^{a+bx}} x^3 dx$	445
3.92	$\int \sqrt{e^{a+bx}} x^2 dx$	449
3.93	$\int \sqrt{e^{a+bx}} x dx$	453
3.94	$\int \sqrt{e^{a+bx}} dx$	456
3.95	$\int \frac{\sqrt{e^{a+bx}}}{x} dx$	459
3.96	$\int \frac{\sqrt{e^{a+bx}}}{x^2} dx$	462
3.97	$\int \frac{\sqrt{e^{a+bx}}}{x^3} dx$	466
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#### 4 Listing of Grading functions

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# Chapter 1

## Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from <https://rulebasedintegration.org>

The number of integrals in this report is [ 98 ]. This is test number [ 53 ].

### 1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12 on windows 10.
3. Maple 2020 (64 bit) on windows 10.
4. Maxima 5.43 on Linux. (via sagemath 8.9)
5. Fricas 1.3.6 on Linux (via sagemath 9.0)
6. Sympy 1.5 under Python 3.7.3 using Anaconda distribution.
7. Giac/Xcas 1.5 on Linux. (via sagemath 8.9)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

## 1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or Hypergeometric2F1 functions. RootSum and RootOf are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100. ( 98 )	% 0. ( 0 )
Mathematica	% 100. ( 98 )	% 0. ( 0 )
Maple	% 79.59 ( 78 )	% 20.41 ( 20 )
Maxima	% 65.31 ( 64 )	% 34.69 ( 34 )
Fricas	% 88.78 ( 87 )	% 11.22 ( 11 )
Sympy	% 38.78 ( 38 )	% 61.22 ( 60 )
Giac	% 57.14 ( 56 )	% 42.86 ( 42 )

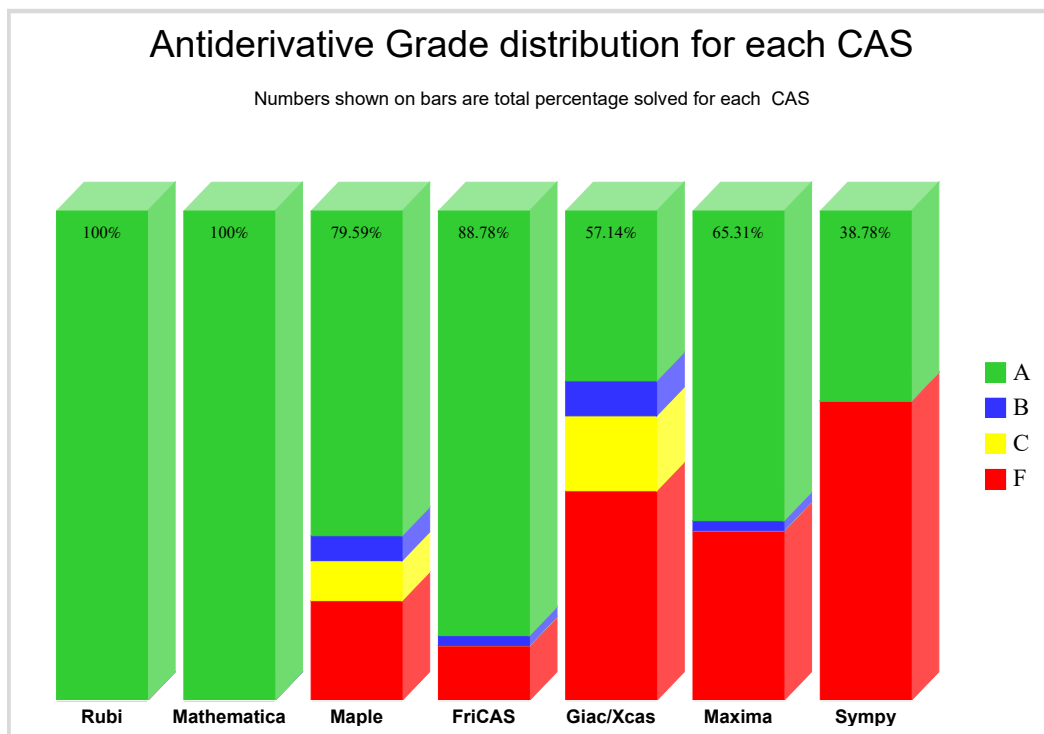
The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons <ol style="list-style-type: none"> <li>1. antiderivative contains a hypergeometric function and the optimal antiderivative does not.</li> <li>2. antiderivative contains a special function and the optimal antiderivative does not.</li> <li>3. antiderivative contains the imaginary unit and the optimal antiderivative does not.</li> </ol>
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

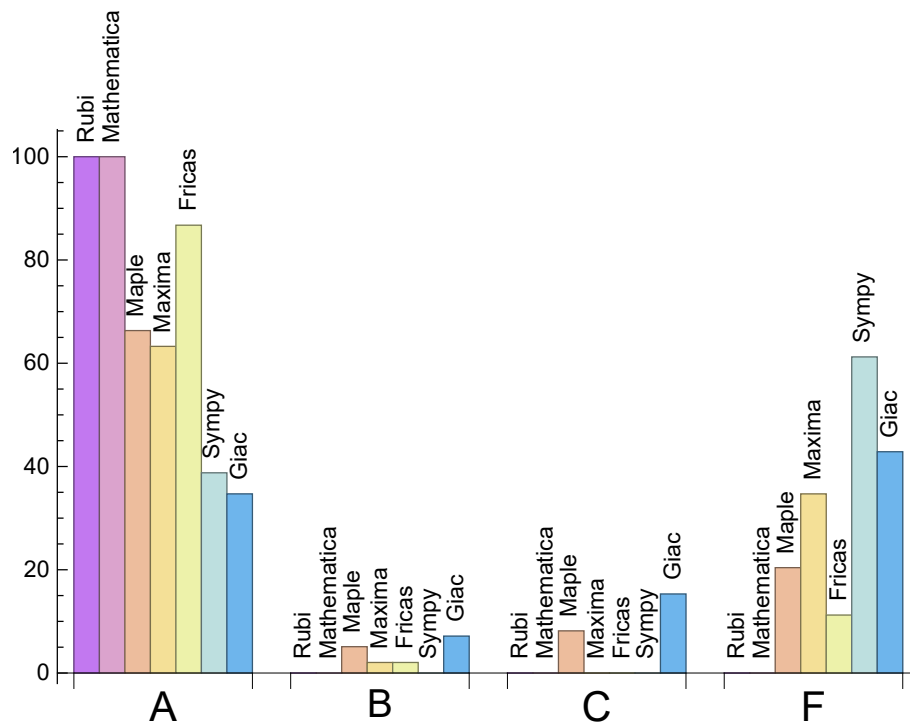
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.	0.	0.	0.
Mathematica	100.	0.	0.	0.
Maple	66.33	5.1	8.16	20.41
Maxima	63.27	2.04	0.	34.69
Fricas	86.73	2.04	0.	11.22
Sympy	38.78	0.	0.	61.22
Giac	34.69	7.14	15.31	42.86

The following is a Bar chart illustration of the data in the above table.





The figure below compares the CAS systems for each grade level.



### 1.3 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.17	134.81	1.	95.5	1.
Mathematica	0.12	86.01	0.72	65.5	0.71
Maple	0.03	177.45	1.83	137.	1.15
Maxima	1.17	153.62	1.27	83.5	1.09
Fricas	1.54	314.39	2.17	205.	2.11
Sympy	5.88	191.18	1.11	124.5	0.87
Giac	1.6	2106.7	12.63	230.	1.41

## 1.4 list of integrals that has no closed form antiderivative

{

## 1.5 list of integrals solved by CAS but has no known antiderivative

**Rubi** {}

**Mathematica** {}

**Maple** {}

**Maxima** {}

**Fricas** {}

**Sympy** {}

**Giac** {}

## 1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

**Rubi** {}

**Mathematica** {}

**Maple** Verification phase not implemented yet.

**Maxima** Verification phase not implemented yet.

**Fricas** Verification phase not implemented yet.

**Sympy** Verification phase not implemented yet.

**Giac** Verification phase not implemented yet.

## 1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime')
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

## 1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

## 1.9 Important notes about some of the results

### 1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using `sagemath` (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See <https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/> for reference.

## 1.9.2 Important note about FriCAS and Giac/X-CAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

## 1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the builtin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via `sagemath`) the following code is used

#see <https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-expression-size-in>

```
def tree(expr):
    if expr.operator() is None:
        return expr
    else:
        return [expr.operator()+map(tree, expr.operands())

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

```
try:
    # 1.7 is a fudge factor since it is low side from actual leaf count
    leafCount = round(1.7*count_ops(anti))

except Exception as ee:
    leafCount =1
```

When these cas systems have a builtin function to find the leaf size of expressions, it will be used instead, and these tests run again.

## 1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



**One record (line) per one integral result. The line is CSV comma separated. It contains 13 fields. This is description of each record (line)**

1. integer, the problem number.
2. integer. 0 or 1 for failed or passed. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be "A", "B", "C", or "F"
13. String. The optimal antiderivative in CAS own syntax.

### High level overview of the CAS independent integration test build system

# Chapter 2

## detailed summary tables of results

### 2.1 List of integrals sorted by grade for each CAS

#### 2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98 }

B grade: { }

C grade: { }

F grade: { }

#### 2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98 }

B grade: { }

C grade: { }

F grade: { }

### 2.1.3 Maple

A grade: { 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 48, 51, 52, 53, 54, 56, 57, 58, 59, 60, 61, 62, 63, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 90, 91, 92, 93, 94 }

B grade: { 64, 95, 96, 97, 98 }

C grade: { 55, 83, 84, 85, 86, 87, 88, 89 }

F grade: { 1, 19, 20, 21, 22, 23, 24, 25, 26, 39, 40, 41, 42, 43, 44, 45, 46, 47, 49, 50 }

### 2.1.4 Maxima

A grade: { 3, 4, 5, 6, 13, 14, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 48, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98 }

B grade: { 2, 12 }

C grade: { }

F grade: { 1, 7, 8, 9, 10, 11, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 39, 40, 41, 42, 43, 44, 45, 46, 47, 49, 50, 78, 79, 80, 81, 82 }

### 2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 23, 24, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 83, 90, 91, 92, 93, 94, 95, 96, 97, 98 }

B grade: { 81, 82 }

C grade: { }

F grade: { 20, 21, 22, 25, 26, 84, 85, 86, 87, 88, 89 }

### 2.1.6 Sympy

A grade: { 2, 3, 4, 5, 6, 12, 13, 14, 27, 28, 29, 35, 36, 51, 52, 53, 54, 56, 57, 58, 59, 60, 61, 62, 63, 65, 66, 67, 68, 74, 75, 76, 77, 90, 91, 92, 93, 94 }

B grade: { }

C grade: { }



F grade: { 1, 7, 8, 9, 10, 11, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 30, 31, 32, 33, 34, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 55, 64, 69, 70, 71, 72, 73, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 95, 96, 97, 98 }

## 2.1.7 Giac

A grade: { 6, 27, 28, 29, 30, 31, 32, 33, 34, 42, 43, 48, 56, 57, 58, 59, 60, 61, 62, 63, 74, 75, 76, 77, 78, 90, 91, 92, 93, 94, 95, 96, 97, 98 }

B grade: { 39, 40, 41, 79, 80, 81, 82 }

C grade: { 2, 3, 4, 5, 12, 13, 14, 51, 52, 53, 54, 65, 66, 67, 68 }

F grade: { 1, 7, 8, 9, 10, 11, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 35, 36, 37, 38, 44, 45, 46, 47, 49, 50, 55, 64, 69, 70, 71, 72, 73, 83, 84, 85, 86, 87, 88, 89 }

## 2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table, the column **normalized size** is defined as  $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	67	0	0	153	0	0
normalized size	1	1.	1.	0.	0.	2.28	0.	0.
time (sec)	N/A	0.04	0.038	0.047	0.	1.525	0.	0.

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	100	260	417	474	350	10620
normalized size	1	1.	0.71	1.84	2.96	3.36	2.48	75.32
time (sec)	N/A	0.108	0.12	0.009	1.588	1.522	0.537	2.585

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	78	165	278	312	231	6336
normalized size	1	1.	0.71	1.5	2.53	2.84	2.1	57.6
time (sec)	N/A	0.073	0.083	0.007	1.02	1.525	0.299	6.128

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	56	91	166	186	133	3367
normalized size	1	1.	0.71	1.15	2.1	2.35	1.68	42.62
time (sec)	N/A	0.043	0.066	0.007	1.008	1.546	0.236	1.83

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	34	38	81	90	60	1462
normalized size	1	1.	0.71	0.79	1.69	1.88	1.25	30.46
time (sec)	N/A	0.017	0.037	0.003	0.976	1.5	0.234	7.068

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	21	21	27	41	20	27
normalized size	1	1.	1.05	1.05	1.35	2.05	1.	1.35
time (sec)	N/A	0.004	0.005	0.001	0.945	1.511	0.108	3.585

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	56	0	78	0	0
normalized size	1	1.	1.	1.81	0.	2.52	0.	0.
time (sec)	N/A	0.024	0.041	0.033	0.	1.546	0.	0.

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	55	99	0	159	0	0
normalized size	1	1.	0.96	1.74	0.	2.79	0.	0.
time (sec)	N/A	0.043	0.119	0.038	0.	1.539	0.	0.

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	88	155	0	278	0	0
normalized size	1	1.	0.93	1.63	0.	2.93	0.	0.
time (sec)	N/A	0.071	0.157	0.04	0.	1.503	0.	0.

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	99	199	0	425	0	0
normalized size	1	1.	0.77	1.55	0.	3.32	0.	0.
time (sec)	N/A	0.1	0.201	0.049	0.	1.521	0.	0.

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	121	243	0	606	0	0
normalized size	1	1.	0.75	1.51	0.	3.76	0.	0.
time (sec)	N/A	0.134	0.172	0.056	0.	1.567	0.	0.

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	100	260	417	474	350	10620
normalized size	1	1.	0.71	1.84	2.96	3.36	2.48	75.32
time (sec)	N/A	0.122	0.065	0.007	1.058	1.51	0.235	1.493

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	78	165	278	312	231	6336
normalized size	1	1.	0.71	1.5	2.53	2.84	2.1	57.6
time (sec)	N/A	0.086	0.075	0.006	1.127	1.533	0.319	1.407

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	56	91	166	186	133	3367
normalized size	1	1.	0.71	1.15	2.1	2.35	1.68	42.62
time (sec)	N/A	0.045	0.039	0.004	1.15	1.529	0.274	1.284

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	55	99	0	159	0	0
normalized size	1	1.	0.96	1.74	0.	2.79	0.	0.
time (sec)	N/A	0.043	0.06	0.043	0.	1.536	0.	0.

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	88	155	0	278	0	0
normalized size	1	1.	0.93	1.63	0.	2.93	0.	0.
time (sec)	N/A	0.098	0.051	0.059	0.	1.499	0.	0.

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	99	199	0	425	0	0
normalized size	1	1.	0.77	1.55	0.	3.32	0.	0.
time (sec)	N/A	0.134	0.08	0.076	0.	1.575	0.	0.

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	121	243	0	606	0	0
normalized size	1	1.	0.75	1.51	0.	3.76	0.	0.
time (sec)	N/A	0.18	0.034	0.096	0.	1.594	0.	0.

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	72	0	0	158	0	0
normalized size	1	1.	1.	0.	0.	2.19	0.	0.
time (sec)	N/A	0.054	0.013	0.043	0.	1.777	0.	0.

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	71	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.07	0.015	0.158	0.	0.	0.	0.

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	71	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.06	0.014	0.077	0.	0.	0.	0.

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	71	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.057	0.014	0.103	0.	0.	0.	0.

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	67	0	0	153	0	0
normalized size	1	1.	1.	0.	0.	2.28	0.	0.
time (sec)	N/A	0.022	0.012	0.	0.	1.54	0.	0.

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	69	0	0	153	0	0
normalized size	1	1.	1.	0.	0.	2.22	0.	0.
time (sec)	N/A	0.024	0.015	0.045	0.	1.566	0.	0.

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	73	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.057	0.011	0.099	0.	0.	0.	0.

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	73	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.06	0.011	0.075	0.	0.	0.	0.

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	18	32	15	18
normalized size	1	1.	1.	0.93	1.2	2.13	1.	1.2
time (sec)	N/A	0.003	0.004	0.004	1.122	1.5	0.095	1.211

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	16	20	32	15	20
normalized size	1	1.	1.	1.07	1.33	2.13	1.	1.33
time (sec)	N/A	0.003	0.004	0.006	1.051	1.51	0.093	1.191

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	14	18	35	10	18
normalized size	1	1.	1.	0.74	0.95	1.84	0.53	0.95
time (sec)	N/A	0.005	0.005	0.013	1.139	1.492	0.09	1.239

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	36	99	32	254	0	127
normalized size	1	1.	0.27	0.76	0.24	1.94	0.	0.97
time (sec)	N/A	0.147	0.006	0.022	1.229	1.533	0.	1.22

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	36	87	32	219	0	111
normalized size	1	1.	0.33	0.81	0.3	2.03	0.	1.03
time (sec)	N/A	0.094	0.006	0.01	1.224	1.518	0.	1.257

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	36	75	32	188	0	95
normalized size	1	1.	0.42	0.88	0.38	2.21	0.	1.12
time (sec)	N/A	0.066	0.006	0.012	1.217	1.499	0.	1.242

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	30	66	32	153	0	78
normalized size	1	1.	0.48	1.06	0.52	2.47	0.	1.26
time (sec)	N/A	0.044	0.008	0.008	1.25	1.519	0.	1.226

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	30	27	39	96	0	38
normalized size	1	1.	0.79	0.71	1.03	2.53	0.	1.
time (sec)	N/A	0.025	0.006	0.008	1.153	1.529	0.	1.346

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	38	64	32	122	34	0
normalized size	1	1.	0.7	1.19	0.59	2.26	0.63	0.
time (sec)	N/A	0.045	0.015	0.01	1.214	1.549	6.776	0.

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	49	72	32	170	39	0
normalized size	1	1.	0.64	0.94	0.42	2.21	0.51	0.
time (sec)	N/A	0.065	0.037	0.011	1.252	1.54	175.103	0.

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	61	84	32	205	0	0
normalized size	1	1.	0.61	0.84	0.32	2.05	0.	0.
time (sec)	N/A	0.089	0.05	0.012	1.255	1.524	0.	0.



Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	73	96	32	236	0	0
normalized size	1	1.	0.59	0.78	0.26	1.92	0.	0.
time (sec)	N/A	0.108	0.062	0.013	1.297	1.48	0.	0.

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	208	208	72	0	0	516	0	1301
normalized size	1	1.	0.35	0.	0.	2.48	0.	6.25
time (sec)	N/A	0.266	0.03	0.02	0.	1.556	0.	1.378

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	173	173	72	0	0	383	0	779
normalized size	1	1.	0.42	0.	0.	2.21	0.	4.5
time (sec)	N/A	0.166	0.027	0.019	0.	1.542	0.	1.334

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	63	0	0	288	0	408
normalized size	1	1.	0.46	0.	0.	2.09	0.	2.96
time (sec)	N/A	0.119	0.07	0.02	0.	1.916	0.	1.191

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	63	0	0	220	0	170
normalized size	1	1.	0.6	0.	0.	2.1	0.	1.62
time (sec)	N/A	0.077	0.038	0.019	0.	2.089	0.	1.302

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	63	0	0	142	0	78
normalized size	1	1.	0.88	0.	0.	1.97	0.	1.08
time (sec)	N/A	0.045	0.028	0.022	0.	2.046	0.	1.254

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	75	0	0	205	0	0
normalized size	1	1.	0.77	0.	0.	2.11	0.	0.
time (sec)	N/A	0.085	0.074	0.02	0.	1.836	0.	0.

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	92	0	0	327	0	0
normalized size	1	1.	0.71	0.	0.	2.52	0.	0.
time (sec)	N/A	0.119	0.234	0.018	0.	1.573	0.	0.

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	118	0	0	497	0	0
normalized size	1	1.	0.72	0.	0.	3.01	0.	0.
time (sec)	N/A	0.165	0.138	0.019	0.	1.569	0.	0.

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	200	200	144	0	0	679	0	0
normalized size	1	1.	0.72	0.	0.	3.4	0.	0.
time (sec)	N/A	0.203	0.165	0.019	0.	1.59	0.	0.

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	24	145	107	242	0	108
normalized size	1	1.	0.16	0.96	0.71	1.6	0.	0.72
time (sec)	N/A	0.155	0.004	0.135	1.073	1.519	0.	1.161

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	63	0	0	281	0	0
normalized size	1	1.	0.89	0.	0.	3.96	0.	0.
time (sec)	N/A	0.031	0.08	0.018	0.	1.571	0.	0.

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	78	0	0	316	0	0
normalized size	1	1.	0.8	0.	0.	3.22	0.	0.
time (sec)	N/A	0.104	0.134	0.044	0.	1.559	0.	0.

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	34	38	81	90	60	1462
normalized size	1	1.	0.71	0.79	1.69	1.88	1.25	30.46
time (sec)	N/A	0.017	0.023	0.	1.041	1.549	0.129	1.296

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	56	80	158	167	116	3362
normalized size	1	1.	0.41	0.59	1.17	1.24	0.86	24.9
time (sec)	N/A	0.103	0.075	0.003	1.188	1.513	0.156	1.384

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	229	229	84	138	262	269	190	5787
normalized size	1	1.	0.37	0.6	1.14	1.17	0.83	25.27
time (sec)	N/A	0.191	0.111	0.004	1.132	1.512	0.184	1.416

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	348	348	117	212	393	396	284	10024
normalized size	1	1.	0.34	0.61	1.13	1.14	0.82	28.8
time (sec)	N/A	0.312	0.162	0.005	1.065	1.565	0.217	1.512

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	61	334	166	297	0	0
normalized size	1	1.	0.53	2.88	1.43	2.56	0.	0.
time (sec)	N/A	0.175	0.057	0.08	1.236	1.562	0.	0.

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	397	397	121	182	265	301	236	273
normalized size	1	1.	0.3	0.46	0.67	0.76	0.59	0.69
time (sec)	N/A	0.516	0.323	0.004	1.063	1.472	0.192	1.251

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	318	318	130	143	221	242	196	220
normalized size	1	1.	0.41	0.45	0.69	0.76	0.62	0.69
time (sec)	N/A	0.412	0.198	0.005	1.056	1.521	0.176	1.361

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	184	96	102	178	182	148	166
normalized size	1	1.	0.52	0.55	0.97	0.99	0.8	0.9
time (sec)	N/A	0.242	0.122	0.004	1.092	1.502	0.163	1.353

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	41	68	139	128	104	117
normalized size	1	1.	0.51	0.85	1.74	1.6	1.3	1.46
time (sec)	N/A	0.068	0.047	0.003	1.062	1.472	0.146	1.315

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	52	113	93	108	70	128
normalized size	1	1.	0.51	1.11	0.91	1.06	0.69	1.25
time (sec)	N/A	0.156	0.05	0.008	1.152	1.465	15.072	1.32

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	54	92	82	117	99	124
normalized size	1	1.	0.57	0.98	0.87	1.24	1.05	1.32
time (sec)	N/A	0.161	0.065	0.008	1.162	1.468	6.476	1.383

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	68	112	86	146	56	169
normalized size	1	1.	0.52	0.86	0.66	1.12	0.43	1.3
time (sec)	N/A	0.213	0.072	0.009	1.272	1.48	6.583	1.334

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	198	198	81	167	85	182	53	247
normalized size	1	1.	0.41	0.84	0.43	0.92	0.27	1.25
time (sec)	N/A	0.29	0.106	0.008	1.233	1.441	6.899	1.367

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	86	433	166	386	0	0
normalized size	1	1.	0.62	3.12	1.19	2.78	0.	0.
time (sec)	N/A	0.309	0.124	0.097	1.274	1.609	0.	0.

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	414	414	159	250	443	493	323	13437
normalized size	1	1.	0.38	0.6	1.07	1.19	0.78	32.46
time (sec)	N/A	0.672	0.12	0.009	1.061	1.553	0.21	1.86

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	328	328	121	197	354	386	260	9532
normalized size	1	1.	0.37	0.6	1.08	1.18	0.79	29.06
time (sec)	N/A	0.532	0.238	0.007	1.044	1.482	0.194	2.007

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	242	242	91	144	265	288	199	6340
normalized size	1	1.	0.38	0.6	1.1	1.19	0.82	26.2
time (sec)	N/A	0.356	0.15	0.007	1.049	1.573	0.176	1.486

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	58	93	181	192	134	3708
normalized size	1	1.	0.68	1.09	2.13	2.26	1.58	43.62
time (sec)	N/A	0.12	0.069	0.007	1.022	1.487	0.155	1.38

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	54	126	117	180	0	0
normalized size	1	1.	0.56	1.31	1.22	1.88	0.	0.
time (sec)	N/A	0.258	0.126	0.045	1.157	1.528	0.	0.

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	58	135	92	189	0	0
normalized size	1	1.	0.68	1.59	1.08	2.22	0.	0.
time (sec)	N/A	0.277	0.149	0.053	1.193	1.492	0.	0.

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	76	204	100	215	0	0
normalized size	1	1.	0.56	1.5	0.74	1.58	0.	0.
time (sec)	N/A	0.363	0.148	0.059	1.324	1.505	0.	0.

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	217	217	116	290	115	317	0	0
normalized size	1	1.	0.53	1.34	0.53	1.46	0.	0.
time (sec)	N/A	0.459	0.224	0.065	1.21	1.53	0.	0.

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	321	321	156	382	126	421	0	0
normalized size	1	1.	0.49	1.19	0.39	1.31	0.	0.
time (sec)	N/A	0.578	0.297	0.066	1.243	1.501	0.	0.

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	754	754	458	1062	1207	1310	1445	1480
normalized size	1	1.	0.61	1.41	1.6	1.74	1.92	1.96
time (sec)	N/A	0.916	0.696	0.005	1.176	1.506	0.492	1.292

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	495	495	320	640	809	844	899	910
normalized size	1	1.	0.65	1.29	1.63	1.71	1.82	1.84
time (sec)	N/A	0.636	0.445	0.005	1.181	1.435	0.356	1.259

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	271	271	191	297	464	468	447	447
normalized size	1	1.	0.7	1.1	1.71	1.73	1.65	1.65
time (sec)	N/A	0.339	0.253	0.005	1.086	1.503	0.245	1.241

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	50	108	201	192	158	178
normalized size	1	1.	0.49	1.06	1.97	1.88	1.55	1.75
time (sec)	N/A	0.096	0.061	0.004	1.123	1.429	0.164	1.207



Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	277	277	175	489	0	479	0	240
normalized size	1	1.	0.63	1.77	0.	1.73	0.	0.87
time (sec)	N/A	0.338	0.298	0.012	0.	1.521	0.	1.189

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	258	258	163	406	0	729	0	1037
normalized size	1	1.	0.63	1.57	0.	2.83	0.	4.02
time (sec)	N/A	0.379	0.437	0.015	0.	1.534	0.	1.742

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	294	294	267	418	0	1158	0	2419
normalized size	1	1.	0.91	1.42	0.	3.94	0.	8.23
time (sec)	N/A	0.408	0.652	0.015	0.	1.569	0.	1.312

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	396	396	389	511	0	1733	0	4180
normalized size	1	1.	0.98	1.29	0.	4.38	0.	10.56
time (sec)	N/A	0.521	0.807	0.013	0.	1.595	0.	1.294

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	557	557	669	596	0	2442	0	6118
normalized size	1	1.	1.2	1.07	0.	4.38	0.	10.98
time (sec)	N/A	0.684	0.726	0.013	0.	1.643	0.	1.516

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	192	57	90	0	0
normalized size	1	1.	1.	8.	2.38	3.75	0.	0.
time (sec)	N/A	0.146	0.366	0.2	1.443	1.558	0.	0.

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	F(-2)	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	23	198	57	0	0	0
normalized size	1	1.	1.05	9.	2.59	0.	0.	0.
time (sec)	N/A	0.131	0.299	0.098	1.374	0.	0.	0.

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	F(-2)	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	23	198	57	0	0	0
normalized size	1	1.	1.05	9.	2.59	0.	0.	0.
time (sec)	N/A	0.09	0.269	0.097	1.404	0.	0.	0.

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	F(-2)	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	21	186	51	0	0	0
normalized size	1	1.	1.05	9.3	2.55	0.	0.	0.
time (sec)	N/A	0.046	0.171	0.108	1.358	0.	0.	0.

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	F(-2)	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	180	49	0	0	0
normalized size	1	1.	1.	9.47	2.58	0.	0.	0.
time (sec)	N/A	0.129	0.032	0.088	1.386	0.	0.	0.

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	F(-2)	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	23	136	53	0	0	0
normalized size	1	1.	1.05	6.18	2.41	0.	0.	0.
time (sec)	N/A	0.131	0.328	0.105	1.428	0.	0.	0.

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	23	136	53	0	0	0
normalized size	1	1.	1.05	6.18	2.41	0.	0.	0.
time (sec)	N/A	0.131	0.323	0.102	1.381	0.	0.	0.

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	45	43	81	105	51	58
normalized size	1	1.	0.49	0.47	0.89	1.15	0.56	0.64
time (sec)	N/A	0.143	0.021	0.003	1.027	1.442	0.114	1.315

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	37	35	65	85	42	47
normalized size	1	1.	0.51	0.49	0.9	1.18	0.58	0.65
time (sec)	N/A	0.101	0.013	0.003	1.093	1.459	0.108	1.254

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	29	27	49	66	34	36
normalized size	1	1.	0.55	0.51	0.92	1.25	0.64	0.68
time (sec)	N/A	0.064	0.013	0.001	1.023	1.476	0.104	1.226

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	21	19	32	50	26	26
normalized size	1	1.	0.62	0.56	0.94	1.47	0.76	0.76
time (sec)	N/A	0.029	0.01	0.001	1.045	1.501	0.094	1.243

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	14	19	34	14	19
normalized size	1	1.	1.	0.88	1.19	2.12	0.88	1.19
time (sec)	N/A	0.007	0.005	0.001	1.043	1.467	0.083	1.245

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	57	14	31	0	14
normalized size	1	1.	1.	2.11	0.52	1.15	0.	0.52
time (sec)	N/A	0.058	0.016	0.039	1.173	1.493	0.	1.365

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	47	116	18	80	0	39
normalized size	1	1.	0.98	2.42	0.38	1.67	0.	0.81
time (sec)	N/A	0.086	0.027	0.021	1.172	1.431	0.	1.256

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	56	155	20	101	0	62
normalized size	1	1.	0.79	2.18	0.28	1.42	0.	0.87
time (sec)	N/A	0.119	0.042	0.028	1.146	1.508	0.	1.286

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	64	189	20	119	0	85
normalized size	1	1.	0.7	2.05	0.22	1.29	0.	0.92
time (sec)	N/A	0.155	0.053	0.03	1.139	1.467	0.	1.309

## 2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio  $\frac{\text{number of rules}}{\text{integrand size}}$  is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [48] had the largest ratio of [ 0.25 ]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	1	1	1.	17	0.059
2	A	5	2	1.	17	0.118
3	A	4	2	1.	17	0.118
4	A	3	2	1.	17	0.118
5	A	2	2	1.	15	0.133
6	A	1	1	1.	9	0.111
7	A	1	1	1.	17	0.059
8	A	2	2	1.	17	0.118
9	A	3	2	1.	17	0.118
10	A	4	2	1.	17	0.118
11	A	5	2	1.	17	0.118
12	A	6	3	1.	48	0.062
13	A	5	3	1.	37	0.081
14	A	4	3	1.	26	0.115
15	A	3	3	1.	28	0.107

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
16	A	4	3	1.	39	0.077
17	A	5	3	1.	50	0.06
18	A	6	3	1.	61	0.049
19	A	2	2	1.	19	0.105
20	A	2	2	1.	50	0.04
21	A	2	2	1.	39	0.051
22	A	2	2	1.	28	0.071
23	A	1	1	1.	17	0.059
24	A	1	1	1.	19	0.053
25	A	2	2	1.	30	0.067
26	A	2	2	1.	41	0.049
27	A	1	1	1.	7	0.143
28	A	1	1	1.	7	0.143
29	A	1	1	1.	7	0.143
30	A	6	3	1.	13	0.231
31	A	5	3	1.	13	0.231
32	A	4	3	1.	13	0.231
33	A	3	3	1.	13	0.231
34	A	2	2	1.	13	0.154
35	A	3	3	1.	13	0.231
36	A	4	3	1.	13	0.231
37	A	5	3	1.	13	0.231
38	A	6	3	1.	13	0.231
39	A	6	3	1.	19	0.158
40	A	5	3	1.	19	0.158
41	A	4	3	1.	19	0.158
42	A	3	3	1.	19	0.158
43	A	2	2	1.	19	0.105
44	A	3	3	1.	19	0.158
45	A	4	3	1.	19	0.158
46	A	5	3	1.	19	0.158
47	A	6	3	1.	19	0.158

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
48	A	9	3	1.	12	0.25
49	A	1	1	1.	19	0.053
50	A	2	2	1.	21	0.095
51	A	2	2	1.	15	0.133
52	A	8	3	1.	20	0.15
53	A	12	3	1.	25	0.12
54	A	17	3	1.	30	0.1
55	A	6	2	1.	21	0.095
56	A	24	3	1.	21	0.143
57	A	20	3	1.	21	0.143
58	A	11	3	1.	19	0.158
59	A	4	2	1.	18	0.111
60	A	9	4	1.	21	0.19
61	A	8	5	1.	21	0.238
62	A	9	4	1.	21	0.19
63	A	12	3	1.	21	0.143
64	A	5	2	1.	22	0.091
65	A	17	3	1.	22	0.136
66	A	14	3	1.	22	0.136
67	A	11	3	1.	20	0.15
68	A	4	3	1.	19	0.158
69	A	6	4	1.	22	0.182
70	A	6	4	1.	22	0.182
71	A	8	3	1.	22	0.136
72	A	11	3	1.	22	0.136
73	A	14	3	1.	22	0.136
74	A	28	3	1.	25	0.12
75	A	20	3	1.	25	0.12
76	A	13	3	1.	23	0.13
77	A	5	2	1.	18	0.111
78	A	13	4	1.	25	0.16
79	A	11	5	1.	25	0.2

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
80	A	11	5	1.	25	0.2
81	A	13	4	1.	25	0.16
82	A	17	3	1.	25	0.12
83	A	1	1	1.	39	0.026
84	A	1	1	1.	38	0.026
85	A	1	1	1.	36	0.028
86	A	1	1	1.	35	0.029
87	A	1	1	1.	35	0.029
88	A	1	1	1.	38	0.026
89	A	1	1	1.	38	0.026
90	A	5	2	1.	15	0.133
91	A	4	2	1.	15	0.133
92	A	3	2	1.	15	0.133
93	A	2	2	1.	13	0.154
94	A	1	1	1.	11	0.091
95	A	2	2	1.	15	0.133
96	A	3	3	1.	15	0.2
97	A	4	3	1.	15	0.2
98	A	5	3	1.	15	0.2



# Chapter 3

## Listing of integrals

### 3.1 $\int F^{c(a+bx)}(d+ex)^m dx$

Optimal. Leaf size=67

$$\frac{(d+ex)^m F^{c\left(a-\frac{bd}{e}\right)} \left(-\frac{bc \log(F)(d+ex)}{e}\right)^{-m} \text{Gamma}\left(m+1, -\frac{bc \log(F)(d+ex)}{e}\right)}{bc \log(F)}$$

[Out]  $(F^{(c*(a - (b*d)/e)})*(d + e*x)^m*\text{Gamma}[1 + m, -((b*c*(d + e*x)*\text{Log}[F])/e)]) / (b*c*\text{Log}[F]*(-((b*c*(d + e*x)*\text{Log}[F])/e))^m)$

---

**Rubi [A]** time = 0.0399163, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$ , Rules used = {2181}

$$\frac{(d+ex)^m F^{c\left(a-\frac{bd}{e}\right)} \left(-\frac{bc \log(F)(d+ex)}{e}\right)^{-m} \text{Gamma}\left(m+1, -\frac{bc \log(F)(d+ex)}{e}\right)}{bc \log(F)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[F^{c*(a + b*x)}*(d + e*x)^m, x]$

[Out]  $(F^{(c*(a - (b*d)/e)})*(d + e*x)^m*\text{Gamma}[1 + m, -((b*c*(d + e*x)*\text{Log}[F])/e)]) / (b*c*\text{Log}[F]*(-((b*c*(d + e*x)*\text{Log}[F])/e))^m)$

Rule 2181

```
Int[(F_)^((g_)*((e_) + (f_)*(x_)))*((c_) + (d_)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x])]/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !I
ntegerQ[m]
```

### Rubi steps

$$\int F^{c(a+bx)}(d+ex)^m dx = \frac{F^{c\left(a-\frac{bd}{e}\right)}(d+ex)^m \Gamma\left(1+m, -\frac{bc(d+ex)\log(F)}{e}\right) \left(-\frac{bc(d+ex)\log(F)}{e}\right)^{-m}}{bc \log(F)}$$

**Mathematica [A]** time = 0.0382056, size = 67, normalized size = 1.

$$\frac{(d+ex)^m F^{c\left(a-\frac{bd}{e}\right)} \left(-\frac{bc \log(F)(d+ex)}{e}\right)^{-m} \text{Gamma}\left(m+1, -\frac{bc \log(F)(d+ex)}{e}\right)}{bc \log(F)}$$

Antiderivative was successfully verified.

```
[In] Integrate[F^(c*(a + b*x))*(d + e*x)^m, x]
```

```
[Out] (F^(c*(a - (b*d)/e))*(d + e*x)^m*Gamma[1 + m, -((b*c*(d + e*x)*Log[F])/e)])
/(b*c*Log[F]*(-((b*c*(d + e*x)*Log[F])/e))^m)
```

**Maple [F]** time = 0.047, size = 0, normalized size = 0.

$$\int F^{c(bx+a)}(ex+d)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(b*x+a))*(e*x+d)^m, x)
```

```
[Out] int(F^(c*(b*x+a))*(e*x+d)^m, x)
```

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (ex + d)^m F^{(bx+a)c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(c\*(b\*x+a))\*(e\*x+d)^m,x, algorithm="maxima")

[Out] integrate((e\*x + d)^m \* F^((b\*x + a)\*c), x)

**Fricas [A]** time = 1.52543, size = 153, normalized size = 2.28

$$\frac{e^{\left(\frac{em \log\left(-\frac{bc \log(F)}{e}\right) + (bcd - ace) \log(F)}{e}\right)} \Gamma\left(m + 1, -\frac{(bcex + bcd) \log(F)}{e}\right)}{bc \log(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(c\*(b\*x+a))\*(e\*x+d)^m,x, algorithm="fricas")

[Out] e^(-(e\*m\*log(-b\*c\*log(F)/e) + (b\*c\*d - a\*c\*e)\*log(F))/e)\*gamma(m + 1, -(b\*c\*e\*x + b\*c\*d)\*log(F)/e)/(b\*c\*log(F))

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(c\*(b\*x+a))\*(e\*x+d)\*\*m,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (ex + d)^m F^{(bx+a)c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*(e*x+d)^m,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^m*F^((b*x + a)*c), x)
```

### 3.2 $\int F^{c(a+bx)}(d+ex)^4 dx$

**Optimal.** Leaf size=141

$$\frac{12e^2(d+ex)^2F^{c(a+bx)}}{b^3c^3\log^3(F)} - \frac{24e^3(d+ex)F^{c(a+bx)}}{b^4c^4\log^4(F)} - \frac{4e(d+ex)^3F^{c(a+bx)}}{b^2c^2\log^2(F)} + \frac{24e^4F^{c(a+bx)}}{b^5c^5\log^5(F)} + \frac{(d+ex)^4F^{c(a+bx)}}{bc\log(F)}$$

[Out]  $(24e^4F^{c(a+bx)})/(b^5c^5\log[F]^5) - (24e^3F^{c(a+bx)}(d+ex))/(b^4c^4\log[F]^4) + (12e^2F^{c(a+bx)}(d+ex)^2)/(b^3c^3\log[F]^3) - (4eF^{c(a+bx)}(d+ex)^3)/(b^2c^2\log[F]^2) + (F^{c(a+bx)}(d+ex)^4)/(bc\log[F])$

**Rubi [A]** time = 0.108315, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$ , Rules used = {2176, 2194}

$$\frac{12e^2(d+ex)^2F^{c(a+bx)}}{b^3c^3\log^3(F)} - \frac{24e^3(d+ex)F^{c(a+bx)}}{b^4c^4\log^4(F)} - \frac{4e(d+ex)^3F^{c(a+bx)}}{b^2c^2\log^2(F)} + \frac{24e^4F^{c(a+bx)}}{b^5c^5\log^5(F)} + \frac{(d+ex)^4F^{c(a+bx)}}{bc\log(F)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[F^{c(a+bx)}(d+ex)^4, x]$

[Out]  $(24e^4F^{c(a+bx)})/(b^5c^5\log[F]^5) - (24e^3F^{c(a+bx)}(d+ex))/(b^4c^4\log[F]^4) + (12e^2F^{c(a+bx)}(d+ex)^2)/(b^3c^3\log[F]^3) - (4eF^{c(a+bx)}(d+ex)^3)/(b^2c^2\log[F]^2) + (F^{c(a+bx)}(d+ex)^4)/(bc\log[F])$

#### Rule 2176

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Simp[((c + d*x)^m*(b*F^(g*(e + f*x)))^n)/(f*g*n*Log[F]), x] - Dist[(d*m)/(f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m] && !$UseGamma === True
```

#### Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] := Simp[(F^(c*(a + b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned}
\int F^{c(a+bx)}(d+ex)^4 dx &= \frac{F^{c(a+bx)}(d+ex)^4}{bc \log(F)} - \frac{(4e) \int F^{c(a+bx)}(d+ex)^3 dx}{bc \log(F)} \\
&= -\frac{4eF^{c(a+bx)}(d+ex)^3}{b^2c^2 \log^2(F)} + \frac{F^{c(a+bx)}(d+ex)^4}{bc \log(F)} + \frac{(12e^2) \int F^{c(a+bx)}(d+ex)^2 dx}{b^2c^2 \log^2(F)} \\
&= \frac{12e^2F^{c(a+bx)}(d+ex)^2}{b^3c^3 \log^3(F)} - \frac{4eF^{c(a+bx)}(d+ex)^3}{b^2c^2 \log^2(F)} + \frac{F^{c(a+bx)}(d+ex)^4}{bc \log(F)} - \frac{(24e^3) \int F^{c(a+bx)}(d+ex) dx}{b^3c^3 \log^3(F)} \\
&= -\frac{24e^3F^{c(a+bx)}(d+ex)}{b^4c^4 \log^4(F)} + \frac{12e^2F^{c(a+bx)}(d+ex)^2}{b^3c^3 \log^3(F)} - \frac{4eF^{c(a+bx)}(d+ex)^3}{b^2c^2 \log^2(F)} + \frac{F^{c(a+bx)}(d+ex)^4}{bc \log(F)} + \frac{(24e^4)}{b^4} \\
&= \frac{24e^4F^{c(a+bx)}}{b^5c^5 \log^5(F)} - \frac{24e^3F^{c(a+bx)}(d+ex)}{b^4c^4 \log^4(F)} + \frac{12e^2F^{c(a+bx)}(d+ex)^2}{b^3c^3 \log^3(F)} - \frac{4eF^{c(a+bx)}(d+ex)^3}{b^2c^2 \log^2(F)} + \frac{F^{c(a+bx)}(d+ex)^4}{bc \log(F)}
\end{aligned}$$

**Mathematica [A]** time = 0.12032, size = 100, normalized size = 0.71

$$\frac{F^{c(a+bx)} \left( 12b^2c^2e^2 \log^2(F)(d+ex)^2 - 4b^3c^3e \log^3(F)(d+ex)^3 + b^4c^4 \log^4(F)(d+ex)^4 - 24bce^3 \log(F)(d+ex) + 24e^4 \right)}{b^5c^5 \log^5(F)}$$

Antiderivative was successfully verified.

[In] Integrate[F^(c\*(a + b\*x))\*(d + e\*x)^4,x]

[Out] (F^(c\*(a + b\*x))\*(24\*e^4 - 24\*b\*c\*e^3\*(d + e\*x)\*Log[F] + 12\*b^2\*c^2\*e^2\*(d + e\*x)^2\*Log[F]^2 - 4\*b^3\*c^3\*e\*(d + e\*x)^3\*Log[F]^3 + b^4\*c^4\*(d + e\*x)^4\*Log[F]^4))/(b^5\*c^5\*Log[F]^5)

**Maple [A]** time = 0.009, size = 260, normalized size = 1.8

$$\frac{(e^4x^4b^4c^4(\ln(F))^4 + 4(\ln(F))^4b^4c^4de^3x^3 + 6(\ln(F))^4b^4c^4d^2e^2x^2 + 4(\ln(F))^4b^4c^4d^3ex + (\ln(F))^4b^4c^4d^4 - 4(\ln(F))^3b^4c^4de^3x^3 + 12b^2c^2e^2(\ln(F))^2(d+ex)^2 - 4b^3c^3e(\ln(F))^3(d+ex)^3 + b^4c^4(\ln(F))^4(d+ex)^4 - 24bce^3\ln(F)(d+ex) + 24e^4)}{b^5c^5\ln^5(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(c\*(b\*x+a))\*(e\*x+d)^4,x)

[Out] (e^4\*x^4\*b^4\*c^4\*ln(F)^4+4\*ln(F)^4\*b^4\*c^4\*d\*e^3\*x^3+6\*ln(F)^4\*b^4\*c^4\*d^2\*e^2\*x^2+4\*ln(F)^4\*b^4\*c^4\*d^3\*e\*x+ln(F)^4\*b^4\*c^4\*d^4-4\*ln(F)^3\*b^3\*c^3\*e^4

$$\frac{x^3 - 12 \ln(F)^3 b^3 c^3 d^2 e^3 x^2 - 12 \ln(F)^3 b^3 c^3 d^2 e^2 x - 4 \ln(F)^3 b^3 c^3 d^3 e + 12 \ln(F)^2 b^2 c^2 d^2 e^4 x^2 + 24 \ln(F)^2 b^2 c^2 d^2 e^3 x + 12 b^2 c^2 d^2 \ln(F)^2 e^2 d^2 - 24 \ln(F) b^2 c^2 d^2 e^3 x - 24 d^2 e^3 b^2 c^2 \ln(F) + 24 e^4}{b^5 c^5 \ln(F)^5} F^{(c(bx+a))} (e^{x+d})^4$$

**Maxima [B]** time = 1.5879, size = 417, normalized size = 2.96

$$\frac{F^{bcx+ac} d^4}{bc \log(F)} + \frac{4(F^{ac} bcx \log(F) - F^{ac}) F^{bcx} d^3 e}{b^2 c^2 \log(F)^2} + \frac{6(F^{ac} b^2 c^2 x^2 \log(F)^2 - 2 F^{ac} bcx \log(F) + 2 F^{ac}) F^{bcx} d^2 e^2}{b^3 c^3 \log(F)^3} + \frac{4(F^{ac} b^3 c^3 x^3 \log(F)^3 - 3 F^{ac} b^2 c^2 x^2 \log(F)^2 + 6 F^{ac} bcx \log(F) - 2 F^{ac}) F^{bcx} d e^3}{b^4 c^4 \log(F)^4} + \frac{4(F^{ac} b^4 c^4 x^4 \log(F)^4 - 4 F^{ac} b^3 c^3 x^3 \log(F)^3 + 12 F^{ac} b^2 c^2 x^2 \log(F)^2 - 24 F^{ac} bcx \log(F) + 24 F^{ac}) F^{bcx} d^4}{b^5 c^5 \log(F)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(c\*(b\*x+a))\*(e\*x+d)^4,x, algorithm="maxima")

[Out]  $F^{(b*c*x + a*c)} d^4 / (b*c*\log(F)) + 4*(F^{(a*c)}*b*c*x*\log(F) - F^{(a*c)})*F^{(b*c*x)} d^3 e / (b^2*c^2*\log(F)^2) + 6*(F^{(a*c)}*b^2*c^2*x^2*\log(F)^2 - 2*F^{(a*c)}*b*c*x*\log(F) + 2*F^{(a*c)})*F^{(b*c*x)} d^2 e^2 / (b^3*c^3*\log(F)^3) + 4*(F^{(a*c)}*b^3*c^3*x^3*\log(F)^3 - 3*F^{(a*c)}*b^2*c^2*x^2*\log(F)^2 + 6*F^{(a*c)}*b*c*x*\log(F) - 6*F^{(a*c)})*F^{(b*c*x)} d e^3 / (b^4*c^4*\log(F)^4) + (F^{(a*c)}*b^4*c^4*x^4*\log(F)^4 - 4*F^{(a*c)}*b^3*c^3*x^3*\log(F)^3 + 12*F^{(a*c)}*b^2*c^2*x^2*\log(F)^2 - 24*F^{(a*c)}*b*c*x*\log(F) + 24*F^{(a*c)})*F^{(b*c*x)} e^4 / (b^5*c^5*\log(F)^5)$

**Fricas [A]** time = 1.5216, size = 474, normalized size = 3.36

$$\frac{((b^4 c^4 e^4 x^4 + 4 b^4 c^4 d e^3 x^3 + 6 b^4 c^4 d^2 e^2 x^2 + 4 b^4 c^4 d^3 e x + b^4 c^4 d^4) \log(F)^4 + 24 e^4 - 4 (b^3 c^3 e^4 x^3 + 3 b^3 c^3 d e^3 x^2 + 3 b^3 c^3 d^2 e^2 x + 3 b^3 c^3 d^3 e) \log(F)^3 + 12 (b^2 c^2 e^4 x^2 + 2 b^2 c^2 d e^3 x + b^2 c^2 d^2 e^2) \log(F)^2 - 24 (b^2 c^2 e^4 x + b^2 c^2 d e^3) \log(F) + 24 F^{(c(bx+a))} (e^{x+d})^4)}{b^5 c^5 \log(F)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(c\*(b\*x+a))\*(e\*x+d)^4,x, algorithm="fricas")

[Out]  $((b^4*c^4*e^4*x^4 + 4*b^4*c^4*d*e^3*x^3 + 6*b^4*c^4*d^2*e^2*x^2 + 4*b^4*c^4*d^3*e*x + b^4*c^4*d^4)*\log(F)^4 + 24*e^4 - 4*(b^3*c^3*e^4*x^3 + 3*b^3*c^3*d*e^3*x^2 + 3*b^3*c^3*d^2*e^2*x + b^3*c^3*d^3*e)*\log(F)^3 + 12*(b^2*c^2*e^4*x^2 + 2*b^2*c^2*d*e^3*x + b^2*c^2*d^2*e^2)*\log(F)^2 - 24*(b^2*c^2*e^4*x + b^2*c^2*d*e^3)*\log(F) + 24*F^{(c*(b*x+a))} (e^{x+d})^4) / (b^5*c^5*\log(F)^5)$





$$\begin{aligned}
& *b^4*c^4*x^4*\log(\text{abs}(F))^2 - 2*b^4*c^4*x^4*\log(\text{abs}(F))^4 + 12*\pi^2*b^3*c^3* \\
& x^3*\log(\text{abs}(F))*\text{sgn}(F) - 12*\pi^2*b^3*c^3*x^3*\log(\text{abs}(F)) + 8*b^3*c^3*x^3*\log(\text{abs}(F))^3 - 12*\pi^2*b^2*c^2*x^2*\text{sgn}(F) + 12*\pi^2*b^2*c^2*x^2 - 24*b^2*c^2 \\
& *x^2*\log(\text{abs}(F))^2 + 48*b*c*x*\log(\text{abs}(F)) - 48*(5*\pi^4*b^5*c^5*\log(\text{abs}(F)) \\
& *\text{sgn}(F) - 10*\pi^2*b^5*c^5*\log(\text{abs}(F))^3*\text{sgn}(F) - 5*\pi^4*b^5*c^5*\log(\text{abs}(F)) \\
& + 10*\pi^2*b^5*c^5*\log(\text{abs}(F))^3 - 2*b^5*c^5*\log(\text{abs}(F))^5)/((\pi^5*b^5*c^5* \\
& \text{sgn}(F) - 10*\pi^3*b^5*c^5*\log(\text{abs}(F))^2*\text{sgn}(F) + 5*\pi*b^5*c^5*\log(\text{abs}(F))^4* \\
& \text{sgn}(F) - \pi^5*b^5*c^5 + 10*\pi^3*b^5*c^5*\log(\text{abs}(F))^2 - 5*\pi*b^5*c^5*\log(\text{abs}(F))^4)^2 + (5*\pi^4*b^5*c^5*\log(\text{abs}(F))*\text{sgn}(F) - 10*\pi^2*b^5*c^5*\log(\text{abs}(F))^3*\text{sgn}(F) - 5*\pi^4*b^5*c^5*\log(\text{abs}(F)) + 10*\pi^2*b^5*c^5*\log(\text{abs}(F))^3 - 2*b^5*c^5*\log(\text{abs}(F))^5)^2)*\cos(-1/2*\pi*b*c*x*\text{sgn}(F) + 1/2*\pi*b*c*x - 1/2*\pi*a*c*\text{sgn}(F) + 1/2*\pi*a*c) - ((\pi^4*b^4*c^4*x^4*\text{sgn}(F) - 6*\pi^2*b^4*c^4*x^4* \\
& 4*\log(\text{abs}(F))^2*\text{sgn}(F) - \pi^4*b^4*c^4*x^4 + 6*\pi^2*b^4*c^4*x^4*\log(\text{abs}(F))^2 - 2*b^4*c^4*x^4*\log(\text{abs}(F))^4 + 12*\pi^2*b^3*c^3*x^3*\log(\text{abs}(F))*\text{sgn}(F) - 12*\pi^2*b^3*c^3*x^3*\log(\text{abs}(F)) + 8*b^3*c^3*x^3*\log(\text{abs}(F))^3 - 12*\pi^2*b^2*c^2*x^2*\text{sgn}(F) + 12*\pi^2*b^2*c^2*x^2 - 24*b^2*c^2*x^2*\log(\text{abs}(F))^2 + 48*b*c*x*\log(\text{abs}(F)) - 48*(\pi^5*b^5*c^5*\text{sgn}(F) - 10*\pi^3*b^5*c^5*\log(\text{abs}(F))^2*\text{sgn}(F) + 5*\pi*b^5*c^5*\log(\text{abs}(F))^4*\text{sgn}(F) - \pi^5*b^5*c^5 + 10*\pi^3*b^5*c^5*\log(\text{abs}(F))^2 - 5*\pi*b^5*c^5*\log(\text{abs}(F))^4)/((\pi^5*b^5*c^5*\text{sgn}(F) - 10*\pi^3*b^5*c^5*\log(\text{abs}(F))^2*\text{sgn}(F) + 5*\pi*b^5*c^5*\log(\text{abs}(F))^4*\text{sgn}(F) - \pi^5*b^5*c^5 + 10*\pi^3*b^5*c^5*\log(\text{abs}(F))^2 - 5*\pi*b^5*c^5*\log(\text{abs}(F))^4)^2 + (5*\pi^4*b^5*c^5*\log(\text{abs}(F))*\text{sgn}(F) - 10*\pi^2*b^5*c^5*\log(\text{abs}(F))^3*\text{sgn}(F) - 5*\pi^4*b^5*c^5*\log(\text{abs}(F)) + 10*\pi^2*b^5*c^5*\log(\text{abs}(F))^3 - 2*b^5*c^5*\log(\text{abs}(F))^5)^2) + 4*(\pi^3*b^4*c^4*x^4*\log(\text{abs}(F))*\text{sgn}(F) - \pi*b^4*c^4*x^4*\log(\text{abs}(F))^3*\text{sgn}(F) - \pi^3*b^4*c^4*x^4*\log(\text{abs}(F)) + \pi*b^4*c^4*x^4*\log(\text{abs}(F))^3 - \pi^3*b^3*c^3*x^3*\text{sgn}(F) + 3*\pi*b^3*c^3*x^3*\log(\text{abs}(F))^2*\text{sgn}(F) + \pi^3*b^3*c^3*x^3 - 3*\pi*b^3*c^3*x^3*\log(\text{abs}(F))^2 - 6*\pi*b^2*c^2*x^2*\log(\text{abs}(F))*\text{sgn}(F) + 6*\pi*b^2*c^2*x^2*\log(\text{abs}(F)) + 6*\pi*b*c*x*\text{sgn}(F) - 6*\pi*b*c*x) * (5*\pi^4*b^5*c^5*\log(\text{abs}(F))*\text{sgn}(F) - 10*\pi^2*b^5*c^5*\log(\text{abs}(F))^3*\text{sgn}(F) - 5*\pi^4*b^5*c^5*\log(\text{abs}(F)) + 10*\pi^2*b^5*c^5*\log(\text{abs}(F))^3 - 2*b^5*c^5*\log(\text{abs}(F))^5)/((\pi^5*b^5*c^5*\text{sgn}(F) - 10*\pi^3*b^5*c^5*\log(\text{abs}(F))^2*\text{sgn}(F) + 5*\pi*b^5*c^5*\log(\text{abs}(F))^4*\text{sgn}(F) - \pi^5*b^5*c^5 + 10*\pi^3*b^5*c^5*\log(\text{abs}(F))^2 - 5*\pi*b^5*c^5*\log(\text{abs}(F))^4)^2 + (5*\pi^4*b^5*c^5*\log(\text{abs}(F))*\text{sgn}(F) - 10*\pi^2*b^5*c^5*\log(\text{abs}(F))^3*\text{sgn}(F) - 5*\pi^4*b^5*c^5*\log(\text{abs}(F)) + 10*\pi^2*b^5*c^5*\log(\text{abs}(F))^3 - 2*b^5*c^5*\log(\text{abs}(F))^5)^2)*\sin(-1/2*\pi*b*c*x*\text{sgn}(F) + 1/2*\pi*b*c*x - 1/2*\pi*a*c*\text{sgn}(F) + 1/2*\pi*a*c)) * e^(b*c*x*\log(\text{abs}(F)) + a*c*\log(\text{abs}(F)) + 4) + 1/2*I*((-16*I*\pi^4*b^4*c^4*x^4*\text{sgn}(F) + 64*\pi^3*b^4*c^4*x^4*\log(\text{abs}(F))*\text{sgn}(F) + 96*I*\pi^2*b^4*c^4*x^4*\log(\text{abs}(F))^2*\text{sgn}(F) - 64*\pi*b^4*c^4*x^4*\log(\text{abs}(F))^3*\text{sgn}(F) + 16*I*\pi^4*b^4*c^4*x^4 - 64*\pi^3*b^4*c^4*x^4*\log(\text{abs}(F)) - 96*I*\pi^2*b^4*c^4*x^4*\log(\text{abs}(F))^2 + 64*\pi*b^4*c^4*x^4*\log(\text{abs}(F))^3 + 32*I*b^4*c^4*x^4*\log(\text{abs}(F))^4 - 64*\pi^3*b^3*c^3*x^3*\text{sgn}(F) - 192*I*\pi^2*b^3*c^3*x^3*\log(\text{abs}(F))*\text{sgn}(F) + 192*\pi*b^3*c^3*x^3*\log(\text{abs}(F))^2*\text{sgn}(F) + 64*\pi^3*b^3*c^3*x^3 + 192*I*\pi^2*b^3*c^3*x^3*\log(\text{abs}(F)) - 192*\pi*b^3*c^3*x^3*\log(\text{abs}(F))^2 - 128*I*b^3*c^3*x^3*\log(\text{abs}(F))^3 + 192*I*\pi^2*b^2*c^2*x^2*\text{sgn}(F) - 384*\pi*b^2*c^2*x^2*\log(\text{abs}(F))*\text{sgn}(F) - 19
\end{aligned}$$

$$\begin{aligned}
& 2*I\pi^2*b^2*c^2*x^2 + 384*\pi*b^2*c^2*x^2*\log(\text{abs}(F)) + 384*I*b^2*c^2*x^2*\log(\text{abs}(F))^2 + 384*\pi*b*c*x*\text{sgn}(F) - 384*\pi*b*c*x - 768*I*b*c*x*\log(\text{abs}(F)) \\
& + 768*I)*e^{(1/2*I\pi*b*c*x*\text{sgn}(F) - 1/2*I\pi*b*c*x + 1/2*I\pi*a*c*\text{sgn}(F) - 1/2*I\pi*a*c)} / (16*I\pi^5*b^5*c^5*\text{sgn}(F) - 80*\pi^4*b^5*c^5*\log(\text{abs}(F))*\text{sgn}(F) - 160*I\pi^3*b^5*c^5*\log(\text{abs}(F))^2*\text{sgn}(F) + 160*\pi^2*b^5*c^5*\log(\text{abs}(F))^3*\text{sgn}(F) + 80*I\pi*b^5*c^5*\log(\text{abs}(F))^4*\text{sgn}(F) - 16*I\pi^5*b^5*c^5 + 80*\pi^4*b^5*c^5*\log(\text{abs}(F)) + 160*I\pi^3*b^5*c^5*\log(\text{abs}(F))^2 - 160*\pi^2*b^5*c^5*\log(\text{abs}(F))^3 - 80*I\pi*b^5*c^5*\log(\text{abs}(F))^4 + 32*b^5*c^5*\log(\text{abs}(F))^5) - (-16*I\pi^4*b^4*c^4*x^4*\text{sgn}(F) - 64*\pi^3*b^4*c^4*x^4*\log(\text{abs}(F))*\text{sgn}(F) + 96*I\pi^2*b^4*c^4*x^4*\log(\text{abs}(F))^2*\text{sgn}(F) + 64*\pi*b^4*c^4*x^4*\log(\text{abs}(F))^3*\text{sgn}(F) + 16*I\pi^4*b^4*c^4*x^4 + 64*\pi^3*b^4*c^4*x^4*\log(\text{abs}(F)) - 96*I\pi^2*b^4*c^4*x^4*\log(\text{abs}(F))^2 - 64*\pi*b^4*c^4*x^4*\log(\text{abs}(F))^3 + 32*I*b^4*c^4*x^4*\log(\text{abs}(F))^4 + 64*\pi^3*b^3*c^3*x^3*\text{sgn}(F) - 192*I\pi^2*b^3*c^3*x^3*\log(\text{abs}(F))*\text{sgn}(F) - 192*\pi*b^3*c^3*x^3*\log(\text{abs}(F))^2*\text{sgn}(F) - 64*\pi^3*b^3*c^3*x^3 + 192*I\pi^2*b^3*c^3*x^3*\log(\text{abs}(F)) + 192*\pi*b^3*c^3*x^3*\log(\text{abs}(F))^2 - 128*I*b^3*c^3*x^3*\log(\text{abs}(F))^3 + 192*I\pi^2*b^2*c^2*x^2*\text{sgn}(F) + 384*\pi*b^2*c^2*x^2*\log(\text{abs}(F))*\text{sgn}(F) - 192*I\pi^2*b^2*c^2*x^2 - 384*\pi*b^2*c^2*x^2*\log(\text{abs}(F)) + 384*I*b^2*c^2*x^2*\log(\text{abs}(F))^2 - 384*\pi*b*c*x*\text{sgn}(F) + 384*\pi*b*c*x - 768*I*b*c*x*\log(\text{abs}(F)) + 768*I)*e^{(-1/2*I\pi*b*c*x*\text{sgn}(F) + 1/2*I\pi*b*c*x - 1/2*I\pi*a*c*\text{sgn}(F) + 1/2*I\pi*a*c)} / (-16*I\pi^5*b^5*c^5*\text{sgn}(F) - 80*\pi^4*b^5*c^5*\log(\text{abs}(F))*\text{sgn}(F) + 160*I\pi^3*b^5*c^5*\log(\text{abs}(F))^2*\text{sgn}(F) + 160*\pi^2*b^5*c^5*\log(\text{abs}(F))^3*\text{sgn}(F) - 80*I\pi*b^5*c^5*\log(\text{abs}(F))^4*\text{sgn}(F) + 16*I\pi^5*b^5*c^5 + 80*\pi^4*b^5*c^5*\log(\text{abs}(F)) - 160*I\pi^3*b^5*c^5*\log(\text{abs}(F))^2 - 160*\pi^2*b^5*c^5*\log(\text{abs}(F))^3 + 80*I\pi*b^5*c^5*\log(\text{abs}(F))^4 + 32*b^5*c^5*\log(\text{abs}(F))^5)*e^{(b*c*x*\log(\text{abs}(F)) + a*c*\log(\text{abs}(F)) + 4) - 4*((3*\pi^2*b^3*c^3*d*x^3*\log(\text{abs}(F))*\text{sgn}(F) - 3*\pi^2*b^3*c^3*d*x^3*\log(\text{abs}(F)) + 2*b^3*c^3*d*x^3*\log(\text{abs}(F))^3 - 3*\pi^2*b^2*c^2*d*x^2*\text{sgn}(F) + 3*\pi^2*b^2*c^2*d*x^2 - 6*b^2*c^2*d*x^2*\log(\text{abs}(F))^2 + 12*b*c*d*x*\log(\text{abs}(F)) - 12*d)*(pi^4*b^4*c^4*\text{sgn}(F) - 6*\pi^2*b^4*c^4*\log(\text{abs}(F))^2*\text{sgn}(F) - pi^4*b^4*c^4 + 6*\pi^2*b^4*c^4*\log(\text{abs}(F))^2 - 2*b^4*c^4*\log(\text{abs}(F))^4)} / ((pi^4*b^4*c^4*\text{sgn}(F) - 6*\pi^2*b^4*c^4*\log(\text{abs}(F))^2*\text{sgn}(F) - pi^4*b^4*c^4 + 6*\pi^2*b^4*c^4*\log(\text{abs}(F))^2 - 2*b^4*c^4*\log(\text{abs}(F))^4)^2 + 16*(pi^3*b^4*c^4*\log(\text{abs}(F))*\text{sgn}(F) - pi*b^4*c^4*\log(\text{abs}(F))^3*\text{sgn}(F) - pi^3*b^4*c^4*\log(\text{abs}(F)) + pi*b^4*c^4*\log(\text{abs}(F))^3)^2) - 4*(pi^3*b^3*c^3*d*x^3*\text{sgn}(F) - 3*\pi*b^3*c^3*d*x^3*\log(\text{abs}(F))^2*\text{sgn}(F) - pi^3*b^3*c^3*d*x^3 + 3*\pi*b^3*c^3*d*x^3*\log(\text{abs}(F))^2 + 6*\pi*b^2*c^2*d*x^2*\log(\text{abs}(F))*\text{sgn}(F) - 6*\pi*b^2*c^2*d*x^2*\log(\text{abs}(F)) - 6*\pi*b*c*d*x*\text{sgn}(F) + 6*\pi*b*c*d*x*(pi^3*b^4*c^4*\log(\text{abs}(F))*\text{sgn}(F) - pi*b^4*c^4*\log(\text{abs}(F))^3*\text{sgn}(F) - pi^3*b^4*c^4*\log(\text{abs}(F)) + pi*b^4*c^4*\log(\text{abs}(F))^3)} / ((pi^4*b^4*c^4*\text{sgn}(F) - 6*\pi^2*b^4*c^4*\log(\text{abs}(F))^2*\text{sgn}(F) - pi^4*b^4*c^4 + 6*\pi^2*b^4*c^4*\log(\text{abs}(F))^2 - 2*b^4*c^4*\log(\text{abs}(F))^4)^2 + 16*(pi^3*b^4*c^4*\log(\text{abs}(F))*\text{sgn}(F) - pi*b^4*c^4*\log(\text{abs}(F))^3*\text{sgn}(F) - pi^3*b^4*c^4*\log(\text{abs}(F)) + pi*b^4*c^4*\log(\text{abs}(F))^3)^2)*\cos(-1/2*\pi*b*c*x*\text{sgn}(F) + 1/2*\pi*b*c*x - 1/2*\pi*a*c*\text{sgn}(F) + 1/2*\pi*a*c) - ((pi^3*b^3*c^3*d*x^3*\text{sgn}(F) - 3*\pi*b^3*c^3*d*x^3*\log(\text{abs}(F))^2*\text{sgn}(F) - pi^3*b^3*c^3*d*x^3 + 3*\pi*b^3*c^3*d*x^3*\log(\text{abs}(F))^2 + 6*\pi*b^2*c^2*d*x^2*\log(\text{abs}(F)) + 6*\pi*b^2*c^2*d*x^2*\log(\text{abs}(F))^2 + 12*b*c*d*x*\log(\text{abs}(F)) + 12*d*(pi^4*b^4*c^4*\text{sgn}(F) - 6*\pi^2*b^4*c^4*\log(\text{abs}(F))^2*\text{sgn}(F) - pi^4*b^4*c^4 + 6*\pi^2*b^4*c^4*\log(\text{abs}(F))^2 - 2*b^4*c^4*\log(\text{abs}(F))^4)} / ((pi^4*b^4*c^4*\text{sgn}(F) - 6*\pi^2*b^4*c^4*\log(\text{abs}(F))^2*\text{sgn}(F) - pi^4*b^4*c^4 + 6*\pi^2*b^4*c^4*\log(\text{abs}(F))^2 - 2*b^4*c^4*\log(\text{abs}(F))^4)^2 + 16*(pi^3*b^4*c^4*\log(\text{abs}(F))*\text{sgn}(F) - pi*b^4*c^4*\log(\text{abs}(F))^3*\text{sgn}(F) - pi^3*b^4*c^4*\log(\text{abs}(F)) + pi*b^4*c^4*\log(\text{abs}(F))^3)^2)
\end{aligned}$$

$$\begin{aligned}
& g(\text{abs}(F)) * \text{sgn}(F) - 6 * \pi * b^2 * c^2 * d * x^2 * \log(\text{abs}(F)) - 6 * \pi * b * c * d * x * \text{sgn}(F) + 6 \\
& * \pi * b * c * d * x * (\pi^4 * b^4 * c^4 * \text{sgn}(F) - 6 * \pi^2 * b^4 * c^4 * \log(\text{abs}(F))^2 * \text{sgn}(F) - \pi^4 * b^4 * c^4 \\
& + 6 * \pi^2 * b^4 * c^4 * \log(\text{abs}(F))^2 - 2 * b^4 * c^4 * \log(\text{abs}(F))^4) / ((\pi^4 * b^4 * c^4 * \text{sgn}(F) - 6 * \pi^2 * b^4 * c^4 * \log(\text{abs}(F))^2 * \text{sgn}(F) - \pi^4 * b^4 * c^4 \\
& + 6 * \pi^2 * b^4 * c^4 * \log(\text{abs}(F))^2 - 2 * b^4 * c^4 * \log(\text{abs}(F))^4)^2 + 16 * (\pi^3 * b^4 * c^4 * \log(\text{abs}(F)) * \text{sgn}(F) - \pi * b^4 * c^4 * \log(\text{abs}(F))^3 * \text{sgn}(F) - \pi^3 * b^4 * c^4 * \log(\text{abs}(F)) \\
& + \pi * b^4 * c^4 * \log(\text{abs}(F))^3)^2) + 4 * (3 * \pi^2 * b^3 * c^3 * d * x^3 * \log(\text{abs}(F)) * \text{sgn}(F) - 3 * \pi^2 * b^3 * c^3 * d * x^3 * \log(\text{abs}(F))^3 - 3 \\
& * \pi^2 * b^2 * c^2 * d * x^2 * \text{sgn}(F) + 3 * \pi^2 * b^2 * c^2 * d * x^2 - 6 * b^2 * c^2 * d * x^2 * \log(\text{abs}(F))^2 + 12 * b * c * d * x * \log(\text{abs}(F)) - 12 * d) * (\pi^3 * b^4 * c^4 * \log(\text{abs}(F)) * \text{sgn}(F) - \\
& \pi * b^4 * c^4 * \log(\text{abs}(F))^3 * \text{sgn}(F) - \pi^3 * b^4 * c^4 * \log(\text{abs}(F)) + \pi * b^4 * c^4 * \log(\text{abs}(F))^3) / ((\pi^4 * b^4 * c^4 * \text{sgn}(F) - 6 * \pi^2 * b^4 * c^4 * \log(\text{abs}(F))^2 * \text{sgn}(F) - \pi^4 * b^4 * c^4 \\
& + 6 * \pi^2 * b^4 * c^4 * \log(\text{abs}(F))^2 - 2 * b^4 * c^4 * \log(\text{abs}(F))^4)^2 + 16 * (\pi^3 * b^4 * c^4 * \log(\text{abs}(F)) * \text{sgn}(F) - \pi * b^4 * c^4 * \log(\text{abs}(F))^3 * \text{sgn}(F) - \pi^3 * b^4 * c^4 * \log(\text{abs}(F)) \\
& + \pi * b^4 * c^4 * \log(\text{abs}(F))^3)^2) * \sin(-1/2 * \pi * b * c * x * \text{sgn}(F) + 1/2 * \pi * b * c * x - 1/2 * \pi * a * c * \text{sgn}(F) + 1/2 * \pi * a * c) * e^{(b * c * x * \log(\text{abs}(F)) + \\
& a * c * \log(\text{abs}(F)) + 3) - 1/2 * I * ((32 * \pi^3 * b^3 * c^3 * d * x^3 * \text{sgn}(F) + 96 * I * \pi^2 * b^3 * c^3 * d * x^3 * \log(\text{abs}(F)) * \text{sgn}(F) - 96 * \pi * b^3 * c^3 * d * x^3 * \log(\text{abs}(F))^2 * \text{sgn}(F) - \\
& 32 * \pi^3 * b^3 * c^3 * d * x^3 - 96 * I * \pi^2 * b^3 * c^3 * d * x^3 * \log(\text{abs}(F)) + 96 * \pi * b^3 * c^3 * d * x^3 * \log(\text{abs}(F))^2 + 64 * I * b^3 * c^3 * d * x^3 * \log(\text{abs}(F))^3 - 96 * I * \pi^2 * b^2 * c^2 * d * x^2 * \text{sgn}(F) + 192 * \pi * b^2 * c^2 * d * x^2 * \log(\text{abs}(F)) * \text{sgn}(F) + 96 * I * \pi^2 * b^2 * c^2 * d * x^2 - 192 * \pi * b^2 * c^2 * d * x^2 * \log(\text{abs}(F)) - 192 * I * b^2 * c^2 * d * x^2 * \log(\text{abs}(F))^2 - 192 * \pi * b * c * d * x * \text{sgn}(F) + 192 * \pi * b * c * d * x + 384 * I * b * c * d * x * \log(\text{abs}(F)) - 384 * I * d) * e^{(1/2 * I * \pi * b * c * x * \text{sgn}(F) - 1/2 * I * \pi * b * c * x + 1/2 * I * \pi * a * c * \text{sgn}(F) - 1/2 * I * \pi * a * c) / (8 * \pi^4 * b^4 * c^4 * \text{sgn}(F) + 32 * I * \pi^3 * b^4 * c^4 * \log(\text{abs}(F)) * \text{sgn}(F) - 48 * \pi^2 * b^4 * c^4 * \log(\text{abs}(F))^2 * \text{sgn}(F) - 32 * I * \pi * b^4 * c^4 * \log(\text{abs}(F))^3 * \text{sgn}(F) - 8 * \pi^4 * b^4 * c^4 - 32 * I * \pi^3 * b^4 * c^4 * \log(\text{abs}(F)) + 48 * \pi^2 * b^4 * c^4 * \log(\text{abs}(F))^2 + 32 * I * \pi * b^4 * c^4 * \log(\text{abs}(F))^3 - 16 * b^4 * c^4 * \log(\text{abs}(F))^4) + (32 * \pi^3 * b^3 * c^3 * d * x^3 * \text{sgn}(F) - 96 * I * \pi^2 * b^3 * c^3 * d * x^3 * \log(\text{abs}(F)) * \text{sgn}(F) - 96 * \pi * b^3 * c^3 * d * x^3 * \log(\text{abs}(F))^2 * \text{sgn}(F) - 32 * \pi^3 * b^3 * c^3 * d * x^3 + 96 * I * \pi^2 * b^3 * c^3 * d * x^3 * \log(\text{abs}(F)) + 96 * \pi * b^3 * c^3 * d * x^3 * \log(\text{abs}(F))^2 - 64 * I * b^3 * c^3 * d * x^3 * \log(\text{abs}(F))^3 + 96 * I * \pi^2 * b^2 * c^2 * d * x^2 * \text{sgn}(F) + 192 * \pi * b^2 * c^2 * d * x^2 * \log(\text{abs}(F)) * \text{sgn}(F) - 96 * I * \pi^2 * b^2 * c^2 * d * x^2 - 192 * \pi * b^2 * c^2 * d * x^2 * \log(\text{abs}(F)) + 192 * I * b^2 * c^2 * d * x^2 * \log(\text{abs}(F))^2 - 192 * \pi * b * c * d * x * \text{sgn}(F) + 192 * \pi * b * c * d * x - 384 * I * b * c * d * x * \log(\text{abs}(F)) + 384 * I * d) * e^{(-1/2 * I * \pi * b * c * x * \text{sgn}(F) + 1/2 * I * \pi * b * c * x - 1/2 * I * \pi * a * c * \text{sgn}(F) + 1/2 * I * \pi * a * c) / (8 * \pi^4 * b^4 * c^4 * \text{sgn}(F) - 32 * I * \pi^3 * b^4 * c^4 * \log(\text{abs}(F)) * \text{sgn}(F) - 48 * \pi^2 * b^4 * c^4 * \log(\text{abs}(F))^2 * \text{sgn}(F) + 32 * I * \pi * b^4 * c^4 * \log(\text{abs}(F))^3 * \text{sgn}(F) - 8 * \pi^4 * b^4 * c^4 + 32 * I * \pi^3 * b^4 * c^4 * \log(\text{abs}(F)) + 48 * \pi^2 * b^4 * c^4 * \log(\text{abs}(F))^2 - 32 * I * \pi * b^4 * c^4 * \log(\text{abs}(F))^3 - 16 * b^4 * c^4 * \log(\text{abs}(F))^4) * e^{(b * c * x * \log(\text{abs}(F)) + a * c * \log(\text{abs}(F)) + 3) - 6 * ((2 * (\pi * b^2 * c^2 * d^2 * x^2 * \log(\text{abs}(F)) * \text{sgn}(F) - \pi * b^2 * c^2 * d^2 * x^2 * \log(\text{abs}(F)) - \pi * b * c * d^2 * x * \text{sgn}(F) + \pi * b * c * d^2 * x) * (\pi^3 * b^3 * c^3 * \text{sgn}(F) - 3 * \pi * b^3 * c^3 * \log(\text{abs}(F))^2 * \text{sgn}(F) - \pi^3 * b^3 * c^3 + 3 * \pi * b^3 * c^3 * \log(\text{abs}(F))^2) / ((\pi^3 * b^3 * c^3 * \text{sgn}(F) - 3 * \pi * b^3 * c^3 * \log(\text{abs}(F))^2 * \text{sgn}(F) - \pi^3 * b^3 * c^3 + 3 * \pi * b^3 * c^3 * \log(\text{abs}(F))^2) + (3 * \pi^2 * b^3 * c^3 * \log(\text{abs}(F)) * \text{sgn}(F) - 3 * \pi
\end{aligned}$$

$$\begin{aligned}
& ^2*b^3*c^3*\log(\text{abs}(F)) + 2*b^3*c^3*\log(\text{abs}(F))^3)^2) - (\pi^2*b^2*c^2*d^2*x^2 \\
& * \text{sgn}(F) - \pi^2*b^2*c^2*d^2*x^2 + 2*b^2*c^2*d^2*x^2*\log(\text{abs}(F))^2 - 4*b*c*d^2*x*\log(\text{abs}(F)) + 4*d^2) * (3*\pi^2*b^3*c^3*\log(\text{abs}(F))*\text{sgn}(F) - 3*\pi^2*b^3*c^3 \\
& *\log(\text{abs}(F)) + 2*b^3*c^3*\log(\text{abs}(F))^3) / ((\pi^3*b^3*c^3*\text{sgn}(F) - 3*\pi*b^3*c^3 \\
& * \log(\text{abs}(F))^2*\text{sgn}(F) - \pi^3*b^3*c^3 + 3*\pi*b^3*c^3*\log(\text{abs}(F))^2)^2 + ( \\
& 3*\pi^2*b^3*c^3*\log(\text{abs}(F))*\text{sgn}(F) - 3*\pi^2*b^3*c^3*\log(\text{abs}(F)) + 2*b^3*c^3* \\
& \log(\text{abs}(F))^3)^2)) * \cos(-1/2*\pi*b*c*x*\text{sgn}(F) + 1/2*\pi*b*c*x - 1/2*\pi*a*c*\text{sgn} \\
& (F) + 1/2*\pi*a*c) - ((\pi^2*b^2*c^2*d^2*x^2*\text{sgn}(F) - \pi^2*b^2*c^2*d^2*x^2 + \\
& 2*b^2*c^2*d^2*x^2*\log(\text{abs}(F))^2 - 4*b*c*d^2*x*\log(\text{abs}(F)) + 4*d^2) * (\pi^3*b^3 \\
& *c^3*\text{sgn}(F) - 3*\pi*b^3*c^3*\log(\text{abs}(F))^2*\text{sgn}(F) - \pi^3*b^3*c^3 + 3*\pi*b^3*c^3 \\
& * \log(\text{abs}(F))^2) / ((\pi^3*b^3*c^3*\text{sgn}(F) - 3*\pi*b^3*c^3*\log(\text{abs}(F))^2*\text{sgn}(F) \\
& ) - \pi^3*b^3*c^3 + 3*\pi*b^3*c^3*\log(\text{abs}(F))^2)^2 + (3*\pi^2*b^3*c^3*\log(\text{abs}( \\
& F))*\text{sgn}(F) - 3*\pi^2*b^3*c^3*\log(\text{abs}(F)) + 2*b^3*c^3*\log(\text{abs}(F))^3)^2) + 2*( \\
& \pi*b^2*c^2*d^2*x^2*\log(\text{abs}(F))*\text{sgn}(F) - \pi*b^2*c^2*d^2*x^2*\log(\text{abs}(F)) - \pi \\
& *b*c*d^2*x*\text{sgn}(F) + \pi*b*c*d^2*x) * (3*\pi^2*b^3*c^3*\log(\text{abs}(F))*\text{sgn}(F) - 3*\pi \\
& ^2*b^3*c^3*\log(\text{abs}(F)) + 2*b^3*c^3*\log(\text{abs}(F))^3) / ((\pi^3*b^3*c^3*\text{sgn}(F) - 3 \\
& *\pi*b^3*c^3*\log(\text{abs}(F))^2*\text{sgn}(F) - \pi^3*b^3*c^3 + 3*\pi*b^3*c^3*\log(\text{abs}(F))^2 \\
& )^2 + (3*\pi^2*b^3*c^3*\log(\text{abs}(F))*\text{sgn}(F) - 3*\pi^2*b^3*c^3*\log(\text{abs}(F)) + 2* \\
& b^3*c^3*\log(\text{abs}(F))^3)^2)) * \sin(-1/2*\pi*b*c*x*\text{sgn}(F) + 1/2*\pi*b*c*x - 1/2*\pi \\
& *a*c*\text{sgn}(F) + 1/2*\pi*a*c) * e^{(b*c*x*\log(\text{abs}(F)) + a*c*\log(\text{abs}(F)) + 2) + 1/ \\
& 2*I*((24*I*\pi^2*b^2*c^2*d^2*x^2*\text{sgn}(F) - 48*\pi*b^2*c^2*d^2*x^2*\log(\text{abs}(F))* \\
& \text{sgn}(F) - 24*I*\pi^2*b^2*c^2*d^2*x^2 + 48*\pi*b^2*c^2*d^2*x^2*\log(\text{abs}(F)) + 48 \\
& *I*b^2*c^2*d^2*x^2*\log(\text{abs}(F))^2 + 48*\pi*b*c*d^2*x*\text{sgn}(F) - 48*\pi*b*c*d^2*x \\
& - 96*I*b*c*d^2*x*\log(\text{abs}(F)) + 96*I*d^2) * e^{(1/2*I*\pi*b*c*x*\text{sgn}(F) - 1/2*I* \\
& \pi*b*c*x + 1/2*I*\pi*a*c*\text{sgn}(F) - 1/2*I*\pi*a*c) / (-4*I*\pi^3*b^3*c^3*\text{sgn}(F) + \\
& 12*\pi^2*b^3*c^3*\log(\text{abs}(F))*\text{sgn}(F) + 12*I*\pi*b^3*c^3*\log(\text{abs}(F))^2*\text{sgn}(F) + \\
& 4*I*\pi^3*b^3*c^3 - 12*\pi^2*b^3*c^3*\log(\text{abs}(F)) - 12*I*\pi*b^3*c^3*\log(\text{abs}(F) \\
& ))^2 + 8*b^3*c^3*\log(\text{abs}(F))^3) - (24*I*\pi^2*b^2*c^2*d^2*x^2*\text{sgn}(F) + 48*\pi \\
& *b^2*c^2*d^2*x^2*\log(\text{abs}(F))*\text{sgn}(F) - 24*I*\pi^2*b^2*c^2*d^2*x^2 - 48*\pi*b^2 \\
& *c^2*d^2*x^2*\log(\text{abs}(F)) + 48*I*b^2*c^2*d^2*x^2*\log(\text{abs}(F))^2 - 48*\pi*b*c*d^2 \\
& *x*\text{sgn}(F) + 48*\pi*b*c*d^2*x - 96*I*b*c*d^2*x*\log(\text{abs}(F)) + 96*I*d^2) * e^{(- \\
& 1/2*I*\pi*b*c*x*\text{sgn}(F) + 1/2*I*\pi*b*c*x - 1/2*I*\pi*a*c*\text{sgn}(F) + 1/2*I*\pi*a*c \\
& ) / (4*I*\pi^3*b^3*c^3*\text{sgn}(F) + 12*\pi^2*b^3*c^3*\log(\text{abs}(F))*\text{sgn}(F) - 12*I*\pi*b^3 \\
& *c^3*\log(\text{abs}(F))^2*\text{sgn}(F) - 4*I*\pi^3*b^3*c^3 - 12*\pi^2*b^3*c^3*\log(\text{abs}(F) \\
& ) + 12*I*\pi*b^3*c^3*\log(\text{abs}(F))^2 + 8*b^3*c^3*\log(\text{abs}(F))^3)) * e^{(b*c*x*\log( \\
& \text{abs}(F)) + a*c*\log(\text{abs}(F)) + 2) + 4*(2*((b*c*d^3*x*\log(\text{abs}(F)) - d^3) * (\pi^2*b^2 \\
& *c^2*\text{sgn}(F) - \pi^2*b^2*c^2 + 2*b^2*c^2*\log(\text{abs}(F))^2) / ((\pi^2*b^2*c^2*\text{sgn} \\
& (F) - \pi^2*b^2*c^2 + 2*b^2*c^2*\log(\text{abs}(F))^2)^2 + 4*(\pi*b^2*c^2*\log(\text{abs}(F)) \\
& )*\text{sgn}(F) - \pi*b^2*c^2*\log(\text{abs}(F)))^2) + (\pi*b*c*d^3*x*\text{sgn}(F) - \pi*b*c*d^3*x) \\
& * (\pi*b^2*c^2*\log(\text{abs}(F))*\text{sgn}(F) - \pi*b^2*c^2*\log(\text{abs}(F))) / ((\pi^2*b^2*c^2*\text{sgn} \\
& n(F) - \pi^2*b^2*c^2 + 2*b^2*c^2*\log(\text{abs}(F))^2)^2 + 4*(\pi*b^2*c^2*\log(\text{abs}(F) \\
& )*\text{sgn}(F) - \pi*b^2*c^2*\log(\text{abs}(F)))^2)) * \cos(-1/2*\pi*b*c*x*\text{sgn}(F) + 1/2*\pi*b* \\
& c*x - 1/2*\pi*a*c*\text{sgn}(F) + 1/2*\pi*a*c) + ((\pi*b*c*d^3*x*\text{sgn}(F) - \pi*b*c*d^3*x) \\
& * (\pi^2*b^2*c^2*\text{sgn}(F) - \pi^2*b^2*c^2 + 2*b^2*c^2*\log(\text{abs}(F))^2) / ((\pi^2*b^2 \\
& *c^2*\text{sgn}(F) - \pi^2*b^2*c^2 + 2*b^2*c^2*\log(\text{abs}(F))^2)^2 + 4*(\pi*b^2*c^2*\log
\end{aligned}$$

$$\begin{aligned}
& g(\text{abs}(F)) * \text{sgn}(F) - \pi * b^2 * c^2 * \log(\text{abs}(F)))^2 - 4 * (b * c * d^3 * x * \log(\text{abs}(F)) - \\
& d^3) * (\pi * b^2 * c^2 * \log(\text{abs}(F)) * \text{sgn}(F) - \pi * b^2 * c^2 * \log(\text{abs}(F))) / ((\pi^2 * b^2 * c^2 * \\
& 2 * \text{sgn}(F) - \pi^2 * b^2 * c^2 + 2 * b^2 * c^2 * \log(\text{abs}(F))^2)^2 + 4 * (\pi * b^2 * c^2 * \log(\text{abs}(F)) * \text{sgn}(F) - \pi * b^2 * c^2 * \log(\text{abs}(F)))^2) * \sin(-1/2 * \pi * b * c * x * \text{sgn}(F) + 1/2 * \pi * \\
& b * c * x - 1/2 * \pi * a * c * \text{sgn}(F) + 1/2 * \pi * a * c) * e^{(b * c * x * \log(\text{abs}(F)) + a * c * \log(\text{abs}(F)) + 1) - 1/2 * I * ((8 * \pi * b * c * d^3 * x * \text{sgn}(F) - 8 * \pi * b * c * d^3 * x - 16 * I * b * c * d^3 * \\
& x * \log(\text{abs}(F)) + 16 * I * d^3) * e^{(1/2 * I * \pi * b * c * x * \text{sgn}(F) - 1/2 * I * \pi * b * c * x + 1/2 * \\
& I * \pi * a * c * \text{sgn}(F) - 1/2 * I * \pi * a * c) / (2 * \pi^2 * b^2 * c^2 * \text{sgn}(F) + 4 * I * \pi * b^2 * c^2 * \log(\text{abs}(F)) * \text{sgn}(F) - 2 * \pi^2 * b^2 * c^2 - 4 * I * \pi * b^2 * c^2 * \log(\text{abs}(F)) + 4 * b^2 * c^2 * \log(\text{abs}(F))^2) + (8 * \pi * b * c * d^3 * x * \text{sgn}(F) - 8 * \pi * b * c * d^3 * x + 16 * I * b * c * d^3 * x * \log(\text{abs}(F)) - 16 * I * d^3) * e^{(-1/2 * I * \pi * b * c * x * \text{sgn}(F) + 1/2 * I * \pi * b * c * x - 1/2 * I * \pi * a * c * \text{sgn}(F) + 1/2 * I * \pi * a * c) / (2 * \pi^2 * b^2 * c^2 * \text{sgn}(F) - 4 * I * \pi * b^2 * c^2 * \log(\text{abs}(F)) * \text{sgn}(F) - 2 * \pi^2 * b^2 * c^2 + 4 * I * \pi * b^2 * c^2 * \log(\text{abs}(F)) + 4 * b^2 * c^2 * \log(\text{abs}(F))^2)} * e^{(b * c * x * \log(\text{abs}(F)) + a * c * \log(\text{abs}(F)) + 1) + 2 * (2 * b * c * d^4 * \cos(-1/2 * \pi * b * c * x * \text{sgn}(F) + 1/2 * \pi * b * c * x - 1/2 * \pi * a * c * \text{sgn}(F) + 1/2 * \pi * a * c) * \log(\text{abs}(F)) / (4 * b^2 * c^2 * \log(\text{abs}(F))^2 + (\pi * b * c * \text{sgn}(F) - \pi * b * c)^2) - (\pi * b * c * \text{sgn}(F) - \pi * b * c) * d^4 * \sin(-1/2 * \pi * b * c * x * \text{sgn}(F) + 1/2 * \pi * b * c * x - 1/2 * \pi * a * c * \text{sgn}(F) + 1/2 * \pi * a * c) / (4 * b^2 * c^2 * \log(\text{abs}(F))^2 + (\pi * b * c * \text{sgn}(F) - \pi * b * c)^2)} * e^{(b * c * x * \log(\text{abs}(F)) + a * c * \log(\text{abs}(F))) - 1/2 * I * (-2 * I * d^4 * e^{(1/2 * I * \pi * b * c * x * \text{sgn}(F) - 1/2 * I * \pi * b * c * x + 1/2 * I * \pi * a * c * \text{sgn}(F) - 1/2 * I * \pi * a * c) / (I * \pi * b * c * \text{sgn}(F) - I * \pi * b * c + 2 * b * c * \log(\text{abs}(F))) + 2 * I * d^4 * e^{(-1/2 * I * \pi * b * c * x * \text{sgn}(F) + 1/2 * I * \pi * b * c * x - 1/2 * I * \pi * a * c * \text{sgn}(F) + 1/2 * I * \pi * a * c) / (-I * \pi * b * c * \text{sgn}(F) + I * \pi * b * c + 2 * b * c * \log(\text{abs}(F)))} * e^{(b * c * x * \log(\text{abs}(F)) + a * c * \log(\text{abs}(F)))}
\end{aligned}$$

### 3.3 $\int F^{c(a+bx)}(d+ex)^3 dx$

**Optimal.** Leaf size=110

$$\frac{6e^2(d+ex)F^{c(a+bx)}}{b^3c^3\log^3(F)} - \frac{3e(d+ex)^2F^{c(a+bx)}}{b^2c^2\log^2(F)} - \frac{6e^3F^{c(a+bx)}}{b^4c^4\log^4(F)} + \frac{(d+ex)^3F^{c(a+bx)}}{bc\log(F)}$$

[Out]  $(-6e^3F^{c(a+bx)})/(b^4c^4\log[F]^4) + (6e^2F^{c(a+bx)}(d+ex))/(b^3c^3\log[F]^3) - (3eF^{c(a+bx)}(d+ex)^2)/(b^2c^2\log[F]^2) + (F^{c(a+bx)}(d+ex)^3)/(bc\log[F])$

**Rubi [A]** time = 0.0729904, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$ , Rules used = {2176, 2194}

$$\frac{6e^2(d+ex)F^{c(a+bx)}}{b^3c^3\log^3(F)} - \frac{3e(d+ex)^2F^{c(a+bx)}}{b^2c^2\log^2(F)} - \frac{6e^3F^{c(a+bx)}}{b^4c^4\log^4(F)} + \frac{(d+ex)^3F^{c(a+bx)}}{bc\log(F)}$$

Antiderivative was successfully verified.

[In] Int[F^{c(a+bx)}(d+ex)^3,x]

[Out]  $(-6e^3F^{c(a+bx)})/(b^4c^4\log[F]^4) + (6e^2F^{c(a+bx)}(d+ex))/(b^3c^3\log[F]^3) - (3eF^{c(a+bx)}(d+ex)^2)/(b^2c^2\log[F]^2) + (F^{c(a+bx)}(d+ex)^3)/(bc\log[F])$

#### Rule 2176

```
Int[((b_.)*(F_)^((g_.)*((e_.)+(f_.)*(x_))))^(n_.)*((c_.)+(d_.)*(x_))^(m_.), x_Symbol] := Simp[((c+d*x)^m*(b*F^(g*(e+f*x)))^n)/(f*g*n*Log[F]), x] - Dist[(d*m)/(f*g*n*Log[F]), Int[(c+d*x)^(m-1)*(b*F^(g*(e+f*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m] && !$UseGamma == True
```

#### Rule 2194

```
Int[((F_)^((c_.)*((a_.)+(b_.)*(x_))))^(n_.), x_Symbol] := Simp[(F^(c(a+bx)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

#### Rubi steps

$$\begin{aligned}
\int F^{c(a+bx)}(d+ex)^3 dx &= \frac{F^{c(a+bx)}(d+ex)^3}{bc \log(F)} - \frac{(3e) \int F^{c(a+bx)}(d+ex)^2 dx}{bc \log(F)} \\
&= -\frac{3eF^{c(a+bx)}(d+ex)^2}{b^2c^2 \log^2(F)} + \frac{F^{c(a+bx)}(d+ex)^3}{bc \log(F)} + \frac{(6e^2) \int F^{c(a+bx)}(d+ex) dx}{b^2c^2 \log^2(F)} \\
&= \frac{6e^2F^{c(a+bx)}(d+ex)}{b^3c^3 \log^3(F)} - \frac{3eF^{c(a+bx)}(d+ex)^2}{b^2c^2 \log^2(F)} + \frac{F^{c(a+bx)}(d+ex)^3}{bc \log(F)} - \frac{(6e^3) \int F^{c(a+bx)} dx}{b^3c^3 \log^3(F)} \\
&= -\frac{6e^3F^{c(a+bx)}}{b^4c^4 \log^4(F)} + \frac{6e^2F^{c(a+bx)}(d+ex)}{b^3c^3 \log^3(F)} - \frac{3eF^{c(a+bx)}(d+ex)^2}{b^2c^2 \log^2(F)} + \frac{F^{c(a+bx)}(d+ex)^3}{bc \log(F)}
\end{aligned}$$

**Mathematica [A]** time = 0.0834539, size = 78, normalized size = 0.71

$$\frac{F^{c(a+bx)} \left( -3b^2c^2e \log^2(F)(d+ex)^2 + b^3c^3 \log^3(F)(d+ex)^3 + 6bce^2 \log(F)(d+ex) - 6e^3 \right)}{b^4c^4 \log^4(F)}$$

Antiderivative was successfully verified.

[In] Integrate[F^(c\*(a + b\*x))\*(d + e\*x)^3,x]

[Out] (F^(c\*(a + b\*x))\*(-6\*e^3 + 6\*b\*c\*e^2\*(d + e\*x)\*Log[F] - 3\*b^2\*c^2\*e\*(d + e\*x)^2\*Log[F]^2 + b^3\*c^3\*(d + e\*x)^3\*Log[F]^3))/(b^4\*c^4\*Log[F]^4)

**Maple [A]** time = 0.007, size = 165, normalized size = 1.5

$$\frac{(e^3x^3b^3c^3(\ln(F))^3 + 3(\ln(F))^3b^3c^3de^2x^2 + 3(\ln(F))^3b^3c^3d^2ex + b^3c^3(\ln(F))^3d^3 - 3(\ln(F))^2b^2c^2e^3x^2 - 6(\ln(F))^2b^2c^2e^2xd - 3b^2c^2e^2d^2x - 6\ln(F)b^2c^2e^3x^2 + 6\ln(F)b^2c^2e^2xd - 6e^3)F^{c(bx+a)})}{b^4c^4(\ln(F))^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(c\*(b\*x+a))\*(e\*x+d)^3,x)

[Out] (e^3\*x^3\*b^3\*c^3\*ln(F)^3+3\*ln(F)^3\*b^3\*c^3\*d\*e^2\*x^2+3\*ln(F)^3\*b^3\*c^3\*d^2\*e\*x+b^3\*c^3\*ln(F)^3\*d^3-3\*ln(F)^2\*b^2\*c^2\*e^3\*x^2-6\*ln(F)^2\*b^2\*c^2\*d\*e^2\*x-3\*b^2\*c^2\*ln(F)^2\*e\*d^2+6\*ln(F)\*b\*c\*e^3\*x+6\*d\*e^2\*b\*c\*ln(F)-6\*e^3)\*F^(c\*(b\*x+a))/b^4/c^4/ln(F)^4

**Maxima [A]** time = 1.01955, size = 278, normalized size = 2.53

$$\frac{F^{bcx+ac}d^3}{bc \log(F)} + \frac{3(F^{ac}bcx \log(F) - F^{ac})F^{bcx}d^2e}{b^2c^2 \log(F)^2} + \frac{3(F^{ac}b^2c^2x^2 \log(F)^2 - 2F^{ac}bcx \log(F) + 2F^{ac})F^{bcx}de^2}{b^3c^3 \log(F)^3} + \frac{(F^{ac}b^3c^3x^3 \log(F) - 3F^{ac}b^2c^2x^2 \log(F)^2 + 6F^{ac}bcx \log(F) - 6F^{ac})F^{bcx}d^3e^3}{b^4c^4 \log(F)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(c\*(b\*x+a))\*(e\*x+d)^3,x, algorithm="maxima")

[Out]  $F^{(b*c*x + a*c)}*d^3/(b*c*\log(F)) + 3*(F^{(a*c)}*b*c*x*\log(F) - F^{(a*c)})*F^{(b*c*x)}*d^2*e/(b^2*c^2*\log(F)^2) + 3*(F^{(a*c)}*b^2*c^2*x^2*\log(F)^2 - 2*F^{(a*c)}*b*c*x*\log(F) + 2*F^{(a*c)})*F^{(b*c*x)}*d*e^2/(b^3*c^3*\log(F)^3) + (F^{(a*c)}*b^3*c^3*x^3*\log(F)^3 - 3*F^{(a*c)}*b^2*c^2*x^2*\log(F)^2 + 6*F^{(a*c)}*b*c*x*\log(F) - 6*F^{(a*c)})*F^{(b*c*x)}*e^3/(b^4*c^4*\log(F)^4)$

**Fricas [A]** time = 1.5247, size = 312, normalized size = 2.84

$$\frac{((b^3c^3e^3x^3 + 3b^3c^3de^2x^2 + 3b^3c^3d^2ex + b^3c^3d^3) \log(F)^3 - 6e^3 - 3(b^2c^2e^3x^2 + 2b^2c^2de^2x + b^2c^2d^2e) \log(F)^2 + 6(bce^3x - 3b^2c^2e^2x + 3b^2c^2d^2e) \log(F) - 6e^3) F^{(b*c*x + a*c)}}{b^4c^4 \log(F)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(c\*(b\*x+a))\*(e\*x+d)^3,x, algorithm="fricas")

[Out]  $((b^3c^3e^3x^3 + 3b^3c^3d^2e^2x^2 + 3b^3c^3d^2e^2x + b^3c^3d^3) * \log(F)^3 - 6e^3 - 3*(b^2c^2e^3x^2 + 2b^2c^2d^2e^2x + b^2c^2d^2e) * \log(F)^2 + 6*(b*c*e^3*x + b*c*d*e^2) * \log(F)) * F^{(b*c*x + a*c)} / (b^4*c^4*\log(F)^4)$

**Sympy [A]** time = 0.298741, size = 231, normalized size = 2.1

$$\left\{ \begin{array}{l} \frac{F^{(a+bx)}(b^3c^3d^3 \log(F)^3 + 3b^3c^3d^2ex \log(F)^3 + 3b^3c^3d^2e^2x^2 \log(F)^3 + b^3c^3e^3x^3 \log(F)^3 - 3b^2c^2d^2e \log(F)^2 - 6b^2c^2de^2x \log(F)^2 - 3b^2c^2e^3x^2 \log(F)^2 + 6bcde^2 \log(F) + 6e^3)}{b^4c^4 \log(F)^4} \\ d^3x + \frac{3d^2ex^2}{2} + de^2x^3 + \frac{e^3x^4}{4} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(c\*(b\*x+a))\*(e\*x+d)\*\*3,x)



```
[Out] Piecewise((F**(c*(a + b*x))*(b**3*c**3*d**3*log(F)**3 + 3*b**3*c**3*d**2*e*
x*log(F)**3 + 3*b**3*c**3*d**2*e**2*x**2*log(F)**3 + b**3*c**3*e**3*x**3*log(F)
)**3 - 3*b**2*c**2*d**2*e*log(F)**2 - 6*b**2*c**2*d**2*e**2*x*log(F)**2 - 3*b*
*2*c**2*e**3*x**2*log(F)**2 + 6*b*c*d**2*log(F) + 6*b*c*e**3*x*log(F) - 6
*e**3)/(b**4*c**4*log(F)**4), Ne(b**4*c**4*log(F)**4, 0)), (d**3*x + 3*d**2
*e*x**2/2 + d*e**2*x**3 + e**3*x**4/4, True))
```

---

**Giac [C]** time = 6.12779, size = 6336, normalized size = 57.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*(e*x+d)^3,x, algorithm="giac")
```

```
[Out] ((4*(pi^3*b^3*c^3*x^3*sgn(F) - 3*pi*b^3*c^3*x^3*log(abs(F))^2*sgn(F) - pi^3
*b^3*c^3*x^3 + 3*pi*b^3*c^3*x^3*log(abs(F))^2 + 6*pi*b^2*c^2*x^2*log(abs(F)
)*sgn(F) - 6*pi*b^2*c^2*x^2*log(abs(F)) - 6*pi*b*c*x*sgn(F) + 6*pi*b*c*x)*(
pi^3*b^4*c^4*log(abs(F))*sgn(F) - pi*b^4*c^4*log(abs(F))^3*sgn(F) - pi^3*b^
4*c^4*log(abs(F)) + pi*b^4*c^4*log(abs(F))^3)/(pi^4*b^4*c^4*sgn(F) - 6*pi^
2*b^4*c^4*log(abs(F))^2*sgn(F) - pi^4*b^4*c^4 + 6*pi^2*b^4*c^4*log(abs(F))^
2 - 2*b^4*c^4*log(abs(F))^4)^2 + 16*(pi^3*b^4*c^4*log(abs(F))*sgn(F) - pi*b
^4*c^4*log(abs(F))^3*sgn(F) - pi^3*b^4*c^4*log(abs(F)) + pi*b^4*c^4*log(abs
(F))^3)^2 - (pi^4*b^4*c^4*sgn(F) - 6*pi^2*b^4*c^4*log(abs(F))^2*sgn(F) - p
i^4*b^4*c^4 + 6*pi^2*b^4*c^4*log(abs(F))^2 - 2*b^4*c^4*log(abs(F))^4)*(3*pi
^2*b^3*c^3*x^3*log(abs(F))*sgn(F) - 3*pi^2*b^3*c^3*x^3*log(abs(F)) + 2*b^3*
c^3*x^3*log(abs(F))^3 - 3*pi^2*b^2*c^2*x^2*sgn(F) + 3*pi^2*b^2*c^2*x^2 - 6*
b^2*c^2*x^2*log(abs(F))^2 + 12*b*c*x*log(abs(F)) - 12)/((pi^4*b^4*c^4*sgn(F)
) - 6*pi^2*b^4*c^4*log(abs(F))^2*sgn(F) - pi^4*b^4*c^4 + 6*pi^2*b^4*c^4*log
(abs(F))^2 - 2*b^4*c^4*log(abs(F))^4)^2 + 16*(pi^3*b^4*c^4*log(abs(F))*sgn(
F) - pi*b^4*c^4*log(abs(F))^3*sgn(F) - pi^3*b^4*c^4*log(abs(F)) + pi*b^4*c^
4*log(abs(F))^3)^2))*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*s
gn(F) + 1/2*pi*a*c) + ((pi^4*b^4*c^4*sgn(F) - 6*pi^2*b^4*c^4*log(abs(F))^2*
sgn(F) - pi^4*b^4*c^4 + 6*pi^2*b^4*c^4*log(abs(F))^2 - 2*b^4*c^4*log(abs(F)
))^4*(pi^3*b^3*c^3*x^3*sgn(F) - 3*pi*b^3*c^3*x^3*log(abs(F))^2*sgn(F) - pi^
3*b^3*c^3*x^3 + 3*pi*b^3*c^3*x^3*log(abs(F))^2 + 6*pi*b^2*c^2*x^2*log(abs(F)
))*sgn(F) - 6*pi*b^2*c^2*x^2*log(abs(F)) - 6*pi*b*c*x*sgn(F) + 6*pi*b*c*x)/
((pi^4*b^4*c^4*sgn(F) - 6*pi^2*b^4*c^4*log(abs(F))^2*sgn(F) - pi^4*b^4*c^4
+ 6*pi^2*b^4*c^4*log(abs(F))^2 - 2*b^4*c^4*log(abs(F))^4)^2 + 16*(pi^3*b^4*
c^4*log(abs(F))*sgn(F) - pi*b^4*c^4*log(abs(F))^3*sgn(F) - pi^3*b^4*c^4*log
(abs(F)) + pi*b^4*c^4*log(abs(F))^3)^2) + 4*(pi^3*b^4*c^4*log(abs(F))*sgn(F)
) - pi*b^4*c^4*log(abs(F))^3*sgn(F) - pi^3*b^4*c^4*log(abs(F)) + pi*b^4*c^4
```

$$\begin{aligned}
& * \log(\text{abs}(F))^3 * (3 * \pi^2 * b^3 * c^3 * x^3 * \log(\text{abs}(F)) * \text{sgn}(F) - 3 * \pi^2 * b^3 * c^3 * x^3 \\
& * \log(\text{abs}(F)) + 2 * b^3 * c^3 * x^3 * \log(\text{abs}(F))^3 - 3 * \pi^2 * b^2 * c^2 * x^2 * \text{sgn}(F) + 3 * \\
& \pi^2 * b^2 * c^2 * x^2 - 6 * b^2 * c^2 * x^2 * \log(\text{abs}(F))^2 + 12 * b * c * x * \log(\text{abs}(F)) - 12) \\
& / ((\pi^4 * b^4 * c^4 * \text{sgn}(F) - 6 * \pi^2 * b^4 * c^4 * \log(\text{abs}(F))^2 * \text{sgn}(F) - \pi^4 * b^4 * c^4 \\
& + 6 * \pi^2 * b^4 * c^4 * \log(\text{abs}(F))^2 - 2 * b^4 * c^4 * \log(\text{abs}(F))^4)^2 + 16 * (\pi^3 * b^4 \\
& * c^4 * \log(\text{abs}(F)) * \text{sgn}(F) - \pi * b^4 * c^4 * \log(\text{abs}(F))^3 * \text{sgn}(F) - \pi^3 * b^4 * c^4 * \log \\
& (\text{abs}(F)) + \pi * b^4 * c^4 * \log(\text{abs}(F))^3)^2) * \sin(-1/2 * \pi * b * c * x * \text{sgn}(F) + 1/2 * \pi \\
& * b * c * x - 1/2 * \pi * a * c * \text{sgn}(F) + 1/2 * \pi * a * c) * e^{(b * c * x * \log(\text{abs}(F)) + a * c * \log(\text{abs} \\
& (\text{abs}(F)) + 3) - 1/2 * I * ((8 * \pi^3 * b^3 * c^3 * x^3 * \text{sgn}(F) + 24 * I * \pi^2 * b^3 * c^3 * x^3 * \log(\text{abs} \\
& (\text{abs}(F)) * \text{sgn}(F) - 24 * \pi * b^3 * c^3 * x^3 * \log(\text{abs}(F))^2 * \text{sgn}(F) - 8 * \pi^3 * b^3 * c^3 * x^3 \\
& - 24 * I * \pi^2 * b^3 * c^3 * x^3 * \log(\text{abs}(F)) + 24 * \pi * b^3 * c^3 * x^3 * \log(\text{abs}(F))^2 + 1 \\
& 6 * I * b^3 * c^3 * x^3 * \log(\text{abs}(F))^3 - 24 * I * \pi^2 * b^2 * c^2 * x^2 * \text{sgn}(F) + 48 * \pi * b^2 * c^2 \\
& 2 * x^2 * \log(\text{abs}(F)) * \text{sgn}(F) + 24 * I * \pi^2 * b^2 * c^2 * x^2 - 48 * \pi * b^2 * c^2 * x^2 * \log(\text{abs} \\
& (\text{abs}(F)) - 48 * I * b^2 * c^2 * x^2 * \log(\text{abs}(F))^2 - 48 * \pi * b * c * x * \text{sgn}(F) + 48 * \pi * b * c * x + \\
& 96 * I * b * c * x * \log(\text{abs}(F)) - 96 * I) * e^{(1/2 * I * \pi * b * c * x * \text{sgn}(F) - 1/2 * I * \pi * b * c * x + \\
& 1/2 * I * \pi * a * c * \text{sgn}(F) - 1/2 * I * \pi * a * c) / (8 * \pi^4 * b^4 * c^4 * \text{sgn}(F) + 32 * I * \pi^3 * b^4 \\
& * c^4 * \log(\text{abs}(F)) * \text{sgn}(F) - 48 * \pi^2 * b^4 * c^4 * \log(\text{abs}(F))^2 * \text{sgn}(F) - 32 * I * \pi * b^4 \\
& 4 * c^4 * \log(\text{abs}(F))^3 * \text{sgn}(F) - 8 * \pi^4 * b^4 * c^4 - 32 * I * \pi^3 * b^4 * c^4 * \log(\text{abs}(F)) \\
& + 48 * \pi^2 * b^4 * c^4 * \log(\text{abs}(F))^2 + 32 * I * \pi * b^4 * c^4 * \log(\text{abs}(F))^3 - 16 * b^4 * c^4 \\
& 4 * \log(\text{abs}(F))^4) + (8 * \pi^3 * b^3 * c^3 * x^3 * \text{sgn}(F) - 24 * I * \pi^2 * b^3 * c^3 * x^3 * \log(\text{abs} \\
& (\text{abs}(F)) * \text{sgn}(F) - 24 * \pi * b^3 * c^3 * x^3 * \log(\text{abs}(F))^2 * \text{sgn}(F) - 8 * \pi^3 * b^3 * c^3 * x^3 \\
& 3 + 24 * I * \pi^2 * b^3 * c^3 * x^3 * \log(\text{abs}(F)) + 24 * \pi * b^3 * c^3 * x^3 * \log(\text{abs}(F))^2 - 1 \\
& 6 * I * b^3 * c^3 * x^3 * \log(\text{abs}(F))^3 + 24 * I * \pi^2 * b^2 * c^2 * x^2 * \text{sgn}(F) + 48 * \pi * b^2 * c^2 \\
& 2 * x^2 * \log(\text{abs}(F)) * \text{sgn}(F) - 24 * I * \pi^2 * b^2 * c^2 * x^2 - 48 * \pi * b^2 * c^2 * x^2 * \log(\text{abs} \\
& (\text{abs}(F)) + 48 * I * b^2 * c^2 * x^2 * \log(\text{abs}(F))^2 - 48 * \pi * b * c * x * \text{sgn}(F) + 48 * \pi * b * c * x - \\
& 96 * I * b * c * x * \log(\text{abs}(F)) + 96 * I) * e^{(-1/2 * I * \pi * b * c * x * \text{sgn}(F) + 1/2 * I * \pi * b * c * x \\
& - 1/2 * I * \pi * a * c * \text{sgn}(F) + 1/2 * I * \pi * a * c) / (8 * \pi^4 * b^4 * c^4 * \text{sgn}(F) - 32 * I * \pi^3 * b^4 \\
& 4 * c^4 * \log(\text{abs}(F)) * \text{sgn}(F) - 48 * \pi^2 * b^4 * c^4 * \log(\text{abs}(F))^2 * \text{sgn}(F) + 32 * I * \pi * b^4 \\
& 4 * c^4 * \log(\text{abs}(F))^3 * \text{sgn}(F) - 8 * \pi^4 * b^4 * c^4 + 32 * I * \pi^3 * b^4 * c^4 * \log(\text{abs}(F)) \\
& ) + 48 * \pi^2 * b^4 * c^4 * \log(\text{abs}(F))^2 - 32 * I * \pi * b^4 * c^4 * \log(\text{abs}(F))^3 - 16 * b^4 * c^4 \\
& c^4 * \log(\text{abs}(F))^4) * e^{(b * c * x * \log(\text{abs}(F)) + a * c * \log(\text{abs}(F)) + 3) + 3 * (((\pi^2 \\
& * b^2 * c^2 * d * x^2 * \text{sgn}(F) - \pi^2 * b^2 * c^2 * d * x^2 + 2 * b^2 * c^2 * d * x^2 * \log(\text{abs}(F))^2 \\
& - 4 * b * c * d * x * \log(\text{abs}(F)) + 4 * d) * (3 * \pi^2 * b^3 * c^3 * \log(\text{abs}(F)) * \text{sgn}(F) - 3 * \pi^2 * \\
& b^3 * c^3 * \log(\text{abs}(F)) + 2 * b^3 * c^3 * \log(\text{abs}(F))^3) / ((\pi^3 * b^3 * c^3 * \text{sgn}(F) - 3 * \pi \\
& * b^3 * c^3 * \log(\text{abs}(F))^2 * \text{sgn}(F) - \pi^3 * b^3 * c^3 + 3 * \pi * b^3 * c^3 * \log(\text{abs}(F))^2)^2 \\
& + (3 * \pi^2 * b^3 * c^3 * \log(\text{abs}(F)) * \text{sgn}(F) - 3 * \pi^2 * b^3 * c^3 * \log(\text{abs}(F)) + 2 * b^3 \\
& * c^3 * \log(\text{abs}(F))^3)^2) - 2 * (\pi^3 * b^3 * c^3 * \text{sgn}(F) - 3 * \pi * b^3 * c^3 * \log(\text{abs}(F))^2 * \text{sgn}(F) - \pi^3 * b^3 * c^3 \\
& + 3 * \pi * b^3 * c^3 * \log(\text{abs}(F))^2) * (\pi * b^2 * c^2 * d * x^2 * \log(\text{abs}(F)) * \text{sgn}(F) - \pi * b^2 * c^2 * d * x^2 * \log(\text{abs}(F)) - \pi * b * c \\
& * d * x * \text{sgn}(F) + \pi * b * c * d * x) / ((\pi^3 * b^3 * c^3 * \text{sgn}(F) - 3 * \pi * b^3 * c^3 * \log(\text{abs}(F))^2 * \text{sgn}(F) - \pi^3 * b^3 * c^3 \\
& c^3 + 3 * \pi * b^3 * c^3 * \log(\text{abs}(F))^2)^2 + (3 * \pi^2 * b^3 * c^3 * \log(\text{abs}(F)) * \text{sgn}(F) - \\
& 3 * \pi^2 * b^3 * c^3 * \log(\text{abs}(F)) + 2 * b^3 * c^3 * \log(\text{abs}(F))^3)^2) * \cos(-1/2 * \pi * b * c * x \\
& * \text{sgn}(F) + 1/2 * \pi * b * c * x - 1/2 * \pi * a * c * \text{sgn}(F) + 1/2 * \pi * a * c) + ((\pi^3 * b^3 * c^3 * \text{sgn}(F) - 3 * \pi * b^3 * c^3 * \log(\text{abs}(F))^2 * \text{sgn}(F) - \pi^3 * b^3 * c^3 \\
& c^3 + 3 * \pi * b^3 * c^3 * \log(\text{abs}(F))^2) * (\pi^2 * b^2 * c^2 * d * x^2 * \text{sgn}(F) - \pi^2 * b^2 * c^2 * d * x^2 + 2 * b^2 * c^2 * d * x
\end{aligned}$$

$$\begin{aligned}
& ^2 \log(\operatorname{abs}(F))^2 - 4*b*c*d*x*\log(\operatorname{abs}(F)) + 4*d)/((\pi^3*b^3*c^3*\operatorname{sgn}(F) - 3*\pi \\
& i*b^3*c^3*\log(\operatorname{abs}(F))^2*\operatorname{sgn}(F) - \pi^3*b^3*c^3 + 3*\pi*i*b^3*c^3*\log(\operatorname{abs}(F))^2) \\
& ^2 + (3*\pi^2*b^3*c^3*\log(\operatorname{abs}(F))*\operatorname{sgn}(F) - 3*\pi^2*b^3*c^3*\log(\operatorname{abs}(F)) + 2*b^3 \\
& c^3*\log(\operatorname{abs}(F))^3)^2) + 2*(3*\pi^2*b^3*c^3*\log(\operatorname{abs}(F))*\operatorname{sgn}(F) - 3*\pi^2*b^3 \\
& *c^3*\log(\operatorname{abs}(F)) + 2*b^3*c^3*\log(\operatorname{abs}(F))^3)*( \pi*b^2*c^2*d*x^2*\log(\operatorname{abs}(F))*\operatorname{sgn}(F) \\
& - \pi*b^2*c^2*d*x^2*\log(\operatorname{abs}(F)) - \pi*b*c*d*x*\operatorname{sgn}(F) + \pi*b*c*d*x)/((\pi^3*b^3*c^3*\operatorname{sgn}(F) - 3*\pi \\
& i*b^3*c^3*\log(\operatorname{abs}(F))^2*\operatorname{sgn}(F) - \pi^3*b^3*c^3 + 3*\pi*i*b^3*c^3*\log(\operatorname{abs}(F))^2)^2 + (3*\pi^2*b^3*c^3*\log(\operatorname{abs}(F))*\operatorname{sgn}(F) - 3*\pi^2*b^3 \\
& *c^3*\log(\operatorname{abs}(F)) + 2*b^3*c^3*\log(\operatorname{abs}(F))^3)^2)*\sin(-1/2*\pi*b*c*x*\operatorname{sgn}(F) + \\
& 1/2*\pi*b*c*x - 1/2*\pi*a*c*\operatorname{sgn}(F) + 1/2*\pi*a*c))*e^{(b*c*x*\log(\operatorname{abs}(F)) + a*c*\log(\operatorname{abs}(F)) + 2) + 1/2*I*((12*I*\pi^2*b^2*c^2*d*x^2*\operatorname{sgn}(F) - 24*\pi*b^2*c^2*d \\
& *x^2*\log(\operatorname{abs}(F))*\operatorname{sgn}(F) - 12*I*\pi^2*b^2*c^2*d*x^2 + 24*\pi*b^2*c^2*d*x^2*\log(\operatorname{abs}(F)) + 24*I*b^2*c^2*d*x^2*\log(\operatorname{abs}(F))^2 + 24*\pi*b*c*d*x*\operatorname{sgn}(F) - 24*\pi*b \\
& c*d*x - 48*I*b*c*d*x*\log(\operatorname{abs}(F)) + 48*I*d)*e^{(1/2*I*\pi*b*c*x*\operatorname{sgn}(F) - 1/2 \\
& *I*\pi*b*c*x + 1/2*I*\pi*a*c*\operatorname{sgn}(F) - 1/2*I*\pi*a*c)/(-4*I*\pi^3*b^3*c^3*\operatorname{sgn}(F) \\
& + 12*\pi^2*b^3*c^3*\log(\operatorname{abs}(F))*\operatorname{sgn}(F) + 12*I*\pi*b^3*c^3*\log(\operatorname{abs}(F))^2*\operatorname{sgn}(F) \\
& ) + 4*I*\pi^3*b^3*c^3 - 12*\pi^2*b^3*c^3*\log(\operatorname{abs}(F)) - 12*I*\pi*b^3*c^3*\log(\operatorname{abs}(F))^2 + 8*b^3*c^3*\log(\operatorname{abs}(F))^3 - (12*I*\pi^2*b^2*c^2*d*x^2*\operatorname{sgn}(F) + 24*\pi \\
& i*b^2*c^2*d*x^2*\log(\operatorname{abs}(F))*\operatorname{sgn}(F) - 12*I*\pi^2*b^2*c^2*d*x^2 - 24*\pi*b^2*c^2 \\
& *d*x^2*\log(\operatorname{abs}(F)) + 24*I*b^2*c^2*d*x^2*\log(\operatorname{abs}(F))^2 - 24*\pi*b*c*d*x*\operatorname{sgn}(F) + 24*\pi*b*c*d*x - 48*I*b*c*d*x*\log(\operatorname{abs}(F)) + 48*I*d)*e^{(-1/2*I*\pi*b*c*x*\operatorname{sgn}(F) + 1/2*I*\pi*b*c*x - 1/2*I*\pi*a*c*\operatorname{sgn}(F) + 1/2*I*\pi*a*c)/ (4*I*\pi^3*b^3 \\
& *c^3*\operatorname{sgn}(F) + 12*\pi^2*b^3*c^3*\log(\operatorname{abs}(F))*\operatorname{sgn}(F) - 12*I*\pi*b^3*c^3*\log(\operatorname{abs}(F))^2*\operatorname{sgn}(F) - 4*I*\pi^3*b^3*c^3 - 12*\pi^2*b^3*c^3*\log(\operatorname{abs}(F)) + 12*I*\pi*b^3 \\
& *c^3*\log(\operatorname{abs}(F))^2 + 8*b^3*c^3*\log(\operatorname{abs}(F))^3)}*e^{(b*c*x*\log(\operatorname{abs}(F)) + a*c*\log(\operatorname{abs}(F)) + 2) + 3*(2*((\pi*b*c*d^2*x*\operatorname{sgn}(F) - \pi*b*c*d^2*x)*( \pi*b^2*c^2*\log(\operatorname{abs}(F))*\operatorname{sgn}(F) - \pi*b^2*c^2*\log(\operatorname{abs}(F))) / ((\pi^2*b^2*c^2*\operatorname{sgn}(F) - \pi^2*b^2 \\
& *c^2 + 2*b^2*c^2*\log(\operatorname{abs}(F))^2)^2 + 4*(\pi*b^2*c^2*\log(\operatorname{abs}(F))*\operatorname{sgn}(F) - \pi*b^2 \\
& *c^2*\log(\operatorname{abs}(F)))^2) + (\pi^2*b^2*c^2*\operatorname{sgn}(F) - \pi^2*b^2*c^2 + 2*b^2*c^2*\log(\operatorname{abs}(F))^2)*(b*c*d^2*x*\log(\operatorname{abs}(F)) - d^2) / ((\pi^2*b^2*c^2*\operatorname{sgn}(F) - \pi^2*b^2 \\
& *c^2 + 2*b^2*c^2*\log(\operatorname{abs}(F))^2)^2 + 4*(\pi*b^2*c^2*\log(\operatorname{abs}(F))*\operatorname{sgn}(F) - \pi*b^2 \\
& *c^2*\log(\operatorname{abs}(F)))^2)*\cos(-1/2*\pi*b*c*x*\operatorname{sgn}(F) + 1/2*\pi*b*c*x - 1/2*\pi*a*c \\
& * \operatorname{sgn}(F) + 1/2*\pi*a*c) + ((\pi^2*b^2*c^2*\operatorname{sgn}(F) - \pi^2*b^2*c^2 + 2*b^2*c^2*\log(\operatorname{abs}(F))^2)*( \pi*b*c*d^2*x*\operatorname{sgn}(F) - \pi*b*c*d^2*x) / ((\pi^2*b^2*c^2*\operatorname{sgn}(F) - \pi^2*b^2*c^2 + 2*b^2*c^2*\log(\operatorname{abs}(F))^2)^2 + 4*(\pi*b^2*c^2*\log(\operatorname{abs}(F))*\operatorname{sgn}(F) - \pi*b^2*c^2*\log(\operatorname{abs}(F)))^2) - 4*(\pi*b^2*c^2*\log(\operatorname{abs}(F))*\operatorname{sgn}(F) - \pi*b^2*c^2*\log(\operatorname{abs}(F)))*(b*c*d^2*x*\log(\operatorname{abs}(F)) - d^2) / ((\pi^2*b^2*c^2*\operatorname{sgn}(F) - \pi^2*b^2*c^2 + 2*b^2*c^2*\log(\operatorname{abs}(F))^2)^2 + 4*(\pi*b^2*c^2*\log(\operatorname{abs}(F))*\operatorname{sgn}(F) - \pi*b^2*c^2*\log(\operatorname{abs}(F)))^2)*\sin(-1/2*\pi*b*c*x*\operatorname{sgn}(F) + 1/2*\pi*b*c*x - 1/2*\pi \\
& i*a*c*\operatorname{sgn}(F) + 1/2*\pi*a*c))*e^{(b*c*x*\log(\operatorname{abs}(F)) + a*c*\log(\operatorname{abs}(F)) + 1) - 1 \\
& /2*I*((6*\pi*b*c*d^2*x*\operatorname{sgn}(F) - 6*\pi*b*c*d^2*x - 12*I*b*c*d^2*x*\log(\operatorname{abs}(F)) \\
& + 12*I*d^2)*e^{(1/2*I*\pi*b*c*x*\operatorname{sgn}(F) - 1/2*I*\pi*b*c*x + 1/2*I*\pi*a*c*\operatorname{sgn}(F) \\
& - 1/2*I*\pi*a*c) / (2*\pi^2*b^2*c^2*\operatorname{sgn}(F) + 4*I*\pi*b^2*c^2*\log(\operatorname{abs}(F))*\operatorname{sgn}(F) \\
& - 2*\pi^2*b^2*c^2 - 4*I*\pi*b^2*c^2*\log(\operatorname{abs}(F)) + 4*b^2*c^2*\log(\operatorname{abs}(F))^2) + \\
& (6*\pi*b*c*d^2*x*\operatorname{sgn}(F) - 6*\pi*b*c*d^2*x + 12*I*b*c*d^2*x*\log(\operatorname{abs}(F)) - 12*
\end{aligned}$$

$$\begin{aligned}
& I*d^2)*e^{(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c)/(2*pi^2*b^2*c^2*sgn(F) - 4*I*pi*b^2*c^2*log(abs(F))*sgn(F) - 2*pi^2*b^2*c^2 + 4*I*pi*b^2*c^2*log(abs(F)) + 4*b^2*c^2*log(abs(F))^2)}*e^{(b*c*x*log(abs(F)) + a*c*log(abs(F)) + 1) + 2*(2*b*c*d^3*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)*log(abs(F))/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c)^2) - (pi*b*c*sgn(F) - pi*b*c)*d^3*sin(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c)^2))*e^{(b*c*x*log(abs(F)) + a*c*log(abs(F))) - 1/2*I*(-2*I*d^3*e^{(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*a*c)/(I*pi*b*c*sgn(F) - I*pi*b*c + 2*b*c*log(abs(F))) + 2*I*d^3*e^{(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c)/(-I*pi*b*c*sgn(F) + I*pi*b*c + 2*b*c*log(abs(F)))})*e^{(b*c*x*log(abs(F)) + a*c*log(abs(F)))}
\end{aligned}$$

### 3.4 $\int F^{c(a+bx)}(d+ex)^2 dx$

**Optimal.** Leaf size=79

$$-\frac{2e(d+ex)F^{c(a+bx)}}{b^2c^2\log^2(F)} + \frac{2e^2F^{c(a+bx)}}{b^3c^3\log^3(F)} + \frac{(d+ex)^2F^{c(a+bx)}}{bc\log(F)}$$

[Out]  $(2e^2F^{c(a+bx)})/(b^3c^3\text{Log}[F]^3) - (2eF^{c(a+bx)})(d+ex)/(b^2c^2\text{Log}[F]^2) + (F^{c(a+bx)})(d+ex)^2/(bc\text{Log}[F])$

**Rubi [A]** time = 0.042787, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$ , Rules used = {2176, 2194}

$$-\frac{2e(d+ex)F^{c(a+bx)}}{b^2c^2\log^2(F)} + \frac{2e^2F^{c(a+bx)}}{b^3c^3\log^3(F)} + \frac{(d+ex)^2F^{c(a+bx)}}{bc\log(F)}$$

Antiderivative was successfully verified.

[In] Int[F^{c(a+bx)}(d+ex)^2,x]

[Out]  $(2e^2F^{c(a+bx)})/(b^3c^3\text{Log}[F]^3) - (2eF^{c(a+bx)})(d+ex)/(b^2c^2\text{Log}[F]^2) + (F^{c(a+bx)})(d+ex)^2/(bc\text{Log}[F])$

#### Rule 2176

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[((c + d*x)^m*(b*F^(g*(e + f*x)))^n)/(f*g*n*Log[F]), x] - Dist[(d*m)/(f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m] && !$UseGamma == True
```

#### Rule 2194

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))^(n_.), x_Symbol] :> Simp[(F^(c*(a + b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

#### Rubi steps

$$\begin{aligned} \int F^{c(a+bx)}(d+ex)^2 dx &= \frac{F^{c(a+bx)}(d+ex)^2}{bc \log(F)} - \frac{(2e) \int F^{c(a+bx)}(d+ex) dx}{bc \log(F)} \\ &= -\frac{2eF^{c(a+bx)}(d+ex)}{b^2c^2 \log^2(F)} + \frac{F^{c(a+bx)}(d+ex)^2}{bc \log(F)} + \frac{(2e^2) \int F^{c(a+bx)} dx}{b^2c^2 \log^2(F)} \\ &= \frac{2e^2 F^{c(a+bx)}}{b^3c^3 \log^3(F)} - \frac{2eF^{c(a+bx)}(d+ex)}{b^2c^2 \log^2(F)} + \frac{F^{c(a+bx)}(d+ex)^2}{bc \log(F)} \end{aligned}$$

**Mathematica [A]** time = 0.0660184, size = 56, normalized size = 0.71

$$\frac{F^{c(a+bx)} \left( b^2c^2 \log^2(F)(d+ex)^2 - 2bce \log(F)(d+ex) + 2e^2 \right)}{b^3c^3 \log^3(F)}$$

Antiderivative was successfully verified.

[In] Integrate[F^(c\*(a + b\*x))\*(d + e\*x)^2, x]

[Out] (F^(c\*(a + b\*x))\*(2\*e^2 - 2\*b\*c\*e\*(d + e\*x)\*Log[F] + b^2\*c^2\*(d + e\*x)^2\*Log[F]^2))/(b^3\*c^3\*Log[F]^3)

**Maple [A]** time = 0.007, size = 91, normalized size = 1.2

$$\frac{(e^2x^2b^2c^2(\ln(F))^2 + 2(\ln(F))^2b^2c^2dex + b^2c^2(\ln(F))^2d^2 - 2\ln(F)bce^2x - 2\ln(F)bced + 2e^2)F^{c(bx+a)}}{b^3c^3(\ln(F))^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(c\*(b\*x+a))\*(e\*x+d)^2, x)

[Out] (e^2\*x^2\*b^2\*c^2\*ln(F)^2+2\*ln(F)^2\*b^2\*c^2\*d\*e\*x+b^2\*c^2\*ln(F)^2\*d^2-2\*ln(F)\*b\*c\*e^2\*x-2\*ln(F)\*b\*c\*e\*d+2\*e^2)\*F^(c\*(b\*x+a))/b^3/c^3/ln(F)^3

**Maxima [A]** time = 1.00751, size = 166, normalized size = 2.1

$$\frac{F^{bcx+ac}d^2}{bc \log(F)} + \frac{2(F^{ac}bcx \log(F) - F^{ac})F^{bcx}de}{b^2c^2 \log(F)^2} + \frac{(F^{ac}b^2c^2x^2 \log(F)^2 - 2F^{ac}bcx \log(F) + 2F^{ac})F^{bcx}e^2}{b^3c^3 \log(F)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(c\*(b\*x+a))\*(e\*x+d)^2,x, algorithm="maxima")

[Out]  $F^{(b*c*x + a*c)*d^2/(b*c*\log(F)) + 2*(F^{(a*c)*b*c*x*\log(F)} - F^{(a*c)})*F^{(b*c*x)*d*e/(b^2*c^2*\log(F)^2) + (F^{(a*c)*b^2*c^2*x^2*\log(F)^2} - 2*F^{(a*c)*b*c*x*\log(F)} + 2*F^{(a*c)})*F^{(b*c*x)*e^2/(b^3*c^3*\log(F)^3)}$

**Fricas [A]** time = 1.54573, size = 186, normalized size = 2.35

$$\frac{\left(\left(b^2 c^2 e^2 x^2 + 2 b^2 c^2 d e x + b^2 c^2 d^2\right) \log (F)^2 + 2 e^2 - 2\left(b c e^2 x + b c d e\right) \log (F)\right) F^{b c x+a c}}{b^3 c^3 \log (F)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(c\*(b\*x+a))\*(e\*x+d)^2,x, algorithm="fricas")

[Out]  $\left(\left(b^2 c^2 e^2 x^2 + 2 b^2 c^2 d e x + b^2 c^2 d^2\right) * \log (F)^2 + 2 e^2 - 2\left(b c e^2 x + b c d e\right) * \log (F)\right) * F^{(b c x+a c)} / \left(b^3 c^3 * \log (F)^3\right)$

**Sympy [A]** time = 0.235673, size = 133, normalized size = 1.68

$$\begin{cases} \frac{F^{c(a+bx)}\left(b^2 c^2 d^2 \log (F)^2 + 2 b^2 c^2 d e x \log (F)^2 + b^2 c^2 e^2 x^2 \log (F)^2 - 2 b c d e \log (F) - 2 b c e^2 x \log (F) + 2 e^2\right)}{b^3 c^3 \log (F)^3} & \text{for } b^3 c^3 \log (F)^3 \neq 0 \\ d^2 x + d e x^2 + \frac{e^2 x^3}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(c\*(b\*x+a))\*(e\*x+d)\*\*2,x)

[Out] Piecewise((F\*\*(c\*(a + b\*x))\*(b\*\*2\*c\*\*2\*d\*\*2\*log(F)\*\*2 + 2\*b\*\*2\*c\*\*2\*d\*e\*x\*log(F)\*\*2 + b\*\*2\*c\*\*2\*e\*\*2\*x\*\*2\*log(F)\*\*2 - 2\*b\*c\*d\*e\*log(F) - 2\*b\*c\*e\*\*2\*x\*log(F) + 2\*e\*\*2)/(b\*\*3\*c\*\*3\*log(F)\*\*3), Ne(b\*\*3\*c\*\*3\*log(F)\*\*3, 0)), (d\*\*2\*x + d\*e\*x\*\*2 + e\*\*2\*x\*\*3/3, True))

**Giac [C]** time = 1.82994, size = 3367, normalized size = 42.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(c\*(b\*x+a))\*(e\*x+d)^2,x, algorithm="giac")

[Out] (((3\*pi^2\*b^3\*c^3\*log(abs(F))\*sgn(F) - 3\*pi^2\*b^3\*c^3\*log(abs(F)) + 2\*b^3\*c^3\*log(abs(F))^3)\*(pi^2\*b^2\*c^2\*x^2\*sgn(F) - pi^2\*b^2\*c^2\*x^2 + 2\*b^2\*c^2\*x^2\*log(abs(F))^2 - 4\*b\*c\*x\*log(abs(F)) + 4)/((pi^3\*b^3\*c^3\*sgn(F) - 3\*pi\*b^3\*c^3\*log(abs(F))^2\*sgn(F) - pi^3\*b^3\*c^3 + 3\*pi\*b^3\*c^3\*log(abs(F))^2)^2 + (3\*pi^2\*b^3\*c^3\*log(abs(F))\*sgn(F) - 3\*pi^2\*b^3\*c^3\*log(abs(F)) + 2\*b^3\*c^3\*log(abs(F))^3)^2) - 2\*(pi^3\*b^3\*c^3\*sgn(F) - 3\*pi\*b^3\*c^3\*log(abs(F))^2\*sgn(F) - pi^3\*b^3\*c^3 + 3\*pi\*b^3\*c^3\*log(abs(F))^2)\*(pi\*b^2\*c^2\*x^2\*log(abs(F))\*sgn(F) - pi\*b^2\*c^2\*x^2\*log(abs(F)) - pi\*b\*c\*x\*sgn(F) + pi\*b\*c\*x)/((pi^3\*b^3\*c^3\*sgn(F) - 3\*pi\*b^3\*c^3\*log(abs(F))^2\*sgn(F) - pi^3\*b^3\*c^3 + 3\*pi\*b^3\*c^3\*log(abs(F))^2)^2 + (3\*pi^2\*b^3\*c^3\*log(abs(F))\*sgn(F) - 3\*pi^2\*b^3\*c^3\*log(abs(F)) + 2\*b^3\*c^3\*log(abs(F))^3)^2))\*cos(-1/2\*pi\*b\*c\*x\*sgn(F) + 1/2\*pi\*b\*c\*x - 1/2\*pi\*a\*c\*sgn(F) + 1/2\*pi\*a\*c) + ((pi^3\*b^3\*c^3\*sgn(F) - 3\*pi\*b^3\*c^3\*log(abs(F))^2\*sgn(F) - pi^3\*b^3\*c^3 + 3\*pi\*b^3\*c^3\*log(abs(F))^2)\*(pi^2\*b^2\*c^2\*x^2\*sgn(F) - pi^2\*b^2\*c^2\*x^2 + 2\*b^2\*c^2\*x^2\*log(abs(F))^2 - 4\*b\*c\*x\*log(abs(F)) + 4)/((pi^3\*b^3\*c^3\*sgn(F) - 3\*pi\*b^3\*c^3\*log(abs(F))^2\*sgn(F) - pi^3\*b^3\*c^3 + 3\*pi\*b^3\*c^3\*log(abs(F))^2)^2 + (3\*pi^2\*b^3\*c^3\*log(abs(F))\*sgn(F) - 3\*pi^2\*b^3\*c^3\*log(abs(F)) + 2\*b^3\*c^3\*log(abs(F))^3)^2) + 2\*(3\*pi^2\*b^3\*c^3\*log(abs(F))\*sgn(F) - 3\*pi^2\*b^3\*c^3\*log(abs(F)) + 2\*b^3\*c^3\*log(abs(F))^3)\*(pi\*b^2\*c^2\*x^2\*log(abs(F))\*sgn(F) - pi\*b^2\*c^2\*x^2\*log(abs(F)) - pi\*b\*c\*x\*sgn(F) + pi\*b\*c\*x)/((pi^3\*b^3\*c^3\*sgn(F) - 3\*pi\*b^3\*c^3\*log(abs(F))^2\*sgn(F) - pi^3\*b^3\*c^3 + 3\*pi\*b^3\*c^3\*log(abs(F))^2)^2 + (3\*pi^2\*b^3\*c^3\*log(abs(F))\*sgn(F) - 3\*pi^2\*b^3\*c^3\*log(abs(F)) + 2\*b^3\*c^3\*log(abs(F))^3)^2))\*sin(-1/2\*pi\*b\*c\*x\*sgn(F) + 1/2\*pi\*b\*c\*x - 1/2\*pi\*a\*c\*sgn(F) + 1/2\*pi\*a\*c))\*e^(b\*c\*x\*log(abs(F)) + a\*c\*log(abs(F)) + 2) + 1/2\*I\*((4\*I\*pi^2\*b^2\*c^2\*x^2\*sgn(F) - 8\*pi\*b^2\*c^2\*x^2\*log(abs(F))\*sgn(F) - 4\*I\*pi^2\*b^2\*c^2\*x^2 + 8\*pi\*b^2\*c^2\*x^2\*log(abs(F)) + 8\*I\*b^2\*c^2\*x^2\*log(abs(F))^2 + 8\*pi\*b\*c\*x\*sgn(F) - 8\*pi\*b\*c\*x - 16\*I\*b\*c\*x\*log(abs(F)) + 16\*I)\*e^(1/2\*I\*pi\*b\*c\*x\*sgn(F) - 1/2\*I\*pi\*b\*c\*x + 1/2\*I\*pi\*a\*c\*sgn(F) - 1/2\*I\*pi\*a\*c)/(-4\*I\*pi^3\*b^3\*c^3\*sgn(F) + 12\*pi^2\*b^3\*c^3\*log(abs(F))\*sgn(F) + 12\*I\*pi\*b^3\*c^3\*log(abs(F))^2\*sgn(F) + 4\*I\*pi^3\*b^3\*c^3 - 12\*pi^2\*b^3\*c^3\*log(abs(F)) - 12\*I\*pi\*b^3\*c^3\*log(abs(F))^2 + 8\*b^3\*c^3\*log(abs(F))^3) - (4\*I\*pi^2\*b^2\*c^2\*x^2\*sgn(F) + 8\*pi\*b^2\*c^2\*x^2\*log(abs(F))\*sgn(F) - 4\*I\*pi^2\*b^2\*c^2\*x^2 - 8\*pi\*b^2\*c^2\*x^2\*log(abs(F)) + 8\*I\*b^2\*c^2\*x^2\*log(abs(F))^2 - 8\*pi\*b\*c\*x\*sgn(F) + 8\*pi\*b\*c\*x - 16\*I\*b\*c\*x\*log(abs(F)) + 16\*I)\*e^(-1/2\*I\*pi\*b\*c\*x\*sgn(F) + 1/2\*I\*pi\*b\*c\*x - 1/2\*I\*pi\*a\*c\*sgn(F) + 1/2\*I\*pi\*a\*c)/(4\*I\*pi^3\*b^3\*c^3\*sgn(F) + 12\*pi^2\*b^3\*c^3\*log(abs(F))\*sgn(F) - 12\*I\*pi\*b^3\*c^3\*log(abs(F))^2\*sgn(F) - 4\*I\*pi^3\*b^3\*c^3 - 12\*pi^2\*b^3\*c^3\*log(abs(F)) + 12\*I\*pi\*b^3\*c^3\*log(abs(F))^2 + 8\*b^3\*c^3\*log(abs(F))^3))\*e^(b\*c\*x\*log(abs(F)) + a\*c\*log(abs(F)) + 2) + 2\*(2\*((pi\*b^2\*c^2\*log(abs(F))\*sgn(F) - pi\*b^2\*c^2\*log(abs(F)))\*(pi\*b\*c\*d\*x\*sgn(F) - pi\*b\*c\*d\*x)/((pi^2\*b^2\*c^2\*sgn(F) - pi^2\*b^2\*c^2 + 2



$$\begin{aligned}
& *b^2*c^2*\log(\text{abs}(F))^2)^2 + 4*(\pi*b^2*c^2*\log(\text{abs}(F))*\text{sgn}(F) - \pi*b^2*c^2*\log(\text{abs}(F)))^2) + (\pi^2*b^2*c^2*\text{sgn}(F) - \pi^2*b^2*c^2 + 2*b^2*c^2*\log(\text{abs}(F)))^2)*(b*c*d*x*\log(\text{abs}(F)) - d)/((\pi^2*b^2*c^2*\text{sgn}(F) - \pi^2*b^2*c^2 + 2*b^2*c^2*\log(\text{abs}(F))^2)^2 + 4*(\pi*b^2*c^2*\log(\text{abs}(F))*\text{sgn}(F) - \pi*b^2*c^2*\log(\text{abs}(F)))^2))*\cos(-1/2*\pi*b*c*x*\text{sgn}(F) + 1/2*\pi*b*c*x - 1/2*\pi*a*c*\text{sgn}(F) + 1/2*\pi*a*c) + ((\pi^2*b^2*c^2*\text{sgn}(F) - \pi^2*b^2*c^2 + 2*b^2*c^2*\log(\text{abs}(F)))^2)*( \pi*b*c*d*x*\text{sgn}(F) - \pi*b*c*d*x)/((\pi^2*b^2*c^2*\text{sgn}(F) - \pi^2*b^2*c^2 + 2*b^2*c^2*\log(\text{abs}(F))^2)^2 + 4*(\pi*b^2*c^2*\log(\text{abs}(F))*\text{sgn}(F) - \pi*b^2*c^2*\log(\text{abs}(F)))^2) - 4*(\pi*b^2*c^2*\log(\text{abs}(F))*\text{sgn}(F) - \pi*b^2*c^2*\log(\text{abs}(F))))*(b*c*d*x*\log(\text{abs}(F)) - d)/((\pi^2*b^2*c^2*\text{sgn}(F) - \pi^2*b^2*c^2 + 2*b^2*c^2*\log(\text{abs}(F))^2)^2 + 4*(\pi*b^2*c^2*\log(\text{abs}(F))*\text{sgn}(F) - \pi*b^2*c^2*\log(\text{abs}(F))))^2))*\sin(-1/2*\pi*b*c*x*\text{sgn}(F) + 1/2*\pi*b*c*x - 1/2*\pi*a*c*\text{sgn}(F) + 1/2*\pi*a*c))*e^{(b*c*x*\log(\text{abs}(F)) + a*c*\log(\text{abs}(F)) + 1) - 1/2*I*((4*\pi*b*c*d*x*\text{sgn}(F) - 4*\pi*b*c*d*x - 8*I*b*c*d*x*\log(\text{abs}(F)) + 8*I*d)*e^{(1/2*I*\pi*b*c*x*\text{sgn}(F) - 1/2*I*\pi*b*c*x + 1/2*I*\pi*a*c*\text{sgn}(F) - 1/2*I*\pi*a*c)/(2*\pi^2*b^2*c^2*\text{sgn}(F) + 4*I*\pi*b^2*c^2*\log(\text{abs}(F))*\text{sgn}(F) - 2*\pi^2*b^2*c^2 - 4*I*\pi*b^2*c^2*\log(\text{abs}(F)) + 4*b^2*c^2*\log(\text{abs}(F))^2) + (4*\pi*b*c*d*x*\text{sgn}(F) - 4*\pi*b*c*d*x + 8*I*b*c*d*x*\log(\text{abs}(F)) - 8*I*d)*e^{(-1/2*I*\pi*b*c*x*\text{sgn}(F) + 1/2*I*\pi*b*c*x - 1/2*I*\pi*a*c*\text{sgn}(F) + 1/2*I*\pi*a*c)/(2*\pi^2*b^2*c^2*\text{sgn}(F) - 4*I*\pi*b^2*c^2*\log(\text{abs}(F))*\text{sgn}(F) - 2*\pi^2*b^2*c^2 + 4*I*\pi*b^2*c^2*\log(\text{abs}(F))) + 4*b^2*c^2*\log(\text{abs}(F))^2))*e^{(b*c*x*\log(\text{abs}(F)) + a*c*\log(\text{abs}(F)) + 1) + 2*(2*b*c*d^2*\cos(-1/2*\pi*b*c*x*\text{sgn}(F) + 1/2*\pi*b*c*x - 1/2*\pi*a*c*\text{sgn}(F) + 1/2*\pi*a*c)*\log(\text{abs}(F))/(4*b^2*c^2*\log(\text{abs}(F))^2 + (\pi*b*c*\text{sgn}(F) - \pi*b*c)^2) - (\pi*b*c*\text{sgn}(F) - \pi*b*c)*d^2*\sin(-1/2*\pi*b*c*x*\text{sgn}(F) + 1/2*\pi*b*c*x - 1/2*\pi*a*c*\text{sgn}(F) + 1/2*\pi*a*c)/(4*b^2*c^2*\log(\text{abs}(F))^2 + (\pi*b*c*\text{sgn}(F) - \pi*b*c)^2))*e^{(b*c*x*\log(\text{abs}(F)) + a*c*\log(\text{abs}(F))) - 1/2*I*(-2*I*d^2*e^{(1/2*I*\pi*b*c*x*\text{sgn}(F) - 1/2*I*\pi*b*c*x + 1/2*I*\pi*a*c*\text{sgn}(F) - 1/2*I*\pi*a*c)/(I*\pi*b*c*\text{sgn}(F) - I*\pi*b*c + 2*b*c*\log(\text{abs}(F))) + 2*I*d^2*e^{(-1/2*I*\pi*b*c*x*\text{sgn}(F) + 1/2*I*\pi*b*c*x - 1/2*I*\pi*a*c*\text{sgn}(F) + 1/2*I*\pi*a*c)/(-I*\pi*b*c*\text{sgn}(F) + I*\pi*b*c + 2*b*c*\log(\text{abs}(F)))})*e^{(b*c*x*\log(\text{abs}(F)) + a*c*\log(\text{abs}(F)))}
\end{aligned}$$

### 3.5 $\int F^{c(a+bx)}(d+ex) dx$

**Optimal.** Leaf size=48

$$\frac{(d+ex)F^{c(a+bx)}}{bc \log(F)} - \frac{eF^{c(a+bx)}}{b^2c^2 \log^2(F)}$$

[Out]  $-\left(\frac{eF^{c(a+bx)}}{b^2c^2 \log^2(F)}\right) + \frac{F^{c(a+bx)}(d+ex)}{bc \log(F)}$

**Rubi [A]** time = 0.0166666, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$ , Rules used = {2176, 2194}

$$\frac{(d+ex)F^{c(a+bx)}}{bc \log(F)} - \frac{eF^{c(a+bx)}}{b^2c^2 \log^2(F)}$$

Antiderivative was successfully verified.

[In] Int[F<sup>c(a+bx)</sup>(d+ex), x]

[Out]  $-\left(\frac{eF^{c(a+bx)}}{b^2c^2 \log^2(F)}\right) + \frac{F^{c(a+bx)}(d+ex)}{bc \log(F)}$

#### Rule 2176

```
Int[((b_.)*(F_)^((g_.)*((e_.)+(f_.)*(x_))))^(n_.)*((c_.)+(d_.)*(x_))^(m_.), x_Symbol] :> Simp[((c+d*x)^m*(b*F^(g*(e+f*x)))^n)/(f*g*n*Log[F]), x] - Dist[(d*m)/(f*g*n*Log[F]), Int[(c+d*x)^(m-1)*(b*F^(g*(e+f*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m] && !$UseGamma == True
```

#### Rule 2194

```
Int[((F_)^((c_.)*((a_.)+(b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a+b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

#### Rubi steps

$$\int F^{c(a+bx)}(d+ex) dx = \frac{F^{c(a+bx)}(d+ex)}{bc \log(F)} - \frac{e \int F^{c(a+bx)} dx}{bc \log(F)}$$

$$= -\frac{eF^{c(a+bx)}}{b^2c^2 \log^2(F)} + \frac{F^{c(a+bx)}(d+ex)}{bc \log(F)}$$

**Mathematica [A]** time = 0.0374469, size = 34, normalized size = 0.71

$$\frac{F^{c(a+bx)}(bc \log(F)(d+ex) - e)}{b^2c^2 \log^2(F)}$$

Antiderivative was successfully verified.

[In] Integrate[F^(c\*(a + b\*x))\*(d + e\*x), x]

[Out] (F^(c\*(a + b\*x))\*(-e + b\*c\*(d + e\*x)\*Log[F]))/(b^2\*c^2\*Log[F]^2)

**Maple [A]** time = 0.003, size = 38, normalized size = 0.8

$$\frac{(\ln(F) bcex + \ln(F) bcd - e) F^{c(bx+a)}}{b^2c^2 (\ln(F))^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(c\*(b\*x+a))\*(e\*x+d), x)

[Out] (ln(F)\*b\*c\*e\*x+ln(F)\*b\*c\*d-e)\*F^(c\*(b\*x+a))/b^2/c^2/ln(F)^2

**Maxima [A]** time = 0.975917, size = 81, normalized size = 1.69

$$\frac{F^{bcx+ac}d}{bc \log(F)} + \frac{(F^{ac}bcx \log(F) - F^{ac})F^{bcx}e}{b^2c^2 \log(F)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(c\*(b\*x+a))\*(e\*x+d), x, algorithm="maxima")

[Out]  $F^{(b*c*x + a*c)*d/(b*c*\log(F)) + (F^{(a*c)}*b*c*x*\log(F) - F^{(a*c)})*F^{(b*c*x)}*e/(b^2*c^2*\log(F)^2)$

**Fricas [A]** time = 1.49956, size = 90, normalized size = 1.88

$$\frac{((bcex + bcd) \log(F) - e)F^{bcx+ac}}{b^2c^2 \log(F)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(c*(b*x+a))*(e*x+d),x, algorithm="fricas")`

[Out]  $((b*c*e*x + b*c*d)*\log(F) - e)*F^{(b*c*x + a*c)}/(b^2*c^2*\log(F)^2)$

**Sympy [A]** time = 0.234251, size = 60, normalized size = 1.25

$$\begin{cases} \frac{F^{c(a+bx)}(bcd \log(F) + bcex \log(F) - e)}{b^2c^2 \log(F)^2} & \text{for } b^2c^2 \log(F)^2 \neq 0 \\ dx + \frac{ex^2}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F**(c*(b*x+a))*(e*x+d),x)`

[Out] `Piecewise((F**(c*(a + b*x))*(b*c*d*log(F) + b*c*e*x*log(F) - e)/(b**2*c**2*log(F)**2), Ne(b**2*c**2*log(F)**2, 0)), (d*x + e*x**2/2, True))`

**Giac [C]** time = 7.06795, size = 1462, normalized size = 30.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(c*(b*x+a))*(e*x+d),x, algorithm="giac")`

[Out]  $(2*((\pi*b^2*c^2*\log(\text{abs}(F))*\text{sgn}(F) - \pi*b^2*c^2*\log(\text{abs}(F)))*(\pi*b*c*x*\text{sgn}(F) - \pi*b*c*x))/((\pi^2*b^2*c^2*\text{sgn}(F) - \pi^2*b^2*c^2 + 2*b^2*c^2*\log(\text{abs}(F)))$

$$\begin{aligned}
&^2)^2 + 4*(\pi*b^2*c^2*\log(\text{abs}(F))*\text{sgn}(F) - \pi*b^2*c^2*\log(\text{abs}(F)))^2) + (\pi \\
&^2*b^2*c^2*\text{sgn}(F) - \pi^2*b^2*c^2 + 2*b^2*c^2*\log(\text{abs}(F))^2)*(b*c*x*\log(\text{abs}( \\
&F)) - 1)/((\pi^2*b^2*c^2*\text{sgn}(F) - \pi^2*b^2*c^2 + 2*b^2*c^2*\log(\text{abs}(F))^2)^2 \\
&+ 4*(\pi*b^2*c^2*\log(\text{abs}(F))*\text{sgn}(F) - \pi*b^2*c^2*\log(\text{abs}(F)))^2))*\cos(-1/2*\pi \\
&i*b*c*x*\text{sgn}(F) + 1/2*\pi*b*c*x - 1/2*\pi*a*c*\text{sgn}(F) + 1/2*\pi*a*c) + ((\pi^2*b^ \\
&2*c^2*\text{sgn}(F) - \pi^2*b^2*c^2 + 2*b^2*c^2*\log(\text{abs}(F))^2)*(\pi*b*c*x*\text{sgn}(F) - \pi \\
&i*b*c*x)/((\pi^2*b^2*c^2*\text{sgn}(F) - \pi^2*b^2*c^2 + 2*b^2*c^2*\log(\text{abs}(F))^2)^2 \\
&+ 4*(\pi*b^2*c^2*\log(\text{abs}(F))*\text{sgn}(F) - \pi*b^2*c^2*\log(\text{abs}(F)))^2) - 4*(\pi*b^2 \\
&*c^2*\log(\text{abs}(F))*\text{sgn}(F) - \pi*b^2*c^2*\log(\text{abs}(F)))*(b*c*x*\log(\text{abs}(F)) - 1)/( \\
&(\pi^2*b^2*c^2*\text{sgn}(F) - \pi^2*b^2*c^2 + 2*b^2*c^2*\log(\text{abs}(F))^2)^2 + 4*(\pi*b^ \\
&2*c^2*\log(\text{abs}(F))*\text{sgn}(F) - \pi*b^2*c^2*\log(\text{abs}(F)))^2))*\sin(-1/2*\pi*b*c*x*\text{sg} \\
&n(F) + 1/2*\pi*b*c*x - 1/2*\pi*a*c*\text{sgn}(F) + 1/2*\pi*a*c))*e^{(b*c*x*\log(\text{abs}(F)) \\
&+ a*c*\log(\text{abs}(F)) + 1) - 1/2*I*((2*\pi*b*c*x*\text{sgn}(F) - 2*\pi*b*c*x - 4*I*b*c* \\
&x*\log(\text{abs}(F)) + 4*I)*e^{(1/2*I*\pi*b*c*x*\text{sgn}(F) - 1/2*I*\pi*b*c*x + 1/2*I*\pi*a \\
&*c*\text{sgn}(F) - 1/2*I*\pi*a*c)/(2*\pi^2*b^2*c^2*\text{sgn}(F) + 4*I*\pi*b^2*c^2*\log(\text{abs}(F) \\
&))*\text{sgn}(F) - 2*\pi^2*b^2*c^2 - 4*I*\pi*b^2*c^2*\log(\text{abs}(F)) + 4*b^2*c^2*\log(\text{abs} \\
&(F))^2) + (2*\pi*b*c*x*\text{sgn}(F) - 2*\pi*b*c*x + 4*I*b*c*x*\log(\text{abs}(F)) - 4*I)*e^{ \\
&(-1/2*I*\pi*b*c*x*\text{sgn}(F) + 1/2*I*\pi*b*c*x - 1/2*I*\pi*a*c*\text{sgn}(F) + 1/2*I*\pi*a \\
&*c)/(2*\pi^2*b^2*c^2*\text{sgn}(F) - 4*I*\pi*b^2*c^2*\log(\text{abs}(F))*\text{sgn}(F) - 2*\pi^2*b^2 \\
&*c^2 + 4*I*\pi*b^2*c^2*\log(\text{abs}(F)) + 4*b^2*c^2*\log(\text{abs}(F))^2))*e^{(b*c*x*\log( \\
&\text{abs}(F)) + a*c*\log(\text{abs}(F)) + 1) + 2*(2*b*c*d*\cos(-1/2*\pi*b*c*x*\text{sgn}(F) + 1/2* \\
&\pi*b*c*x - 1/2*\pi*a*c*\text{sgn}(F) + 1/2*\pi*a*c)*\log(\text{abs}(F))/(4*b^2*c^2*\log(\text{abs}(F) \\
&))^2 + (\pi*b*c*\text{sgn}(F) - \pi*b*c)^2) - (\pi*b*c*\text{sgn}(F) - \pi*b*c)*d*\sin(-1/2*\pi \\
&b*c*x*\text{sgn}(F) + 1/2*\pi*b*c*x - 1/2*\pi*a*c*\text{sgn}(F) + 1/2*\pi*a*c)/(4*b^2*c^2*\log \\
&(\text{abs}(F))^2 + (\pi*b*c*\text{sgn}(F) - \pi*b*c)^2))*e^{(b*c*x*\log(\text{abs}(F)) + a*c*\log( \\
&\text{abs}(F))) - 1/2*I*(-2*I*d*e^{(1/2*I*\pi*b*c*x*\text{sgn}(F) - 1/2*I*\pi*b*c*x + 1/2*I* \\
&\pi*a*c*\text{sgn}(F) - 1/2*I*\pi*a*c)/(I*\pi*b*c*\text{sgn}(F) - I*\pi*b*c + 2*b*c*\log(\text{abs}(F) \\
&))) + 2*I*d*e^{(-1/2*I*\pi*b*c*x*\text{sgn}(F) + 1/2*I*\pi*b*c*x - 1/2*I*\pi*a*c*\text{sgn}(F) \\
&+ 1/2*I*\pi*a*c)/(-I*\pi*b*c*\text{sgn}(F) + I*\pi*b*c + 2*b*c*\log(\text{abs}(F)))}}*e^{(b*c \\
&*x*\log(\text{abs}(F)) + a*c*\log(\text{abs}(F)))}
\end{aligned}$$

### 3.6 $\int F^{c(a+bx)} dx$

**Optimal.** Leaf size=20

$$\frac{F^{c(a+bx)}}{bc \log(F)}$$

[Out]  $F^{c*(a + b*x)}/(b*c*Log[F])$

**Rubi [A]** time = 0.0036889, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 9,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$ , Rules used = {2194}

$$\frac{F^{c(a+bx)}}{bc \log(F)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[F^{c*(a + b*x)}, x]$

[Out]  $F^{c*(a + b*x)}/(b*c*Log[F])$

**Rule 2194**

$\text{Int}[(F_)^{((c_.)*((a_.) + (b_.)*(x_)))^{(n_.)}, x\_Symbol] \rightarrow \text{Simp}[(F^{c*(a + b*x)})^n/(b*c*n*Log[F]), x] /; \text{FreeQ}[\{F, a, b, c, n\}, x]$

**Rubi steps**

$$\int F^{c(a+bx)} dx = \frac{F^{c(a+bx)}}{bc \log(F)}$$

**Mathematica [A]** time = 0.0048603, size = 21, normalized size = 1.05

$$\frac{F^{ac+bcx}}{bc \log(F)}$$

Antiderivative was successfully verified.

[In] Integrate[F^(c\*(a + b\*x)),x]

[Out]  $F^{(a*c + b*c*x)/(b*c*\text{Log}[F])}$

**Maple [A]** time = 0.001, size = 21, normalized size = 1.1

$$\frac{F^{c(bx+a)}}{bc \ln(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(c\*(b\*x+a)),x)

[Out]  $F^{(c*(b*x+a))/b/c/\ln(F)}$

**Maxima [A]** time = 0.944983, size = 27, normalized size = 1.35

$$\frac{F^{(bx+a)c}}{bc \log(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(c\*(b\*x+a)),x, algorithm="maxima")

[Out]  $F^{((b*x + a)*c)/(b*c*\log(F))}$

**Fricas [A]** time = 1.51071, size = 41, normalized size = 2.05

$$\frac{F^{bcx+ac}}{bc \log(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(c\*(b\*x+a)),x, algorithm="fricas")

[Out]  $F^{(b*c*x + a*c)/(b*c*\log(F))}$

---

**Sympy [A]** time = 0.108263, size = 20, normalized size = 1.

$$\begin{cases} \frac{F^{c(a+bx)}}{bc \log(F)} & \text{for } bc \log(F) \neq 0 \\ x & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(c\*(b\*x+a)),x)

[Out] Piecewise((F\*\*(c\*(a + b\*x))/(b\*c\*log(F)), Ne(b\*c\*log(F), 0)), (x, True))

---

**Giac [A]** time = 3.58521, size = 27, normalized size = 1.35

$$\frac{F^{(bx+a)c}}{bc \log(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(c\*(b\*x+a)),x, algorithm="giac")

[Out] F^((b\*x + a)\*c)/(b\*c\*log(F))



$$3.7 \quad \int \frac{F^{c(a+bx)}}{d+ex} dx$$

**Optimal.** Leaf size=31

$$\frac{F^{c\left(a-\frac{bd}{e}\right)} \text{Ei}\left(\frac{bc(d+ex)\log(F)}{e}\right)}{e}$$

[Out]  $(F^{(c*(a - (b*d)/e)}) * \text{ExpIntegralEi}[(b*c*(d + e*x)*\text{Log}[F])/e])/e$

**Rubi [A]** time = 0.0238996, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$ , Rules used = {2178}

$$\frac{F^{c\left(a-\frac{bd}{e}\right)} \text{Ei}\left(\frac{bc(d+ex)\log(F)}{e}\right)}{e}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[F^{(c*(a + b*x))}/(d + e*x), x]$

[Out]  $(F^{(c*(a - (b*d)/e)}) * \text{ExpIntegralEi}[(b*c*(d + e*x)*\text{Log}[F])/e])/e$

### Rule 2178

$\text{Int}[(F_)^{((g_.) * ((e_.) + (f_.) * (x_)))} / ((c_.) + (d_.) * (x_)), x\_Symbol] \rightarrow \text{Simp}[(F^{(g*(e - (c*f)/d)}) * \text{ExpIntegralEi}[(f*g*(c + d*x)*\text{Log}[F])/d])/d, x] /;$  FreeQ[{F, c, d, e, f, g}, x] && !\$UseGamma == True

### Rubi steps

$$\int \frac{F^{c(a+bx)}}{d+ex} dx = \frac{F^{c\left(a-\frac{bd}{e}\right)} \text{Ei}\left(\frac{bc(d+ex)\log(F)}{e}\right)}{e}$$

**Mathematica [A]** time = 0.0407943, size = 31, normalized size = 1.

$$\frac{F^{c\left(a-\frac{bd}{e}\right)} \text{Ei}\left(\frac{bc(d+ex)\log(F)}{e}\right)}{e}$$

Antiderivative was successfully verified.

[In] Integrate[F^(c\*(a + b\*x))/(d + e\*x),x]

[Out] (F^(c\*(a - (b\*d)/e))\*ExpIntegralEi[(b\*c\*(d + e\*x)\*Log[F])/e])/e

**Maple [A]** time = 0.033, size = 56, normalized size = 1.8

$$-\frac{1}{e}F^{\frac{c(ae-bd)}{e}}\text{Ei}\left(1, -bcx \ln(F) - ac \ln(F) - \frac{-\ln(F)ace + \ln(F)bcd}{e}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(c\*(b\*x+a))/(e\*x+d),x)

[Out] -1/e\*F^(c\*(a\*e-b\*d)/e)\*Ei(1, -b\*c\*x\*ln(F)-a\*c\*ln(F)-(-ln(F)\*a\*c\*e+ln(F)\*b\*c\*d)/e)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{(bx+a)c}}{ex + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(c\*(b\*x+a))/(e\*x+d),x, algorithm="maxima")

[Out] integrate(F^((b\*x + a)\*c)/(e\*x + d), x)

**Fricas [A]** time = 1.54641, size = 78, normalized size = 2.52

$$\frac{\text{Ei}\left(\frac{(bcex+bcd)\log(F)}{e}\right)}{F^{\frac{bcd-ace}{e}}e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))/(e*x+d),x, algorithm="fricas")
```

```
[Out] Ei((b*c*e*x + b*c*d)*log(F)/e)/(F^((b*c*d - a*c*e)/e)*e)
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{c(a+bx)}}{d+ex} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(c*(b*x+a))/(e*x+d),x)
```

```
[Out] Integral(F**(c*(a + b*x))/(d + e*x), x)
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{(bx+a)c}}{ex+d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate(F^((b*x + a)*c)/(e*x + d), x)
```

### 3.8 $\int \frac{F^{c(a+bx)}}{(d+ex)^2} dx$

**Optimal.** Leaf size=57

$$\frac{bc \log(F) F^{c\left(a - \frac{bd}{e}\right)} \text{Ei}\left(\frac{bc(d+ex) \log(F)}{e}\right)}{e^2} - \frac{F^{c(a+bx)}}{e(d+ex)}$$

[Out]  $-(F^{c(a+bx)})/(e(d+ex)) + (b*c*F^{c(a-(b*d)/e)})*\text{ExpIntegralEi}[(b*c*(d+e*x)*\text{Log}[F])/e]*\text{Log}[F])/e^2$

**Rubi [A]** time = 0.043363, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$ , Rules used = {2177, 2178}

$$\frac{bc \log(F) F^{c\left(a - \frac{bd}{e}\right)} \text{Ei}\left(\frac{bc(d+ex) \log(F)}{e}\right)}{e^2} - \frac{F^{c(a+bx)}}{e(d+ex)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[F^{c(a+bx)}]/(d+ex)^2, x]$

[Out]  $-(F^{c(a+bx)})/(e(d+ex)) + (b*c*F^{c(a-(b*d)/e)})*\text{ExpIntegralEi}[(b*c*(d+e*x)*\text{Log}[F])/e]*\text{Log}[F])/e^2$

#### Rule 2177

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_), x_Symbol] := Simp[((c + d*x)^(m + 1)*(b*F^(g*(e + f*x)))^n)/(d*(m + 1))
, x] - Dist[(f*g*n*Log[F])/(d*(m + 1)), Int[(c + d*x)^(m + 1)*(b*F^(g*(e +
f*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && LtQ[m, -1] && Int
egerQ[2*m] && !$UseGamma === True
```

#### Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] := Si
mp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; F
reeQ[{F, c, d, e, f, g}, x] && !$UseGamma === True
```

#### Rubi steps

$$\int \frac{F^{c(a+bx)}}{(d+ex)^2} dx = -\frac{F^{c(a+bx)}}{e(d+ex)} + \frac{(bc \log(F)) \int \frac{F^{c(a+bx)}}{d+ex} dx}{e}$$

$$= -\frac{F^{c(a+bx)}}{e(d+ex)} + \frac{bc F^{c\left(a-\frac{bd}{e}\right)} \text{Ei}\left(\frac{bc(d+ex) \log(F)}{e}\right) \log(F)}{e^2}$$

**Mathematica [A]** time = 0.118915, size = 55, normalized size = 0.96

$$\frac{F^{ac} \left( bc \log(F) F^{-\frac{bcd}{e}} \text{Ei}\left(\frac{bc(d+ex) \log(F)}{e}\right) - \frac{e F^{bcx}}{d+ex} \right)}{e^2}$$

Antiderivative was successfully verified.

[In] Integrate[F^(c\*(a + b\*x))/(d + e\*x)^2,x]

[Out] (F^(a\*c)\*(-(e\*F^(b\*c\*x))/(d + e\*x)) + (b\*c\*ExpIntegralEi[(b\*c\*(d + e\*x)\*Log[F])/e]\*Log[F])/F^((b\*c\*d)/e))/e^2

**Maple [A]** time = 0.038, size = 99, normalized size = 1.7

$$-\frac{bc \ln(F) F^{bcx} F^{ac}}{e^2} \left( bcx \ln(F) + \frac{\ln(F) bcd}{e} \right)^{-1} - \frac{bc \ln(F)}{e^2} F^{\frac{c(ae-bd)}{e}} \text{Ei}\left(1, -bcx \ln(F) - ac \ln(F) - \frac{-\ln(F) ace + \ln(F) bcd}{e}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(c\*(b\*x+a))/(e\*x+d)^2,x)

[Out] -b\*c\*ln(F)/e^2\*F^(b\*c\*x)\*F^(a\*c)/(b\*c\*x\*ln(F)+1/e\*ln(F)\*b\*c\*d)-b\*c\*ln(F)/e^2\*F^(c\*(a\*e-b\*d)/e)\*Ei(1,-b\*c\*x\*ln(F)-a\*c\*ln(F)-(-ln(F)\*a\*c\*e+ln(F)\*b\*c\*d)/e)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{(bx+a)c}}{(ex+d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(c\*(b\*x+a))/(e\*x+d)^2,x, algorithm="maxima")

[Out] integrate(F^((b\*x + a)\*c)/(e\*x + d)^2, x)

**Fricas [A]** time = 1.53936, size = 159, normalized size = 2.79

$$F^{bcx+ac} e - \frac{(bcex+bcd) \operatorname{Ei}\left(\frac{(bcex+bcd)\log(F)}{e}\right) \log(F)}{F^{\frac{bcd-ace}{e}}}$$


---


$$- \frac{e^3 x + d e^2}{e^3 x + d e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(c\*(b\*x+a))/(e\*x+d)^2,x, algorithm="fricas")

[Out]  $-(F^{(b*c*x + a*c)*e} - (b*c*e*x + b*c*d)*\operatorname{Ei}((b*c*e*x + b*c*d)*\log(F)/e)*\log(F)/F^{((b*c*d - a*c*e)/e)})/(e^3*x + d*e^2)$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{c(a+bx)}}{(d+ex)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(c\*(b\*x+a))/(e\*x+d)\*\*2,x)

[Out] Integral(F\*\*(c\*(a + b\*x))/(d + e\*x)\*\*2, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{(bx+a)c}}{(ex+d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] integrate(F^((b*x + a)*c)/(e*x + d)^2, x)
```

### 3.9 $\int \frac{F^{c(a+bx)}}{(d+ex)^3} dx$

**Optimal.** Leaf size=95

$$\frac{b^2 c^2 \log^2(F) F^{c\left(a-\frac{bd}{e}\right)} \text{Ei}\left(\frac{bc(d+ex)\log(F)}{e}\right)}{2e^3} - \frac{bc \log(F) F^{c(a+bx)}}{2e^2(d+ex)} - \frac{F^{c(a+bx)}}{2e(d+ex)^2}$$

[Out]  $-F^{(c*(a + b*x))}/(2*e*(d + e*x)^2) - (b*c*F^{(c*(a + b*x))*\text{Log}[F]})/(2*e^2*(d + e*x)) + (b^2*c^2*F^{(c*(a - (b*d)/e))*\text{ExpIntegralEi}[(b*c*(d + e*x)*\text{Log}[F])/e]*\text{Log}[F]^2)/(2*e^3)$

**Rubi [A]** time = 0.0707388, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$ , Rules used = {2177, 2178}

$$\frac{b^2 c^2 \log^2(F) F^{c\left(a-\frac{bd}{e}\right)} \text{Ei}\left(\frac{bc(d+ex)\log(F)}{e}\right)}{2e^3} - \frac{bc \log(F) F^{c(a+bx)}}{2e^2(d+ex)} - \frac{F^{c(a+bx)}}{2e(d+ex)^2}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[F^{(c*(a + b*x))}/(d + e*x)^3, x]$

[Out]  $-F^{(c*(a + b*x))}/(2*e*(d + e*x)^2) - (b*c*F^{(c*(a + b*x))*\text{Log}[F]})/(2*e^2*(d + e*x)) + (b^2*c^2*F^{(c*(a - (b*d)/e))*\text{ExpIntegralEi}[(b*c*(d + e*x)*\text{Log}[F])/e]*\text{Log}[F]^2)/(2*e^3)$

#### Rule 2177

$\text{Int}[(b_*)*(F_*)^{((g_*)*((e_*) + (f_*)*(x_*)))}^{(n_*)*((c_*) + (d_*)*(x_*))}^{(m_*)}, x\_Symbol] \rightarrow \text{Simp}[(c + d*x)^{(m + 1)}*(b*F^{(g*(e + f*x))})^n/(d*(m + 1)), x] - \text{Dist}[(f*g*n*\text{Log}[F])/(d*(m + 1)), \text{Int}[(c + d*x)^{(m + 1)}*(b*F^{(g*(e + f*x))})^n, x], x] /;$  FreeQ[{F, b, c, d, e, f, g, n}, x] && LtQ[m, -1] && IntegerQ[2\*m] && !\$UseGamma == True

#### Rule 2178

$\text{Int}[(F_*)^{((g_*)*((e_*) + (f_*)*(x_*)))}/((c_*) + (d_*)*(x_*)), x\_Symbol] \rightarrow \text{Simp}[(F^{(g*(e - (c*f)/d))*\text{ExpIntegralEi}[(f*g*(c + d*x)*\text{Log}[F])/d]})/d, x] /;$  FreeQ[{F, c, d, e, f, g}, x] && !\$UseGamma == True



Rubi steps

$$\begin{aligned}
\int \frac{F^{c(a+bx)}}{(d+ex)^3} dx &= -\frac{F^{c(a+bx)}}{2e(d+ex)^2} + \frac{(bc \log(F)) \int \frac{F^{c(a+bx)}}{(d+ex)^2} dx}{2e} \\
&= -\frac{F^{c(a+bx)}}{2e(d+ex)^2} - \frac{bcF^{c(a+bx)} \log(F)}{2e^2(d+ex)} + \frac{(b^2c^2 \log^2(F)) \int \frac{F^{c(a+bx)}}{d+ex} dx}{2e^2} \\
&= -\frac{F^{c(a+bx)}}{2e(d+ex)^2} - \frac{bcF^{c(a+bx)} \log(F)}{2e^2(d+ex)} + \frac{b^2c^2 F^{c\left(a-\frac{bd}{e}\right)} \text{Ei}\left(\frac{bc(d+ex) \log(F)}{e}\right) \log^2(F)}{2e^3}
\end{aligned}$$

**Mathematica [A]** time = 0.157105, size = 88, normalized size = 0.93

$$\frac{F^{c\left(a-\frac{bd}{e}\right)} \left( b^2c^2 \log^2(F)(d+ex)^2 \text{Ei}\left(\frac{bc(d+ex) \log(F)}{e}\right) - eF^{\frac{bc(d+ex)}{e}} (bc \log(F)(d+ex) + e) \right)}{2e^3(d+ex)^2}$$

Antiderivative was successfully verified.

[In] Integrate[F^(c\*(a + b\*x))/(d + e\*x)^3, x]

[Out] (F^(c\*(a - (b\*d)/e))\*(b^2\*c^2\*(d + e\*x)^2\*ExpIntegralEi[(b\*c\*(d + e\*x)\*Log[F])/e]\*Log[F]^2 - eF^((b\*c\*(d + e\*x))/e)\*(e + b\*c\*(d + e\*x)\*Log[F]))/(2\*e^3\*(d + e\*x)^2)

**Maple [A]** time = 0.04, size = 155, normalized size = 1.6

$$-\frac{b^2c^2 (\ln(F))^2 F^{bcx} F^{ac}}{2e^3} \left( bcx \ln(F) + \frac{\ln(F) bcd}{e} \right)^{-2} - \frac{b^2c^2 (\ln(F))^2 F^{bcx} F^{ac}}{2e^3} \left( bcx \ln(F) + \frac{\ln(F) bcd}{e} \right)^{-1} - \frac{b^2c^2 (\ln(F))^2}{2e^3} F^{\frac{c(a+bx)}{e}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(c\*(b\*x+a))/(e\*x+d)^3, x)

[Out] -1/2\*b^2\*c^2\*ln(F)^2/e^3\*F^(b\*c\*x)\*F^(a\*c)/(b\*c\*x\*ln(F)+1/e\*ln(F)\*b\*c\*d)^2-1/2\*b^2\*c^2\*ln(F)^2/e^3\*F^(b\*c\*x)\*F^(a\*c)/(b\*c\*x\*ln(F)+1/e\*ln(F)\*b\*c\*d)-1/2\*b^2\*c^2\*ln(F)^2/e^3\*F^(c\*(a\*e-b\*d)/e)\*Ei(1, -b\*c\*x\*ln(F)-a\*c\*ln(F)-(-ln(F)\*a\*c\*e+ln(F)\*b\*c\*d)/e)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{(bx+a)c}}{(ex+d)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(c\*(b\*x+a))/(e\*x+d)^3,x, algorithm="maxima")

[Out] integrate(F^((b\*x + a)\*c)/(e\*x + d)^3, x)

---

**Fricas [A]** time = 1.50316, size = 278, normalized size = 2.93

$$\frac{\frac{(b^2c^2e^2x^2+2b^2c^2dex+b^2c^2d^2)\text{Ei}\left(\frac{(bcex+bcd)\log(F)}{e}\right)\log(F)^2}{F^{\frac{bcd-ace}{e}}}}{2(e^5x^2+2de^4x+d^2e^3)} - (e^2 + (bce^2x + bcde)\log(F))F^{bcx+ac}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(c\*(b\*x+a))/(e\*x+d)^3,x, algorithm="fricas")

[Out] 1/2\*((b^2\*c^2\*e^2\*x^2 + 2\*b^2\*c^2\*d\*e\*x + b^2\*c^2\*d^2)\*Ei((b\*c\*e\*x + b\*c\*d)\*log(F)/e)\*log(F)^2/F^((b\*c\*d - a\*c\*e)/e) - (e^2 + (b\*c\*e^2\*x + b\*c\*d\*e)\*log(F))\*F^(b\*c\*x + a\*c))/(e^5\*x^2 + 2\*d\*e^4\*x + d^2\*e^3)

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{c(a+bx)}}{(d+ex)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(c\*(b\*x+a))/(e\*x+d)\*\*3,x)

[Out] Integral(F\*\*(c\*(a + b\*x))/(d + e\*x)\*\*3, x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{(bx+a)c}}{(ex+d)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(c\*(b\*x+a))/(e\*x+d)^3,x, algorithm="giac")

[Out] integrate(F^((b\*x + a)\*c)/(e\*x + d)^3, x)

### 3.10 $\int \frac{F^{c(a+bx)}}{(d+ex)^4} dx$

**Optimal.** Leaf size=128

$$\frac{b^3 c^3 \log^3(F) F^{c\left(a-\frac{bd}{e}\right)} \text{Ei}\left(\frac{bc(d+ex)\log(F)}{e}\right)}{6e^4} - \frac{b^2 c^2 \log^2(F) F^{c(a+bx)}}{6e^3(d+ex)} - \frac{bc \log(F) F^{c(a+bx)}}{6e^2(d+ex)^2} - \frac{F^{c(a+bx)}}{3e(d+ex)^3}$$

[Out]  $-F^{c*(a + b*x)} / (3*e*(d + e*x)^3) - (b*c*F^{c*(a + b*x)}*Log[F]) / (6*e^2*(d + e*x)^2) - (b^2*c^2*F^{c*(a + b*x)}*Log[F]^2) / (6*e^3*(d + e*x)) + (b^3*c^3*F^{c*(a - (b*d)/e)}*ExpIntegralEi[(b*c*(d + e*x)*Log[F])/e]*Log[F]^3) / (6*e^4)$

**Rubi [A]** time = 0.100237, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$ , Rules used = {2177, 2178}

$$\frac{b^3 c^3 \log^3(F) F^{c\left(a-\frac{bd}{e}\right)} \text{Ei}\left(\frac{bc(d+ex)\log(F)}{e}\right)}{6e^4} - \frac{b^2 c^2 \log^2(F) F^{c(a+bx)}}{6e^3(d+ex)} - \frac{bc \log(F) F^{c(a+bx)}}{6e^2(d+ex)^2} - \frac{F^{c(a+bx)}}{3e(d+ex)^3}$$

Antiderivative was successfully verified.

[In] Int[F^(c\*(a + b\*x))/(d + e\*x)^4, x]

[Out]  $-F^{c*(a + b*x)} / (3*e*(d + e*x)^3) - (b*c*F^{c*(a + b*x)}*Log[F]) / (6*e^2*(d + e*x)^2) - (b^2*c^2*F^{c*(a + b*x)}*Log[F]^2) / (6*e^3*(d + e*x)) + (b^3*c^3*F^{c*(a - (b*d)/e)}*ExpIntegralEi[(b*c*(d + e*x)*Log[F])/e]*Log[F]^3) / (6*e^4)$

#### Rule 2177

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Simp[((c + d*x)^(m + 1)*(b*F^(g*(e + f*x)))^n)/(d*(m + 1)), x] - Dist[(f*g*n*Log[F])/(d*(m + 1)), Int[(c + d*x)^(m + 1)*(b*F^(g*(e + f*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && LtQ[m, -1] && IntegerQ[2*m] && !$UseGamma === True
```

#### Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; F
```

```
reeQ[{F, c, d, e, f, g}, x] && !$UseGamma === True
```

### Rubi steps

$$\begin{aligned}
 \int \frac{F^{c(a+bx)}}{(d+ex)^4} dx &= -\frac{F^{c(a+bx)}}{3e(d+ex)^3} + \frac{(bc \log(F)) \int \frac{F^{c(a+bx)}}{(d+ex)^3} dx}{3e} \\
 &= -\frac{F^{c(a+bx)}}{3e(d+ex)^3} - \frac{bcF^{c(a+bx)} \log(F)}{6e^2(d+ex)^2} + \frac{(b^2c^2 \log^2(F)) \int \frac{F^{c(a+bx)}}{(d+ex)^2} dx}{6e^2} \\
 &= -\frac{F^{c(a+bx)}}{3e(d+ex)^3} - \frac{bcF^{c(a+bx)} \log(F)}{6e^2(d+ex)^2} - \frac{b^2c^2F^{c(a+bx)} \log^2(F)}{6e^3(d+ex)} + \frac{(b^3c^3 \log^3(F)) \int \frac{F^{c(a+bx)}}{d+ex} dx}{6e^3} \\
 &= -\frac{F^{c(a+bx)}}{3e(d+ex)^3} - \frac{bcF^{c(a+bx)} \log(F)}{6e^2(d+ex)^2} - \frac{b^2c^2F^{c(a+bx)} \log^2(F)}{6e^3(d+ex)} + \frac{b^3c^3F^{c\left(a-\frac{bd}{e}\right)} \text{Ei}\left(\frac{bc(d+ex)\log(F)}{e}\right) \log^3(F)}{6e^4}
 \end{aligned}$$

**Mathematica [A]** time = 0.201428, size = 99, normalized size = 0.77

$$\frac{F^{ac} \left( b^3 c^3 \log^3(F) F^{-\frac{bcd}{e}} \text{Ei}\left(\frac{bc(d+ex)\log(F)}{e}\right) - \frac{eF^{bcx}(b^2c^2 \log^2(F)(d+ex)^2 + bce \log(F)(d+ex) + 2e^2)}{(d+ex)^3} \right)}{6e^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[F^(c*(a + b*x))/(d + e*x)^4, x]
```

```
[Out] (F^(a*c)*((b^3*c^3*ExpIntegralEi[(b*c*(d + e*x)*Log[F])/e]*Log[F]^3)/F^((b*c*d)/e) - (e*F^(b*c*x)*(2*e^2 + b*c*e*(d + e*x)*Log[F] + b^2*c^2*(d + e*x)^2*Log[F]^2))/(d + e*x)^3))/(6*e^4)
```

**Maple [A]** time = 0.049, size = 199, normalized size = 1.6

$$-\frac{b^3c^3(\ln(F))^3 F^{bcx} F^{ac}}{3e^4} \left( bcx \ln(F) + \frac{\ln(F) bcd}{e} \right)^{-3} - \frac{b^3c^3(\ln(F))^3 F^{bcx} F^{ac}}{6e^4} \left( bcx \ln(F) + \frac{\ln(F) bcd}{e} \right)^{-2} - \frac{b^3c^3(\ln(F))^3 F^{bcx} F^{ac}}{6e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(b*x+a))/(e*x+d)^4, x)
```

[Out]  $-1/3*b^3*c^3*\ln(F)^3/e^4*F^{(b*c*x)*F^{(a*c)}}/(b*c*x*\ln(F)+1/e*\ln(F)*b*c*d)^3-1/6*b^3*c^3*\ln(F)^3/e^4*F^{(b*c*x)*F^{(a*c)}}/(b*c*x*\ln(F)+1/e*\ln(F)*b*c*d)^2-1/6*b^3*c^3*\ln(F)^3/e^4*F^{(b*c*x)*F^{(a*c)}}/(b*c*x*\ln(F)+1/e*\ln(F)*b*c*d)-1/6*b^3*c^3*\ln(F)^3/e^4*F^{(c*(a*e-b*d)/e)*Ei(1,-b*c*x*\ln(F)-a*c*\ln(F)-(-\ln(F))*c*e+\ln(F)*b*c*d)/e}$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{(bx+a)c}}{(ex+d)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(c\*(b\*x+a))/(e\*x+d)^4,x, algorithm="maxima")

[Out] integrate(F^((b\*x + a)\*c)/(e\*x + d)^4, x)

**Fricas [A]** time = 1.52107, size = 425, normalized size = 3.32

$$\frac{(b^3c^3e^3x^3+3b^3c^3de^2x^2+3b^3c^3d^2ex+b^3c^3d^3)Ei\left(\frac{(bcex+bcd)\log(F)}{e}\right)\log(F)^3}{F\frac{bcd-ace}{e}} - \left(2e^3 + (b^2c^2e^3x^2 + 2b^2c^2de^2x + b^2c^2d^2e)\log(F)^2 + (bce^3x + bc\right)$$


---


$$6(e^7x^3 + 3de^6x^2 + 3d^2e^5x + d^3e^4)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(c\*(b\*x+a))/(e\*x+d)^4,x, algorithm="fricas")

[Out]  $1/6*((b^3*c^3*e^3*x^3 + 3*b^3*c^3*d*e^2*x^2 + 3*b^3*c^3*d^2*e*x + b^3*c^3*d^3)*Ei((b*c*e*x + b*c*d)*\log(F)/e)*\log(F)^3/F^{((b*c*d - a*c*e)/e)} - (2*e^3 + (b^2*c^2*e^3*x^2 + 2*b^2*c^2*d*e^2*x + b^2*c^2*d^2*e)*\log(F)^2 + (b*c*e^3*x + b*c*d*e^2)*\log(F))*F^{(b*c*x + a*c)}/(e^7*x^3 + 3*d*e^6*x^2 + 3*d^2*e^5*x + d^3*e^4)$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{c(a+bx)}}{(d+ex)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(c\*(b\*x+a))/(e\*x+d)\*\*4,x)

[Out] Integral(F\*\*(c\*(a + b\*x))/(d + e\*x)\*\*4, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{(bx+a)c}}{(ex+d)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(c\*(b\*x+a))/(e\*x+d)^4,x, algorithm="giac")

[Out] integrate(F^((b\*x + a)\*c)/(e\*x + d)^4, x)

### 3.11 $\int \frac{F^{c(a+bx)}}{(d+ex)^5} dx$

**Optimal.** Leaf size=161

$$\frac{b^4 c^4 \log^4(F) F^{c\left(a-\frac{bd}{e}\right)} \text{Ei}\left(\frac{bc(d+ex)\log(F)}{e}\right)}{24e^5} - \frac{b^3 c^3 \log^3(F) F^{c(a+bx)}}{24e^4(d+ex)} - \frac{b^2 c^2 \log^2(F) F^{c(a+bx)}}{24e^3(d+ex)^2} - \frac{bc \log(F) F^{c(a+bx)}}{12e^2(d+ex)^3} - \frac{F^{c(a+bx)}}{4e(d+ex)^4}$$

[Out]  $-F^{c(a+bx)}/(4e(d+ex)^4) - (b^3 c^3 \log^3(F) F^{c(a+bx)})/(12e^2(d+ex)^3) - (b^2 c^2 \log^2(F) F^{c(a+bx)})/(24e^3(d+ex)^2) - (bc \log(F) F^{c(a+bx)})/(24e^4(d+ex)) + (b^4 c^4 F^{c(a-\frac{bd}{e})}) \text{ExpIntegralEi}[(b^3 c^3 \log^3(F) F^{c(a+bx)})/e] \text{Log}[F]^4/(24e^5)$

**Rubi [A]** time = 0.134198, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$ , Rules used = {2177, 2178}

$$\frac{b^4 c^4 \log^4(F) F^{c\left(a-\frac{bd}{e}\right)} \text{Ei}\left(\frac{bc(d+ex)\log(F)}{e}\right)}{24e^5} - \frac{b^3 c^3 \log^3(F) F^{c(a+bx)}}{24e^4(d+ex)} - \frac{b^2 c^2 \log^2(F) F^{c(a+bx)}}{24e^3(d+ex)^2} - \frac{bc \log(F) F^{c(a+bx)}}{12e^2(d+ex)^3} - \frac{F^{c(a+bx)}}{4e(d+ex)^4}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[F^{c(a+bx)}/(d+ex)^5, x]$

[Out]  $-F^{c(a+bx)}/(4e(d+ex)^4) - (b^3 c^3 \log^3(F) F^{c(a+bx)})/(12e^2(d+ex)^3) - (b^2 c^2 \log^2(F) F^{c(a+bx)})/(24e^3(d+ex)^2) - (bc \log(F) F^{c(a+bx)})/(24e^4(d+ex)) + (b^4 c^4 F^{c(a-\frac{bd}{e})}) \text{ExpIntegralEi}[(b^3 c^3 \log^3(F) F^{c(a+bx)})/e] \text{Log}[F]^4/(24e^5)$

#### Rule 2177

```
Int[((b_.)*(F_)^((g_.)*((e_.)+(f_.)*(x_))))^(n_.)*((c_.)+(d_.)*(x_))^(m_.), x_Symbol] := Simp[((c+d*x)^(m+1)*(b*F^(g*(e+f*x)))^n)/(d*(m+1)), x] - Dist[(f*g*n*Log[F])/(d*(m+1)), Int[(c+d*x)^(m+1)*(b*F^(g*(e+f*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && LtQ[m, -1] && IntegerQ[2*m] && !UseGamma == True
```

#### Rule 2178

```
Int[(F_)^((g_.)*((e_.)+(f_.)*(x_)))/((c_.)+(d_.)*(x_)), x_Symbol] := Simp[(F^(g*(e-(c*f)/d))*ExpIntegralEi[(f*g*(c+d*x)*Log[F])/d])/d, x] /; F
```



```
reeQ[{F, c, d, e, f, g}, x] && !$UseGamma === True
```

### Rubi steps

$$\begin{aligned}
 \int \frac{F^{c(a+bx)}}{(d+ex)^5} dx &= -\frac{F^{c(a+bx)}}{4e(d+ex)^4} + \frac{(bc \log(F)) \int \frac{F^{c(a+bx)}}{(d+ex)^4} dx}{4e} \\
 &= -\frac{F^{c(a+bx)}}{4e(d+ex)^4} - \frac{bcF^{c(a+bx)} \log(F)}{12e^2(d+ex)^3} + \frac{(b^2c^2 \log^2(F)) \int \frac{F^{c(a+bx)}}{(d+ex)^3} dx}{12e^2} \\
 &= -\frac{F^{c(a+bx)}}{4e(d+ex)^4} - \frac{bcF^{c(a+bx)} \log(F)}{12e^2(d+ex)^3} - \frac{b^2c^2F^{c(a+bx)} \log^2(F)}{24e^3(d+ex)^2} + \frac{(b^3c^3 \log^3(F)) \int \frac{F^{c(a+bx)}}{(d+ex)^2} dx}{24e^3} \\
 &= -\frac{F^{c(a+bx)}}{4e(d+ex)^4} - \frac{bcF^{c(a+bx)} \log(F)}{12e^2(d+ex)^3} - \frac{b^2c^2F^{c(a+bx)} \log^2(F)}{24e^3(d+ex)^2} - \frac{b^3c^3F^{c(a+bx)} \log^3(F)}{24e^4(d+ex)} + \frac{(b^4c^4 \log^4(F)) \int \frac{F^{c(a+bx)}}{d+ex} dx}{24e^4} \\
 &= -\frac{F^{c(a+bx)}}{4e(d+ex)^4} - \frac{bcF^{c(a+bx)} \log(F)}{12e^2(d+ex)^3} - \frac{b^2c^2F^{c(a+bx)} \log^2(F)}{24e^3(d+ex)^2} - \frac{b^3c^3F^{c(a+bx)} \log^3(F)}{24e^4(d+ex)} + \frac{b^4c^4F^{c(a-\frac{bd}{e})} \text{Ei}\left(\frac{bc(d+ex)}{e}\right)}{24e^5}
 \end{aligned}$$

**Mathematica [A]** time = 0.171768, size = 121, normalized size = 0.75

$$\frac{F^{ac} \left( b^4 c^4 \log^4(F) F^{-\frac{bcd}{e}} \text{Ei}\left(\frac{bc(d+ex) \log(F)}{e}\right) - \frac{e F^{bcx} (b^2 c^2 e \log^2(F)(d+ex)^2 + b^3 c^3 \log^3(F)(d+ex)^3 + 2bce^2 \log(F)(d+ex) + 6e^3)}{(d+ex)^4} \right)}{24e^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[F^(c*(a + b*x))/(d + e*x)^5,x]
```

```
[Out] (F^(a*c))*((b^4*c^4*ExpIntegralEi[(b*c*(d + e*x)*Log[F])/e]*Log[F]^4)/F^((b*c*d)/e) - (e*F^(b*c*x)*(6*e^3 + 2*b*c*e^2*(d + e*x)*Log[F] + b^2*c^2*e*(d + e*x)^2*Log[F]^2 + b^3*c^3*(d + e*x)^3*Log[F]^3))/(d + e*x)^4))/(24*e^5)
```

**Maple [A]** time = 0.056, size = 243, normalized size = 1.5

$$-\frac{b^4 c^4 (\ln(F))^4 F^{bcx} F^{ac}}{4e^5} \left( bcx \ln(F) + \frac{\ln(F) bcd}{e} \right)^{-4} - \frac{b^4 c^4 (\ln(F))^4 F^{bcx} F^{ac}}{12e^5} \left( bcx \ln(F) + \frac{\ln(F) bcd}{e} \right)^{-3} - \frac{b^4 c^4 (\ln(F))^4 F^{bcx} F^{ac}}{24e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(c\*(b\*x+a))/(e\*x+d)^5,x)

[Out] 
$$-1/4*b^4*c^4*\ln(F)^4/e^5*F^(b*c*x)*F^(a*c)/(b*c*x*\ln(F)+1/e*\ln(F)*b*c*d)^4-1/12*b^4*c^4*\ln(F)^4/e^5*F^(b*c*x)*F^(a*c)/(b*c*x*\ln(F)+1/e*\ln(F)*b*c*d)^3-1/24*b^4*c^4*\ln(F)^4/e^5*F^(b*c*x)*F^(a*c)/(b*c*x*\ln(F)+1/e*\ln(F)*b*c*d)^2-1/24*b^4*c^4*\ln(F)^4/e^5*F^(b*c*x)*F^(a*c)/(b*c*x*\ln(F)+1/e*\ln(F)*b*c*d)-1/24*b^4*c^4*\ln(F)^4/e^5*F^(c*(a*e-b*d)/e)*Ei(1,-b*c*x*\ln(F)-a*c*\ln(F)-(-\ln(F))*a*c*e+\ln(F)*b*c*d)/e)$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{(bx+a)c}}{(ex+d)^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(c\*(b\*x+a))/(e\*x+d)^5,x, algorithm="maxima")

[Out] integrate(F^((b\*x + a)\*c)/(e\*x + d)^5, x)

**Fricas [A]** time = 1.56745, size = 606, normalized size = 3.76

$$\frac{(b^4c^4e^4x^4+4b^4c^4de^3x^3+6b^4c^4d^2e^2x^2+4b^4c^4d^3ex+b^4c^4d^4)Ei\left(\frac{(bcex+bcd)\log(F)}{e}\right)\log(F)^4}{F^{\frac{bcd-ace}{e}}}-\left(6e^4+(b^3c^3e^4x^3+3b^3c^3de^3x^2+3b^3c^3d^2e^2x+b^3c^3d^3e)\log(F)^3+(b^2c^2e^4x^2+2b^2c^2d^2e^3x+b^2c^2d^2e^2)\log(F)^2+2*(b*c*e^4*x+b*c*d*e^3)*\log(F)*F^(b*c*x+a*c)\right)/(e^9x^4+4*d*e^8x^3+6*d^2*e^7x^2+4*d^3*e^6x+d^4*e^5)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(c\*(b\*x+a))/(e\*x+d)^5,x, algorithm="fricas")

[Out] 
$$1/24*((b^4*c^4*e^4*x^4+4*b^4*c^4*d*e^3*x^3+6*b^4*c^4*d^2*e^2*x^2+4*b^4*c^4*d^3*e*x+b^4*c^4*d^4)*Ei((b*c*e*x+b*c*d)*\log(F)/e)*\log(F)^4/F^((b*c*d-a*c*e)/e)-(6*e^4+(b^3*c^3*e^4*x^3+3*b^3*c^3*d*e^3*x^2+3*b^3*c^3*d^2*e^2*x+b^3*c^3*d^3*e)*\log(F)^3+(b^2*c^2*e^4*x^2+2*b^2*c^2*d^2*e^3*x+b^2*c^2*d^2*e^2)*\log(F)^2+2*(b*c*e^4*x+b*c*d*e^3)*\log(F))*F^(b*c*x+a*c))/(e^9*x^4+4*d*e^8*x^3+6*d^2*e^7*x^2+4*d^3*e^6*x+d^4*e^5)$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(c\*(b\*x+a))/(e\*x+d)\*\*5,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{(bx+a)c}}{(ex+d)^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(c\*(b\*x+a))/(e\*x+d)^5,x, algorithm="giac")

[Out] integrate(F^((b\*x + a)\*c)/(e\*x + d)^5, x)

$$3.12 \quad \int F^{c(a+bx)} \left( d^4 + 4d^3 ex + 6d^2 e^2 x^2 + 4de^3 x^3 + e^4 x^4 \right) dx$$

**Optimal.** Leaf size=141

$$\frac{12e^2(d+ex)^2 F^{c(a+bx)}}{b^3 c^3 \log^3(F)} - \frac{24e^3(d+ex) F^{c(a+bx)}}{b^4 c^4 \log^4(F)} - \frac{4e(d+ex)^3 F^{c(a+bx)}}{b^2 c^2 \log^2(F)} + \frac{24e^4 F^{c(a+bx)}}{b^5 c^5 \log^5(F)} + \frac{(d+ex)^4 F^{c(a+bx)}}{bc \log(F)}$$

[Out] (24\*e^4\*F^(c\*(a + b\*x)))/(b^5\*c^5\*Log[F]^5) - (24\*e^3\*F^(c\*(a + b\*x))\*(d + e\*x))/(b^4\*c^4\*Log[F]^4) + (12\*e^2\*F^(c\*(a + b\*x))\*(d + e\*x)^2)/(b^3\*c^3\*Log[F]^3) - (4\*e\*F^(c\*(a + b\*x))\*(d + e\*x)^3)/(b^2\*c^2\*Log[F]^2) + (F^(c\*(a + b\*x))\*(d + e\*x)^4)/(b\*c\*Log[F])

**Rubi [A]** time = 0.122031, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 48,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$ , Rules used = {2187, 2176, 2194}

$$\frac{12e^2(d+ex)^2 F^{c(a+bx)}}{b^3 c^3 \log^3(F)} - \frac{24e^3(d+ex) F^{c(a+bx)}}{b^4 c^4 \log^4(F)} - \frac{4e(d+ex)^3 F^{c(a+bx)}}{b^2 c^2 \log^2(F)} + \frac{24e^4 F^{c(a+bx)}}{b^5 c^5 \log^5(F)} + \frac{(d+ex)^4 F^{c(a+bx)}}{bc \log(F)}$$

Antiderivative was successfully verified.

[In] Int[F^(c\*(a + b\*x))\*(d^4 + 4\*d^3\*e\*x + 6\*d^2\*e^2\*x^2 + 4\*d\*e^3\*x^3 + e^4\*x^4), x]

[Out] (24\*e^4\*F^(c\*(a + b\*x)))/(b^5\*c^5\*Log[F]^5) - (24\*e^3\*F^(c\*(a + b\*x))\*(d + e\*x))/(b^4\*c^4\*Log[F]^4) + (12\*e^2\*F^(c\*(a + b\*x))\*(d + e\*x)^2)/(b^3\*c^3\*Log[F]^3) - (4\*e\*F^(c\*(a + b\*x))\*(d + e\*x)^3)/(b^2\*c^2\*Log[F]^2) + (F^(c\*(a + b\*x))\*(d + e\*x)^4)/(b\*c\*Log[F])

### Rule 2187

```
Int[((a_.) + (b_.)*((F_)^((g_.)*(v_)))^(n_.))^(p_.)*(u_)^(m_.), x_Symbol] :
> Int[NormalizePowerOfLinear[u, x]^m*(a + b*(F^(g*ExpandToSum[v, x]))^n)^p,
x] /; FreeQ[{F, a, b, g, n, p}, x] && LinearQ[v, x] && PowerOfLinearQ[u, x]
] && !(LinearMatchQ[v, x] && PowerOfLinearMatchQ[u, x]) && IntegerQ[m]
```

### Rule 2176

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[((c + d*x)^m*(b*F^(g*(e + f*x)))^n)/(f*g*n*Log[F]),
x] - Dist[(d*m)/(f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
```

, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2\*m] && !\$UseGamma === True

### Rule 2194

Int[((F\_)^((c\_.)\*((a\_.) + (b\_.)\*(x\_))))^(n\_.), x\_Symbol] :> Simp[(F^(c\*(a + b\*x)))^n/(b\*c\*n\*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]

### Rubi steps

$$\begin{aligned}
 \int F^{c(a+bx)} (d^4 + 4d^3ex + 6d^2e^2x^2 + 4de^3x^3 + e^4x^4) dx &= \int F^{c(a+bx)}(d+ex)^4 dx \\
 &= \frac{F^{c(a+bx)}(d+ex)^4}{bc \log(F)} - \frac{(4e) \int F^{c(a+bx)}(d+ex)^3 dx}{bc \log(F)} \\
 &= -\frac{4eF^{c(a+bx)}(d+ex)^3}{b^2c^2 \log^2(F)} + \frac{F^{c(a+bx)}(d+ex)^4}{bc \log(F)} + \frac{(12e^2) \int F^{c(a+bx)}(d+ex)^2 dx}{b^2c^2 \log^2(F)} \\
 &= \frac{12e^2F^{c(a+bx)}(d+ex)^2}{b^3c^3 \log^3(F)} - \frac{4eF^{c(a+bx)}(d+ex)^3}{b^2c^2 \log^2(F)} + \frac{F^{c(a+bx)}(d+ex)^4}{bc \log(F)} \\
 &= -\frac{24e^3F^{c(a+bx)}(d+ex)}{b^4c^4 \log^4(F)} + \frac{12e^2F^{c(a+bx)}(d+ex)^2}{b^3c^3 \log^3(F)} - \frac{4eF^{c(a+bx)}(d+ex)^3}{b^2c^2 \log^2(F)} \\
 &= \frac{24e^4F^{c(a+bx)}}{b^5c^5 \log^5(F)} - \frac{24e^3F^{c(a+bx)}(d+ex)}{b^4c^4 \log^4(F)} + \frac{12e^2F^{c(a+bx)}(d+ex)^2}{b^3c^3 \log^3(F)} - \frac{4eF^{c(a+bx)}(d+ex)^3}{b^2c^2 \log^2(F)}
 \end{aligned}$$

**Mathematica [A]** time = 0.0651696, size = 100, normalized size = 0.71

$$\frac{F^{c(a+bx)} (12b^2c^2e^2 \log^2(F)(d+ex)^2 - 4b^3c^3e \log^3(F)(d+ex)^3 + b^4c^4 \log^4(F)(d+ex)^4 - 24bce^3 \log(F)(d+ex) + 24e^4)}{b^5c^5 \log^5(F)}$$

Antiderivative was successfully verified.

[In] Integrate[F^(c\*(a + b\*x))\*(d^4 + 4\*d^3\*e\*x + 6\*d^2\*e^2\*x^2 + 4\*d\*e^3\*x^3 + e^4\*x^4), x]

[Out] (F^(c\*(a + b\*x))\*(24\*e^4 - 24\*b\*c\*e^3\*(d + e\*x)\*Log[F] + 12\*b^2\*c^2\*e^2\*(d + e\*x)^2\*Log[F]^2 - 4\*b^3\*c^3\*e\*(d + e\*x)^3\*Log[F]^3 + b^4\*c^4\*(d + e\*x)^4\*Log[F]^4))/(b^5\*c^5\*Log[F]^5)

**Maple [A]** time = 0.007, size = 260, normalized size = 1.8

$$\frac{(e^4 x^4 b^4 c^4 (\ln(F))^4 + 4 (\ln(F))^4 b^4 c^4 d e^3 x^3 + 6 (\ln(F))^4 b^4 c^4 d^2 e^2 x^2 + 4 (\ln(F))^4 b^4 c^4 d^3 e x + (\ln(F))^4 b^4 c^4 d^4 - 4 (\ln(F))^3 b^4 c^4 d^4) F^{c(bx+a)}}{b^5 c^5 \ln(F)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(c\*(b\*x+a))\*(e^4\*x^4+4\*d\*e^3\*x^3+6\*d^2\*e^2\*x^2+4\*d^3\*e\*x+d^4), x)

[Out] (e^4\*x^4\*b^4\*c^4\*ln(F)^4+4\*ln(F)^4\*b^4\*c^4\*d\*e^3\*x^3+6\*ln(F)^4\*b^4\*c^4\*d^2\*e^2\*x^2+4\*ln(F)^4\*b^4\*c^4\*d^3\*e\*x+ln(F)^4\*b^4\*c^4\*d^4-4\*ln(F)^3\*b^3\*c^3\*e^4\*x^3-12\*ln(F)^3\*b^3\*c^3\*d\*e^3\*x^2-12\*ln(F)^3\*b^3\*c^3\*d^2\*e^2\*x-4\*ln(F)^3\*b^3\*c^3\*d^3\*e+12\*ln(F)^2\*b^2\*c^2\*e^4\*x^2+24\*ln(F)^2\*b^2\*c^2\*d\*e^3\*x+12\*b^2\*c^2\*ln(F)^2\*e^2\*d^2-24\*ln(F)\*b\*c\*e^4\*x-24\*d\*e^3\*b\*c\*ln(F)+24\*e^4)\*F^(c\*(b\*x+a))/b^5/c^5/ln(F)^5

**Maxima [B]** time = 1.05755, size = 417, normalized size = 2.96

$$\frac{F^{bcx+ac} d^4}{bc \log(F)} + \frac{4(F^{ac} bcx \log(F) - F^{ac}) F^{bcx} d^3 e}{b^2 c^2 \log(F)^2} + \frac{6(F^{ac} b^2 c^2 x^2 \log(F)^2 - 2 F^{ac} bcx \log(F) + 2 F^{ac}) F^{bcx} d^2 e^2}{b^3 c^3 \log(F)^3} + \frac{4(F^{ac} b^3 c^3 x^3 \log(F)^3 - 3 F^{ac} b^2 c^2 x^2 \log(F)^2 + 6 F^{ac} b c x \log(F) - 6 F^{ac}) F^{bcx} d e^3}{b^4 c^4 \log(F)^4} + \frac{(F^{ac}) b^4 c^4 x^4 \log(F)^4 - 4 F^{ac} b^3 c^3 x^3 \log(F)^3 + 12 F^{ac} b^2 c^2 x^2 \log(F)^2 - 24 F^{ac} b c x \log(F) + 24 F^{ac}) F^{bcx} e^4}{b^5 c^5 \log(F)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(c\*(b\*x+a))\*(e^4\*x^4+4\*d\*e^3\*x^3+6\*d^2\*e^2\*x^2+4\*d^3\*e\*x+d^4), x, algorithm="maxima")

[Out] F^(b\*c\*x + a\*c)\*d^4/(b\*c\*log(F)) + 4\*(F^(a\*c)\*b\*c\*x\*log(F) - F^(a\*c))\*F^(b\*c\*x)\*d^3\*e/(b^2\*c^2\*log(F)^2) + 6\*(F^(a\*c)\*b^2\*c^2\*x^2\*log(F)^2 - 2\*F^(a\*c)\*b\*c\*x\*log(F) + 2\*F^(a\*c))\*F^(b\*c\*x)\*d^2\*e^2/(b^3\*c^3\*log(F)^3) + 4\*(F^(a\*c)\*b^3\*c^3\*x^3\*log(F)^3 - 3\*F^(a\*c)\*b^2\*c^2\*x^2\*log(F)^2 + 6\*F^(a\*c)\*b\*c\*x\*log(F) - 6\*F^(a\*c))\*F^(b\*c\*x)\*d\*e^3/(b^4\*c^4\*log(F)^4) + (F^(a\*c)\*b^4\*c^4\*x^4\*log(F)^4 - 4\*F^(a\*c)\*b^3\*c^3\*x^3\*log(F)^3 + 12\*F^(a\*c)\*b^2\*c^2\*x^2\*log(F)^2 - 24\*F^(a\*c)\*b\*c\*x\*log(F) + 24\*F^(a\*c))\*F^(b\*c\*x)\*e^4/(b^5\*c^5\*log(F)^5)

**Fricas [A]** time = 1.51039, size = 474, normalized size = 3.36

$$\frac{((b^4 c^4 e^4 x^4 + 4 b^4 c^4 d e^3 x^3 + 6 b^4 c^4 d^2 e^2 x^2 + 4 b^4 c^4 d^3 e x + b^4 c^4 d^4) \log(F)^4 + 24 e^4 - 4 (b^3 c^3 e^4 x^3 + 3 b^3 c^3 d e^3 x^2 + 3 b^3 c^3 d^2 e^2 x + b^3 c^3 d^3 e) \log(F)^3 + 12 (b^2 c^2 e^4 x^2 + 2 b^2 c^2 d e^3 x + b^2 c^2 d^2 e^2) \log(F)^2 - 24 (b c e^4 x + b c d e^3) \log(F) + 24 e^4) F^{c(bx+a)}}{b^5 c^5 \log(F)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*(e^4*x^4+4*d*e^3*x^3+6*d^2*e^2*x^2+4*d^3*e*x+d^4),x
, algorithm="fricas")
```

```
[Out] ((b^4*c^4*e^4*x^4 + 4*b^4*c^4*d*e^3*x^3 + 6*b^4*c^4*d^2*e^2*x^2 + 4*b^4*c^4
*d^3*e*x + b^4*c^4*d^4)*log(F)^4 + 24*e^4 - 4*(b^3*c^3*e^4*x^3 + 3*b^3*c^3*
d*e^3*x^2 + 3*b^3*c^3*d^2*e^2*x + b^3*c^3*d^3*e)*log(F)^3 + 12*(b^2*c^2*e^4
*x^2 + 2*b^2*c^2*d*e^3*x + b^2*c^2*d^2*e^2)*log(F)^2 - 24*(b*c*e^4*x + b*c*
d*e^3)*log(F))*F^(b*c*x + a*c)/(b^5*c^5*log(F)^5)
```

**Sympy [A]** time = 0.234603, size = 350, normalized size = 2.48

$$\left\{ \frac{F^{c(a+bx)}(b^4c^4d^4\log(F)^4+4b^4c^4d^3ex\log(F)^4+6b^4c^4d^2e^2x^2\log(F)^4+4b^4c^4de^3x^3\log(F)^4+b^4c^4e^4x^4\log(F)^4-4b^3c^3d^3e\log(F)^3-12b^3c^3d^2e^2x\log(F)^3-12b^3c^3de^3x^2\log(F)^3+12b^2c^2d^2e^2x^2\log(F)^2+24e^4-4(b^3c^3e^4x^3+3b^3c^3d^3e)\log(F)^2-24(b^2c^2e^4x^2+2b^2c^2d^2e^3x+b^2c^2d^2e^2)\log(F)-24(bce^4x+bcd^3e)\log(F))F^{bcx+ac}}{b^5c^5\log(F)^5} \right. \\ \left. d^4x + 2d^3ex^2 + 2d^2e^2x^3 + de^3x^4 + \frac{e^4x^5}{5} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(c*(b*x+a))*(e**4*x**4+4*d*e**3*x**3+6*d**2*e**2*x**2+4*d**3*e
*x+d**4),x)
```

```
[Out] Piecewise((F**(c*(a + b*x))*(b**4*c**4*d**4*log(F)**4 + 4*b**4*c**4*d**3*e*
x*log(F)**4 + 6*b**4*c**4*d**2*e**2*x**2*log(F)**4 + 4*b**4*c**4*d*e**3*x**
3*log(F)**4 + b**4*c**4*d**4*x**4*log(F)**4 - 4*b**3*c**3*d**3*e*log(F)**3
- 12*b**3*c**3*d**2*e**2*x*log(F)**3 - 12*b**3*c**3*d*e**3*x**2*log(F)**3 -
4*b**3*c**3*d**4*x**3*log(F)**3 + 12*b**2*c**2*d**2*e**2*log(F)**2 + 24*b*
**2*c**2*d*e**3*x*log(F)**2 + 12*b**2*c**2*d**4*x**2*log(F)**2 - 24*b*c*d*e*
**3*log(F) - 24*b*c*e**4*x*log(F) + 24*e**4)/(b**5*c**5*log(F)**5), Ne(b**5*
c**5*log(F)**5, 0)), (d**4*x + 2*d**3*e*x**2 + 2*d**2*e**2*x**3 + d*e**3*x*
**4 + e**4*x**5/5, True))
```

**Giac [C]** time = 1.4932, size = 10620, normalized size = 75.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*(e^4*x^4+4*d*e^3*x^3+6*d^2*e^2*x^2+4*d^3*e*x+d^4),x
, algorithm="giac")
```





$$\begin{aligned}
& (F))^2 - 5\pi b^5 c^5 \log(\text{abs}(F))^4)^2 + (5\pi^4 b^5 c^5 \log(\text{abs}(F)) \text{sgn}(F) \\
& - 10\pi^2 b^5 c^5 \log(\text{abs}(F))^3 \text{sgn}(F) - 5\pi^4 b^5 c^5 \log(\text{abs}(F)) + 10\pi \\
& i^2 b^5 c^5 \log(\text{abs}(F))^3 - 2b^5 c^5 \log(\text{abs}(F))^5)^2) \sin(-1/2\pi i b c x x \\
& \text{sgn}(F) + 1/2\pi i b c x - 1/2\pi i a c \text{sgn}(F) + 1/2\pi i a c) e^{(b c x \log(\text{abs}(F) \\
& )) + a c \log(\text{abs}(F)) + 4} + 1/2 I * ((-16 I \pi^4 b^4 c^4 x^4 \text{sgn}(F) + 64 \pi^3 \\
& * b^4 c^4 x^4 \log(\text{abs}(F)) \text{sgn}(F) + 96 I \pi^2 b^4 c^4 x^4 \log(\text{abs}(F))^2 \text{sgn}(F) \\
& ) - 64 \pi b^4 c^4 x^4 \log(\text{abs}(F))^3 \text{sgn}(F) + 16 I \pi^4 b^4 c^4 x^4 - 64 \pi i^3 \\
& * b^4 c^4 x^4 \log(\text{abs}(F)) - 96 I \pi^2 b^4 c^4 x^4 \log(\text{abs}(F))^2 + 64 \pi i b^4 \\
& * c^4 x^4 \log(\text{abs}(F))^3 + 32 I b^4 c^4 x^4 \log(\text{abs}(F))^4 - 64 \pi i^3 b^3 c^3 x^3 \\
& ^3 \text{sgn}(F) - 192 I \pi^2 b^3 c^3 x^3 \log(\text{abs}(F)) \text{sgn}(F) + 192 \pi i b^3 c^3 x^3 \\
& \log(\text{abs}(F))^2 \text{sgn}(F) + 64 \pi i^3 b^3 c^3 x^3 + 192 I \pi^2 b^3 c^3 x^3 \log(\text{abs} \\
& (F)) - 192 \pi i b^3 c^3 x^3 \log(\text{abs}(F))^2 - 128 I b^3 c^3 x^3 \log(\text{abs}(F))^3 + \\
& 192 I \pi^2 b^2 c^2 x^2 \text{sgn}(F) - 384 \pi i b^2 c^2 x^2 \log(\text{abs}(F)) \text{sgn}(F) - 19 \\
& 2 I \pi^2 b^2 c^2 x^2 + 384 \pi i b^2 c^2 x^2 \log(\text{abs}(F)) + 384 I b^2 c^2 x^2 \log \\
& (\text{abs}(F))^2 + 384 \pi i b c x x \text{sgn}(F) - 384 \pi i b c x - 768 I b c x \log(\text{abs}(F)) \\
& + 768 I) e^{(1/2 I \pi i b c x x \text{sgn}(F) - 1/2 I \pi i b c x + 1/2 I \pi i a c \text{sgn}(F) - \\
& 1/2 I \pi i a c) / (16 I \pi^5 b^5 c^5 \text{sgn}(F) - 80 \pi^4 b^5 c^5 \log(\text{abs}(F)) \text{sgn}( \\
& F) - 160 I \pi^3 b^5 c^5 \log(\text{abs}(F))^2 \text{sgn}(F) + 160 \pi^2 b^5 c^5 \log(\text{abs}(F)) \\
& ^3 \text{sgn}(F) + 80 I \pi i b^5 c^5 \log(\text{abs}(F))^4 \text{sgn}(F) - 16 I \pi^5 b^5 c^5 + 80 \pi \\
& i^4 b^5 c^5 \log(\text{abs}(F)) + 160 I \pi^3 b^5 c^5 \log(\text{abs}(F))^2 - 160 \pi^2 b^5 c^5 \\
& ^5 \log(\text{abs}(F))^3 - 80 I \pi i b^5 c^5 \log(\text{abs}(F))^4 + 32 b^5 c^5 \log(\text{abs}(F))^5 \\
& ) - (-16 I \pi^4 b^4 c^4 x^4 \text{sgn}(F) - 64 \pi i^3 b^4 c^4 x^4 \log(\text{abs}(F)) \text{sgn}(F) \\
& + 96 I \pi^2 b^4 c^4 x^4 \log(\text{abs}(F))^2 \text{sgn}(F) + 64 \pi i b^4 c^4 x^4 \log(\text{abs}(F) \\
& ))^3 \text{sgn}(F) + 16 I \pi^4 b^4 c^4 x^4 + 64 \pi i^3 b^4 c^4 x^4 \log(\text{abs}(F)) - 96 I \\
& \pi^2 b^4 c^4 x^4 \log(\text{abs}(F))^2 - 64 \pi i b^4 c^4 x^4 \log(\text{abs}(F))^3 + 32 I b^4 c^4 \\
& x^4 \log(\text{abs}(F))^4 + 64 \pi i^3 b^3 c^3 x^3 \text{sgn}(F) - 192 I \pi^2 b^3 c^3 x^3 \\
& x^3 \log(\text{abs}(F)) \text{sgn}(F) - 192 \pi i b^3 c^3 x^3 \log(\text{abs}(F))^2 \text{sgn}(F) - 64 \pi i^3 \\
& b^3 c^3 x^3 + 192 I \pi^2 b^3 c^3 x^3 \log(\text{abs}(F)) + 192 \pi i b^3 c^3 x^3 \log(a \\
& bs(F))^2 - 128 I b^3 c^3 x^3 \log(\text{abs}(F))^3 + 192 I \pi^2 b^2 c^2 x^2 \text{sgn}(F) \\
& + 384 \pi i b^2 c^2 x^2 \log(\text{abs}(F)) \text{sgn}(F) - 192 I \pi^2 b^2 c^2 x^2 - 384 \pi i b \\
& ^2 c^2 x^2 \log(\text{abs}(F)) + 384 I b^2 c^2 x^2 \log(\text{abs}(F))^2 - 384 \pi i b c x x \text{sgn} \\
& (F) + 384 \pi i b c x - 768 I b c x \log(\text{abs}(F)) + 768 I) e^{(-1/2 I \pi i b c x x \text{sg} \\
& n(F) + 1/2 I \pi i b c x - 1/2 I \pi i a c \text{sgn}(F) + 1/2 I \pi i a c) / (-16 I \pi^5 b^5 \\
& * c^5 \text{sgn}(F) - 80 \pi^4 b^5 c^5 \log(\text{abs}(F)) \text{sgn}(F) + 160 I \pi^3 b^5 c^5 \log(a \\
& bs(F))^2 \text{sgn}(F) + 160 \pi^2 b^5 c^5 \log(\text{abs}(F))^3 \text{sgn}(F) - 80 I \pi i b^5 c^5 \log \\
& (\text{abs}(F))^4 \text{sgn}(F) + 16 I \pi^5 b^5 c^5 + 80 \pi i^4 b^5 c^5 \log(\text{abs}(F)) - 160 \\
& * I \pi^3 b^5 c^5 \log(\text{abs}(F))^2 - 160 \pi^2 b^5 c^5 \log(\text{abs}(F))^3 + 80 I \pi i b^5 \\
& c^5 \log(\text{abs}(F))^4 + 32 b^5 c^5 \log(\text{abs}(F))^5) e^{(b c x \log(\text{abs}(F)) + a c \\
& * \log(\text{abs}(F)) + 4) - 4 * (((3 \pi^2 b^3 c^3 d x^3 \log(\text{abs}(F)) \text{sgn}(F) - 3 \pi^2 b \\
& ^3 c^3 d x^3 \log(\text{abs}(F)) + 2 b^3 c^3 d x^3 \log(\text{abs}(F))^3 - 3 \pi^2 b^2 c^2 d \\
& * x^2 \text{sgn}(F) + 3 \pi^2 b^2 c^2 d x^2 - 6 b^2 c^2 d x^2 \log(\text{abs}(F))^2 + 12 b c \\
& * d x \log(\text{abs}(F)) - 12 d) * (\pi^4 b^4 c^4 \text{sgn}(F) - 6 \pi^2 b^4 c^4 \log(\text{abs}(F))^ \\
& 2 \text{sgn}(F) - \pi^4 b^4 c^4 + 6 \pi^2 b^4 c^4 \log(\text{abs}(F))^2 - 2 b^4 c^4 \log(\text{abs}( \\
& F))^4) / ((\pi^4 b^4 c^4 \text{sgn}(F) - 6 \pi^2 b^4 c^4 \log(\text{abs}(F))^2 \text{sgn}(F) - \pi^4 b \\
& ^4 c^4 + 6 \pi^2 b^4 c^4 \log(\text{abs}(F))^2 - 2 b^4 c^4 \log(\text{abs}(F))^4)^2 + 16 * (\pi
\end{aligned}$$

$$\begin{aligned}
& ^3b^4c^4\log(\text{abs}(F))*\text{sgn}(F) - \text{pi}b^4c^4\log(\text{abs}(F))^3*\text{sgn}(F) - \text{pi}^3b^4c^4\log(\text{abs}(F))^4 \\
& + \text{pi}b^4c^4\log(\text{abs}(F))^3)^2) - 4*(\text{pi}^3b^3c^3d*x^3*\text{sgn}(F) - 3*\text{pi}b^3c^3d*x^3*\log(\text{abs}(F))^2*\text{sgn}(F) - \text{pi}^3b^3c^3d*x^3 \\
& + 3*\text{pi}b^3c^3d*x^3*\log(\text{abs}(F))^2 + 6*\text{pi}b^2c^2d*x^2*\log(\text{abs}(F))*\text{sgn}(F) - 6*\text{pi}b^2c^2d*x^2*\log(\text{abs}(F)) \\
& - 6*\text{pi}b*c*d*x*\text{sgn}(F) + 6*\text{pi}b*c*d*x*(\text{pi}^3b^4c^4*\log(\text{abs}(F))*\text{sgn}(F) - \text{pi}b^4c^4\log(\text{abs}(F))^3*\text{sgn}(F) - \text{pi}^3b^4c^4\log(\text{abs}(F))^4 \\
& + \text{pi}b^4c^4\log(\text{abs}(F))^3)/((\text{pi}^4b^4c^4*\text{sgn}(F) - 6*\text{pi}^2b^4c^4*\log(\text{abs}(F))^2*\text{sgn}(F) - \text{pi}^4b^4c^4 + 6*\text{pi}^2b^4c^4*\log(\text{abs}(F))^2 - 2*b^4c^4 \\
& \log(\text{abs}(F))^4)^2 + 16*(\text{pi}^3b^4c^4*\log(\text{abs}(F))*\text{sgn}(F) - \text{pi}b^4c^4\log(\text{abs}(F))^3*\text{sgn}(F) - \text{pi}^3b^4c^4\log(\text{abs}(F)) + \text{pi}b^4c^4\log(\text{abs}(F))^3)^2))* \\
& \cos(-1/2*\text{pi}b*c*x*\text{sgn}(F) + 1/2*\text{pi}b*c*x - 1/2*\text{pi}a*c*\text{sgn}(F) + 1/2*\text{pi}a*c) - \\
& ((\text{pi}^3b^3c^3d*x^3*\text{sgn}(F) - 3*\text{pi}b^3c^3d*x^3*\log(\text{abs}(F))^2*\text{sgn}(F) - \text{pi}^3b^3c^3d*x^3 + 3*\text{pi}b^3c^3d*x^3*\log(\text{abs}(F))^2 + 6*\text{pi}b^2c^2d*x^2*\log(\text{abs}(F))*\text{sgn}(F) - 6*\text{pi}b^2c^2d*x^2*\log(\text{abs}(F)) \\
& - 6*\text{pi}b*c*d*x*\text{sgn}(F) + 6*\text{pi}b*c*d*x*(\text{pi}^4b^4c^4*\text{sgn}(F) - 6*\text{pi}^2b^4c^4*\log(\text{abs}(F))^2*\text{sgn}(F) - \text{pi}^4b^4c^4 + 6*\text{pi}^2b^4c^4*\log(\text{abs}(F))^2 - 2*b^4c^4 \\
& \log(\text{abs}(F))^4)^2 + 16*(\text{pi}^3b^4c^4*\log(\text{abs}(F))*\text{sgn}(F) - \text{pi}b^4c^4\log(\text{abs}(F))^3*\text{sgn}(F) - \text{pi}^3b^4c^4\log(\text{abs}(F)) + \text{pi}b^4c^4\log(\text{abs}(F))^3)^2) + 4*(3*\text{pi}^2b^3c^3d*x^3*\log(\text{abs}(F))*\text{sgn}(F) - 3*\text{pi}^2b^3c^3d*x^3*\log(\text{abs}(F)) + 2*b^3c^3d*x^3*\log(\text{abs}(F))^3 - 3*\text{pi}^2b^2c^2d*x^2*\text{sgn}(F) + 3*\text{pi}^2b^2c^2d*x^2 - 6*b^2c^2d*x^2*\log(\text{abs}(F))^2 + 12*b*c*d*x*\log(\text{abs}(F)) - 12*d*(\text{pi}^3b^4c^4*\log(\text{abs}(F))*\text{sgn}(F) - \text{pi}b^4c^4\log(\text{abs}(F))^3*\text{sgn}(F) - \text{pi}^3b^4c^4\log(\text{abs}(F)) + \text{pi}b^4c^4\log(\text{abs}(F))^3)/((\text{pi}^4b^4c^4*\text{sgn}(F) - 6*\text{pi}^2b^4c^4*\log(\text{abs}(F))^2*\text{sgn}(F) - \text{pi}^4b^4c^4 + 6*\text{pi}^2b^4c^4*\log(\text{abs}(F))^2 - 2*b^4c^4\log(\text{abs}(F))^4)^2 + 16*(\text{pi}^3b^4c^4*\log(\text{abs}(F))*\text{sgn}(F) - \text{pi}b^4c^4\log(\text{abs}(F))^3*\text{sgn}(F) - \text{pi}^3b^4c^4\log(\text{abs}(F)) + \text{pi}b^4c^4\log(\text{abs}(F))^3)^2))*\sin(-1/2*\text{pi}b*c*x*\text{sgn}(F) + 1/2*\text{pi}b*c*x - 1/2*\text{pi}a*c*\text{sgn}(F) + 1/2*\text{pi}a*c))*e^(b*c*x*\log(\text{abs}(F)) + a*c*\log(\text{abs}(F)) + 3) - 1/2*I*((32*\text{pi}^3b^3c^3d*x^3*\text{sgn}(F) + 96*I*\text{pi}^2b^3c^3d*x^3*\log(\text{abs}(F))*\text{sgn}(F) - 96*\text{pi}b^3c^3d*x^3*\log(\text{abs}(F))^2*\text{sgn}(F) - 32*\text{pi}^3b^3c^3d*x^3 - 96*I*\text{pi}^2b^3c^3d*x^3*\log(\text{abs}(F)) + 96*\text{pi}b^3c^3d*x^3*\log(\text{abs}(F))^2 + 64*I*b^3c^3d*x^3*\log(\text{abs}(F))^3 - 96*I*\text{pi}^2b^2c^2d*x^2*\text{sgn}(F) + 192*\text{pi}b^2c^2d*x^2*\log(\text{abs}(F))*\text{sgn}(F) + 96*I*\text{pi}^2b^2c^2d*x^2 - 192*\text{pi}b^2c^2d*x^2*\log(\text{abs}(F)) - 192*I*b^2c^2d*x^2*\log(\text{abs}(F))^2 - 192*\text{pi}b*c*d*x*\text{sgn}(F) + 192*\text{pi}b*c*d*x + 384*I*b*c*d*x*\log(\text{abs}(F)) - 384*I*d))*e^(1/2*I*\text{pi}b*c*x*\text{sgn}(F) - 1/2*I*\text{pi}b*c*x + 1/2*I*\text{pi}a*c*\text{sgn}(F) - 1/2*I*\text{pi}a*c)/(8*\text{pi}^4b^4c^4*\text{sgn}(F) + 32*I*\text{pi}^3b^4c^4*\log(\text{abs}(F))*\text{sgn}(F) - 48*\text{pi}^2b^4c^4*\log(\text{abs}(F))^2*\text{sgn}(F) - 32*I*\text{pi}b^4c^4*\log(\text{abs}(F))^3*\text{sgn}(F) - 8*\text{pi}^4b^4c^4 - 32*I*\text{pi}^3b^4c^4*\log(\text{abs}(F)) + 48*\text{pi}^2b^4c^4*\log(\text{abs}(F))^2 + 32*I*\text{pi}b^4c^4*\log(\text{abs}(F))^3 - 16*b^4c^4*\log(\text{abs}(F))^4) + (32*\text{pi}^3b^3c^3d*x^3*\text{sgn}(F) - 96*I*\text{pi}^2b^3c^3d*x^3*\log(\text{abs}(F))*\text{sgn}(F) - 96*\text{pi}b^3c^3d*x^3*\log(\text{abs}(F))^2*\text{sgn}(F) - 32*\text{pi}^3b^3c^3d*x^3 + 96*I*\text{pi}^2b^3c^3d*x^3*\log(\text{abs}(F)) + 96*\text{pi}b^3c^3d*x^3*\log(\text{abs}(F))^2 - 64*I*b^3c^3d*x^3*\log(\text{abs}(F))^3 + 96*I*\text{pi}^2b^2c^2d*x^2*\text{sgn}(F) + 192*\text{pi}b^2c^2d*
\end{aligned}$$

$$\begin{aligned}
& x^2 \log(\operatorname{abs}(F)) \operatorname{sgn}(F) - 96 I \pi^2 b^2 c^2 d x^2 - 192 \pi b^2 c^2 d x^2 \log(\operatorname{abs}(F)) + 192 I b^2 c^2 d x^2 \log(\operatorname{abs}(F))^2 - 192 \pi b^2 c^2 d x^2 \operatorname{sgn}(F) + 192 I \pi b^2 c^2 d x^2 \operatorname{sgn}(F) \\
& - 384 I \pi b^2 c^2 d x^2 \log(\operatorname{abs}(F)) + 384 I d e^{(-1/2 I \pi b^2 c^2 x \operatorname{sgn}(F) + 1/2 I \pi b^2 c^2 x - 1/2 I \pi a^2 c^2 \operatorname{sgn}(F) + 1/2 I \pi a^2 c^2)} / (8 \pi^4 b^4 c^4 \operatorname{sgn}(F) - 32 I \pi^3 b^4 c^4 \log(\operatorname{abs}(F)) \operatorname{sgn}(F) - 48 \pi^2 b^4 c^4 \log(\operatorname{abs}(F))^2 \operatorname{sgn}(F) \\
& + 32 I \pi b^4 c^4 \log(\operatorname{abs}(F))^3 \operatorname{sgn}(F) - 8 \pi^4 b^4 c^4 + 32 I \pi^3 b^4 c^4 \log(\operatorname{abs}(F)) + 48 \pi^2 b^4 c^4 \log(\operatorname{abs}(F))^2 - 32 I \pi b^4 c^4 \log(\operatorname{abs}(F))^3 - 16 b^4 c^4 \log(\operatorname{abs}(F))^4) e^{(b^2 c^2 x \log(\operatorname{abs}(F)) + a^2 c^2 \log(\operatorname{abs}(F)) + 3)} \\
& - 6 \left( (2 (\pi b^2 c^2 d^2 x^2 \log(\operatorname{abs}(F)) \operatorname{sgn}(F) - \pi b^2 c^2 d^2 x^2 \log(\operatorname{abs}(F)) - \pi b^2 c^2 d^2 x \operatorname{sgn}(F) + \pi b^2 c^2 d^2 x) (\pi^3 b^3 c^3 \operatorname{sgn}(F) - 3 \pi b^3 c^3 \log(\operatorname{abs}(F))^2 \operatorname{sgn}(F) - \pi^3 b^3 c^3 + 3 \pi b^3 c^3 \log(\operatorname{abs}(F))^2) \right. \\
& \left. \right) / ((\pi^3 b^3 c^3 \operatorname{sgn}(F) - 3 \pi b^3 c^3 \log(\operatorname{abs}(F))^2 \operatorname{sgn}(F) - \pi^3 b^3 c^3 + 3 \pi b^3 c^3 \log(\operatorname{abs}(F))^2)^2 + (3 \pi^2 b^3 c^3 \log(\operatorname{abs}(F)) \operatorname{sgn}(F) - 3 \pi^2 b^3 c^3 \log(\operatorname{abs}(F)) + 2 b^3 c^3 \log(\operatorname{abs}(F))^3)^2) - (\pi^2 b^2 c^2 d^2 x^2 \operatorname{sgn}(F) - \pi^2 b^2 c^2 d^2 x^2 + 2 b^2 c^2 d^2 x^2 \log(\operatorname{abs}(F))^2 - 4 b^2 c^2 d^2 x^2 \log(\operatorname{abs}(F)) + 4 d^2) (3 \pi^2 b^3 c^3 \log(\operatorname{abs}(F)) \operatorname{sgn}(F) - 3 \pi^2 b^3 c^3 \log(\operatorname{abs}(F)) + 2 b^3 c^3 \log(\operatorname{abs}(F))^3) / ((\pi^3 b^3 c^3 \operatorname{sgn}(F) - 3 \pi b^3 c^3 \log(\operatorname{abs}(F))^2 \operatorname{sgn}(F) - \pi^3 b^3 c^3 + 3 \pi b^3 c^3 \log(\operatorname{abs}(F))^2)^2 + (3 \pi^2 b^3 c^3 \log(\operatorname{abs}(F)) \operatorname{sgn}(F) - 3 \pi^2 b^3 c^3 \log(\operatorname{abs}(F)) + 2 b^3 c^3 \log(\operatorname{abs}(F))^3)^2) \cos(-1/2 \pi b^2 c^2 x \operatorname{sgn}(F) + 1/2 \pi b^2 c^2 x - 1/2 \pi a^2 c^2 \operatorname{sgn}(F) + 1/2 \pi a^2 c^2) - ((\pi^2 b^2 c^2 d^2 x^2 \operatorname{sgn}(F) - \pi^2 b^2 c^2 d^2 x^2 + 2 b^2 c^2 d^2 x^2 \log(\operatorname{abs}(F))^2 - 4 b^2 c^2 d^2 x^2 \log(\operatorname{abs}(F)) + 4 d^2) (\pi^3 b^3 c^3 \operatorname{sgn}(F) - 3 \pi b^3 c^3 \log(\operatorname{abs}(F))^2 \operatorname{sgn}(F) - \pi^3 b^3 c^3 + 3 \pi b^3 c^3 \log(\operatorname{abs}(F))^2) / ((\pi^3 b^3 c^3 \operatorname{sgn}(F) - 3 \pi b^3 c^3 \log(\operatorname{abs}(F))^2 \operatorname{sgn}(F) - \pi^3 b^3 c^3 + 3 \pi b^3 c^3 \log(\operatorname{abs}(F))^2)^2 + (3 \pi^2 b^3 c^3 \log(\operatorname{abs}(F)) \operatorname{sgn}(F) - 3 \pi^2 b^3 c^3 \log(\operatorname{abs}(F)) + 2 b^3 c^3 \log(\operatorname{abs}(F))^3)^2) + 2 (\pi b^2 c^2 d^2 x^2 \log(\operatorname{abs}(F)) \operatorname{sgn}(F) - \pi b^2 c^2 d^2 x^2 \log(\operatorname{abs}(F)) - \pi b^2 c^2 d^2 x \operatorname{sgn}(F) + \pi b^2 c^2 d^2 x) (3 \pi^2 b^3 c^3 \log(\operatorname{abs}(F)) \operatorname{sgn}(F) - 3 \pi^2 b^3 c^3 \log(\operatorname{abs}(F)) + 2 b^3 c^3 \log(\operatorname{abs}(F))^3) / ((\pi^3 b^3 c^3 \operatorname{sgn}(F) - 3 \pi b^3 c^3 \log(\operatorname{abs}(F))^2 \operatorname{sgn}(F) - \pi^3 b^3 c^3 + 3 \pi b^3 c^3 \log(\operatorname{abs}(F))^2)^2 + (3 \pi^2 b^3 c^3 \log(\operatorname{abs}(F)) \operatorname{sgn}(F) - 3 \pi^2 b^3 c^3 \log(\operatorname{abs}(F)) + 2 b^3 c^3 \log(\operatorname{abs}(F))^3)^2) \sin(-1/2 \pi b^2 c^2 x \operatorname{sgn}(F) + 1/2 \pi b^2 c^2 x - 1/2 \pi a^2 c^2 \operatorname{sgn}(F) + 1/2 \pi a^2 c^2) e^{(b^2 c^2 x \log(\operatorname{abs}(F)) + a^2 c^2 \log(\operatorname{abs}(F)) + 2)} + 1/2 I ((24 I \pi^2 b^2 c^2 d^2 x^2 \operatorname{sgn}(F) - 48 \pi b^2 c^2 d^2 x^2 \log(\operatorname{abs}(F)) \operatorname{sgn}(F) - 24 I \pi^2 b^2 c^2 d^2 x^2 + 48 \pi b^2 c^2 d^2 x^2 \log(\operatorname{abs}(F)) + 48 I b^2 c^2 d^2 x^2 \log(\operatorname{abs}(F))^2 + 48 \pi b^2 c^2 d^2 x \operatorname{sgn}(F) - 48 \pi b^2 c^2 d^2 x - 96 I b^2 c^2 d^2 x \log(\operatorname{abs}(F)) + 96 I d^2) e^{(1/2 I \pi b^2 c^2 x \operatorname{sgn}(F) - 1/2 I \pi b^2 c^2 x + 1/2 I \pi a^2 c^2 \operatorname{sgn}(F) - 1/2 I \pi a^2 c^2)} / (-4 I \pi^3 b^3 c^3 \operatorname{sgn}(F) + 12 \pi^2 b^3 c^3 \log(\operatorname{abs}(F)) \operatorname{sgn}(F) + 12 I \pi b^3 c^3 \log(\operatorname{abs}(F))^2 \operatorname{sgn}(F) + 4 I \pi^3 b^3 c^3 - 12 \pi^2 b^3 c^3 \log(\operatorname{abs}(F)) - 12 I \pi b^3 c^3 \log(\operatorname{abs}(F))^2 + 8 b^3 c^3 \log(\operatorname{abs}(F))^3) - (24 I \pi^2 b^2 c^2 d^2 x^2 \operatorname{sgn}(F) + 48 \pi b^2 c^2 d^2 x^2 \log(\operatorname{abs}(F)) \operatorname{sgn}(F) - 24 I \pi^2 b^2 c^2 d^2 x^2 - 48 \pi b^2 c^2 d^2 x^2 \log(\operatorname{abs}(F)) + 48 I b^2 c^2 d^2 x^2 \log(\operatorname{abs}(F))^2 - 48 \pi b^2 c^2 d^2 x^2 \log(\operatorname{abs}(F)) + 48 I b^2 c^2 d^2 x - 96 I b^2 c^2 d^2 x \log(\operatorname{abs}(F)) + 96 I d^2) e^{(-1/2 I \pi b^2 c^2 x \operatorname{sgn}(F) + 1/2 I \pi b^2 c^2 x - 1/2 I \pi a^2 c^2 \operatorname{sgn}(F) + 1/2 I \pi a^2 c^2)}
\end{aligned}$$

$$\begin{aligned}
&)/(4*I*pi^3*b^3*c^3*sgn(F) + 12*pi^2*b^3*c^3*log(abs(F))*sgn(F) - 12*I*pi*b^3*c^3*log(abs(F))^2*sgn(F) - 4*I*pi^3*b^3*c^3 - 12*pi^2*b^3*c^3*log(abs(F)) \\
&)+ 12*I*pi*b^3*c^3*log(abs(F))^2 + 8*b^3*c^3*log(abs(F))^3))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F)) + 2) + 4*(2*((b*c*d^3*x*log(abs(F)) - d^3)*(pi^2*b^2*c^2*sgn(F) - pi^2*b^2*c^2 + 2*b^2*c^2*log(abs(F))^2)/((pi^2*b^2*c^2*sgn(F) - pi^2*b^2*c^2 + 2*b^2*c^2*log(abs(F))^2)^2 + 4*(pi*b^2*c^2*log(abs(F))*sgn(F) - pi*b^2*c^2*log(abs(F)))^2) + (pi*b*c*d^3*x*sgn(F) - pi*b*c*d^3*x)*(pi*b^2*c^2*log(abs(F))*sgn(F) - pi*b^2*c^2*log(abs(F)))/((pi^2*b^2*c^2*sgn(F) - pi^2*b^2*c^2 + 2*b^2*c^2*log(abs(F))^2)^2 + 4*(pi*b^2*c^2*log(abs(F))*sgn(F) - pi*b^2*c^2*log(abs(F)))^2))*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c) + ((pi*b*c*d^3*x*sgn(F) - pi*b*c*d^3*x)*(pi^2*b^2*c^2*sgn(F) - pi^2*b^2*c^2 + 2*b^2*c^2*log(abs(F))^2)/((pi^2*b^2*c^2*sgn(F) - pi^2*b^2*c^2 + 2*b^2*c^2*log(abs(F))^2)^2 + 4*(pi*b^2*c^2*log(abs(F))*sgn(F) - pi*b^2*c^2*log(abs(F)))^2) - 4*(b*c*d^3*x*log(abs(F)) - d^3)*(pi*b^2*c^2*log(abs(F))*sgn(F) - pi*b^2*c^2*log(abs(F)))/((pi^2*b^2*c^2*sgn(F) - pi^2*b^2*c^2 + 2*b^2*c^2*log(abs(F))^2)^2 + 4*(pi*b^2*c^2*log(abs(F))*sgn(F) - pi*b^2*c^2*log(abs(F)))^2))*sin(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F)) + 1) - 1/2*I*((8*pi*b*c*d^3*x*sgn(F) - 8*pi*b*c*d^3*x - 16*I*b*c*d^3*x*log(abs(F)) + 16*I*d^3)*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*a*c)/(2*pi^2*b^2*c^2*sgn(F) + 4*I*pi*b^2*c^2*log(abs(F))*sgn(F) - 2*pi^2*b^2*c^2 - 4*I*pi*b^2*c^2*log(abs(F)) + 4*b^2*c^2*log(abs(F))^2) + (8*pi*b*c*d^3*x*sgn(F) - 8*pi*b*c*d^3*x + 16*I*b*c*d^3*x*log(abs(F)) - 16*I*d^3)*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c)/(2*pi^2*b^2*c^2*sgn(F) - 4*I*pi*b^2*c^2*log(abs(F))*sgn(F) - 2*pi^2*b^2*c^2 + 4*I*pi*b^2*c^2*log(abs(F)) + 4*b^2*c^2*log(abs(F))^2))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F)) + 1) + 2*(2*b*c*d^4*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)*log(abs(F))/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c)^2) - (pi*b*c*sgn(F) - pi*b*c)*d^4*sin(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c)^2))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))) - 1/2*I*(-2*I*d^4*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*a*c)/(I*pi*b*c*sgn(F) - I*pi*b*c + 2*b*c*log(abs(F))) + 2*I*d^4*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c)/(-I*pi*b*c*sgn(F) + I*pi*b*c + 2*b*c*log(abs(F))))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F)))
\end{aligned}$$

### 3.13 $\int F^{c(a+bx)} (d^3 + 3d^2ex + 3de^2x^2 + e^3x^3) dx$

**Optimal.** Leaf size=110

$$\frac{6e^2(d+ex)F^{c(a+bx)}}{b^3c^3\log^3(F)} - \frac{3e(d+ex)^2F^{c(a+bx)}}{b^2c^2\log^2(F)} - \frac{6e^3F^{c(a+bx)}}{b^4c^4\log^4(F)} + \frac{(d+ex)^3F^{c(a+bx)}}{bc\log(F)}$$

[Out]  $(-6e^3F^{c(a+bx)})/(b^4c^4\log^4(F)) + (6e^2F^{c(a+bx)}(d+ex))/(b^3c^3\log^3(F)) - (3eF^{c(a+bx)}(d+ex)^2)/(b^2c^2\log^2(F)) + (F^{c(a+bx)}(d+ex)^3)/(bc\log(F))$

**Rubi [A]** time = 0.0857957, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 37,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.081$ , Rules used = {2187, 2176, 2194}

$$\frac{6e^2(d+ex)F^{c(a+bx)}}{b^3c^3\log^3(F)} - \frac{3e(d+ex)^2F^{c(a+bx)}}{b^2c^2\log^2(F)} - \frac{6e^3F^{c(a+bx)}}{b^4c^4\log^4(F)} + \frac{(d+ex)^3F^{c(a+bx)}}{bc\log(F)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[F^{c(a+bx)}(d^3 + 3d^2ex + 3de^2x^2 + e^3x^3), x]$

[Out]  $(-6e^3F^{c(a+bx)})/(b^4c^4\log^4(F)) + (6e^2F^{c(a+bx)}(d+ex))/(b^3c^3\log^3(F)) - (3eF^{c(a+bx)}(d+ex)^2)/(b^2c^2\log^2(F)) + (F^{c(a+bx)}(d+ex)^3)/(bc\log(F))$

#### Rule 2187

```
Int[((a_.) + (b_.)*((F_)^((g_.)*(v_)))^(n_.))^(p_.)*(u_)^(m_.), x_Symbol] :
> Int[NormalizePowerOfLinear[u, x]^m*(a + b*(F^(g*ExpandToSum[v, x]))^n)^p,
x] /; FreeQ[{F, a, b, g, n, p}, x] && LinearQ[v, x] && PowerOfLinearQ[u, x]
] && !(LinearMatchQ[v, x] && PowerOfLinearMatchQ[u, x]) && IntegerQ[m]
```

#### Rule 2176

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[((c + d*x)^m*(b*F^(g*(e + f*x)))^n)/(f*g*n*Log[F]),
x] - Dist[(d*m)/(f*g*n*Log[F]), Int[(c + d*x)^(m-1)*(b*F^(g*(e + f*x)))^n,
x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m]
] && !$UseGamma === True
```



[In] int(F^(c\*(b\*x+a))\*(e^3\*x^3+3\*d\*e^2\*x^2+3\*d^2\*e\*x+d^3),x)

[Out] (e^3\*x^3\*b^3\*c^3\*ln(F)^3+3\*ln(F)^3\*b^3\*c^3\*d\*e^2\*x^2+3\*ln(F)^3\*b^3\*c^3\*d^2\*e\*x+b^3\*c^3\*ln(F)^3\*d^3-3\*ln(F)^2\*b^2\*c^2\*e^3\*x^2-6\*ln(F)^2\*b^2\*c^2\*d\*e^2\*x-3\*b^2\*c^2\*ln(F)^2\*e\*d^2+6\*ln(F)\*b\*c\*e^3\*x+6\*d\*e^2\*b\*c\*ln(F)-6\*e^3)\*F^(c\*(b\*x+a))/b^4/c^4/ln(F)^4

**Maxima [A]** time = 1.1271, size = 278, normalized size = 2.53

$$\frac{F^{bcx+ac}d^3}{bc \log(F)} + \frac{3(F^{ac}bcx \log(F) - F^{ac})F^{bcx}d^2e}{b^2c^2 \log(F)^2} + \frac{3(F^{ac}b^2c^2x^2 \log(F)^2 - 2F^{ac}bcx \log(F) + 2F^{ac})F^{bcx}de^2}{b^3c^3 \log(F)^3} + \frac{(F^{ac}b^3c^3x^3 \log(F)^3 - 6F^{ac}b^3c^3d^3)}{b^4c^4 \log(F)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(c\*(b\*x+a))\*(e^3\*x^3+3\*d\*e^2\*x^2+3\*d^2\*e\*x+d^3),x, algorithm="maxima")

[Out] F^(b\*c\*x + a\*c)\*d^3/(b\*c\*log(F)) + 3\*(F^(a\*c)\*b\*c\*x\*log(F) - F^(a\*c))\*F^(b\*c\*x)\*d^2\*e/(b^2\*c^2\*log(F)^2) + 3\*(F^(a\*c)\*b^2\*c^2\*x^2\*log(F)^2 - 2\*F^(a\*c)\*b\*c\*x\*log(F) + 2\*F^(a\*c))\*F^(b\*c\*x)\*d\*e^2/(b^3\*c^3\*log(F)^3) + (F^(a\*c)\*b^3\*c^3\*x^3\*log(F)^3 - 3\*F^(a\*c)\*b^2\*c^2\*x^2\*log(F)^2 + 6\*F^(a\*c)\*b\*c\*x\*log(F) - 6\*F^(a\*c))\*F^(b\*c\*x)\*e^3/(b^4\*c^4\*log(F)^4)

**Fricas [A]** time = 1.53344, size = 312, normalized size = 2.84

$$\frac{((b^3c^3e^3x^3 + 3b^3c^3de^2x^2 + 3b^3c^3d^2ex + b^3c^3d^3) \log(F)^3 - 6e^3 - 3(b^2c^2e^3x^2 + 2b^2c^2de^2x + b^2c^2d^2e) \log(F)^2 + 6(bce^3x^3 - 3b^2c^2d^2e) \log(F) - 6e^3)}{b^4c^4 \log(F)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(c\*(b\*x+a))\*(e^3\*x^3+3\*d\*e^2\*x^2+3\*d^2\*e\*x+d^3),x, algorithm="fricas")

[Out] ((b^3\*c^3\*e^3\*x^3 + 3\*b^3\*c^3\*d\*e^2\*x^2 + 3\*b^3\*c^3\*d^2\*e\*x + b^3\*c^3\*d^3)\*log(F)^3 - 6\*e^3 - 3\*(b^2\*c^2\*e^3\*x^2 + 2\*b^2\*c^2\*d\*e^2\*x + b^2\*c^2\*d^2\*e)\*log(F)^2 + 6\*(b\*c\*e^3\*x + b\*c\*d\*e^2)\*log(F))\*F^(b\*c\*x + a\*c)/(b^4\*c^4\*log(F)^4)

---

**Sympy [A]** time = 0.318774, size = 231, normalized size = 2.1

$$\left\{ \frac{F^{c(ax+bx)}(b^3c^3d^3 \log(F)^3 + 3b^3c^3d^2ex \log(F)^3 + 3b^3c^3de^2x^2 \log(F)^3 + b^3c^3e^3x^3 \log(F)^3 - 3b^2c^2d^2e \log(F)^2 - 6b^2c^2de^2x \log(F)^2 - 3b^2c^2e^3x^2 \log(F)^2 + 6bcde^2 \log(F) + 6c^3d^2e^2x \log(F) + 6c^3de^3x^2 \log(F) + 6c^3e^4x^3 \log(F) + 6c^4e^4x^4 \log(F))}{b^4c^4 \log(F)^4} \right.$$

$$\left. \left( d^3x + \frac{3d^2ex^2}{2} + de^2x^3 + \frac{e^3x^4}{4} \right) \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(c\*(b\*x+a))\*(e\*\*3\*x\*\*3+3\*d\*e\*\*2\*x\*\*2+3\*d\*\*2\*e\*x+d\*\*3),x)

[Out] Piecewise((F\*\*(c\*(a + b\*x))\*(b\*\*3\*c\*\*3\*d\*\*3\*log(F)\*\*3 + 3\*b\*\*3\*c\*\*3\*d\*\*2\*e\*x\*log(F)\*\*3 + 3\*b\*\*3\*c\*\*3\*d\*e\*\*2\*x\*\*2\*log(F)\*\*3 + b\*\*3\*c\*\*3\*e\*\*3\*x\*\*3\*log(F)\*\*3 - 3\*b\*\*2\*c\*\*2\*d\*\*2\*e\*log(F)\*\*2 - 6\*b\*\*2\*c\*\*2\*d\*e\*\*2\*x\*log(F)\*\*2 - 3\*b\*\*2\*c\*\*2\*e\*\*3\*x\*\*2\*log(F)\*\*2 + 6\*b\*c\*d\*e\*\*2\*log(F) + 6\*b\*c\*e\*\*3\*x\*log(F) - 6\*e\*\*3)/(b\*\*4\*c\*\*4\*log(F)\*\*4), Ne(b\*\*4\*c\*\*4\*log(F)\*\*4, 0)), (d\*\*3\*x + 3\*d\*\*2\*e\*x\*\*2/2 + d\*e\*\*2\*x\*\*3 + e\*\*3\*x\*\*4/4, True))

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**Giac [C]** time = 1.40736, size = 6336, normalized size = 57.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(c\*(b\*x+a))\*(e^3\*x^3+3\*d\*e^2\*x^2+3\*d^2\*e\*x+d^3),x, algorithm="giac")

[Out] ((4\*(pi^3\*b^3\*c^3\*x^3\*sgn(F) - 3\*pi\*b^3\*c^3\*x^3\*log(abs(F))^2\*sgn(F) - pi^3\*b^3\*c^3\*x^3 + 3\*pi\*b^3\*c^3\*x^3\*log(abs(F))^2 + 6\*pi\*b^2\*c^2\*x^2\*log(abs(F))\*sgn(F) - 6\*pi\*b^2\*c^2\*x^2\*log(abs(F)) - 6\*pi\*b\*c\*x\*sgn(F) + 6\*pi\*b\*c\*x)\*(pi^3\*b^4\*c^4\*log(abs(F))\*sgn(F) - pi\*b^4\*c^4\*log(abs(F))^3\*sgn(F) - pi^3\*b^4\*c^4\*log(abs(F)) + pi\*b^4\*c^4\*log(abs(F))^3)/((pi^4\*b^4\*c^4\*sgn(F) - 6\*pi^2\*b^4\*c^4\*log(abs(F))^2\*sgn(F) - pi^4\*b^4\*c^4 + 6\*pi^2\*b^4\*c^4\*log(abs(F))^2 - 2\*b^4\*c^4\*log(abs(F))^4)^2 + 16\*(pi^3\*b^4\*c^4\*log(abs(F))\*sgn(F) - pi\*b^4\*c^4\*log(abs(F))^3\*sgn(F) - pi^3\*b^4\*c^4\*log(abs(F)) + pi\*b^4\*c^4\*log(abs(F))^3)^2) - (pi^4\*b^4\*c^4\*sgn(F) - 6\*pi^2\*b^4\*c^4\*log(abs(F))^2\*sgn(F) - pi^4\*b^4\*c^4 + 6\*pi^2\*b^4\*c^4\*log(abs(F))^2 - 2\*b^4\*c^4\*log(abs(F))^4)\*(3\*pi^2\*b^3\*c^3\*x^3\*log(abs(F))\*sgn(F) - 3\*pi^2\*b^3\*c^3\*x^3\*log(abs(F)) + 2\*b^3\*c^3\*x^3\*log(abs(F))^3 - 3\*pi^2\*b^2\*c^2\*x^2\*sgn(F) + 3\*pi^2\*b^2\*c^2\*x^2 - 6\*b^2\*c^2\*x^2\*log(abs(F))^2 + 12\*b\*c\*x\*log(abs(F)) - 12)/((pi^4\*b^4\*c^4\*sgn(F) - 6\*pi^2\*b^4\*c^4\*log(abs(F))^2\*sgn(F) - pi^4\*b^4\*c^4 + 6\*pi^2\*b^4\*c^4\*log



$$\begin{aligned}
& (\text{abs}(F))^2 - 2*b^4*c^4*\log(\text{abs}(F))^4)^2 + 16*(\pi^3*b^4*c^4*\log(\text{abs}(F))*\text{sgn}(F) - \pi*b^4*c^4*\log(\text{abs}(F))^3*\text{sgn}(F) - \pi^3*b^4*c^4*\log(\text{abs}(F)) + \pi*b^4*c^4*\log(\text{abs}(F))^3)^2) * \cos(-1/2*\pi*b*c*x*\text{sgn}(F) + 1/2*\pi*b*c*x - 1/2*\pi*a*c*\text{sgn}(F) + 1/2*\pi*a*c) + ((\pi^4*b^4*c^4*\text{sgn}(F) - 6*\pi^2*b^4*c^4*\log(\text{abs}(F))^2*\text{sgn}(F) - \pi^4*b^4*c^4 + 6*\pi^2*b^4*c^4*\log(\text{abs}(F))^2 - 2*b^4*c^4*\log(\text{abs}(F))^4) * (\pi^3*b^3*c^3*x^3*\text{sgn}(F) - 3*\pi*b^3*c^3*x^3*\log(\text{abs}(F))^2*\text{sgn}(F) - \pi^3*b^3*c^3*x^3 + 3*\pi*b^3*c^3*x^3*\log(\text{abs}(F))^2 + 6*\pi*b^2*c^2*x^2*\log(\text{abs}(F))*\text{sgn}(F) - 6*\pi*b^2*c^2*x^2*\log(\text{abs}(F)) - 6*\pi*b*c*x*\text{sgn}(F) + 6*\pi*b*c*x) / ((\pi^4*b^4*c^4*\text{sgn}(F) - 6*\pi^2*b^4*c^4*\log(\text{abs}(F))^2*\text{sgn}(F) - \pi^4*b^4*c^4 + 6*\pi^2*b^4*c^4*\log(\text{abs}(F))^2 - 2*b^4*c^4*\log(\text{abs}(F))^4)^2 + 16*(\pi^3*b^4*c^4*\log(\text{abs}(F))*\text{sgn}(F) - \pi*b^4*c^4*\log(\text{abs}(F))^3*\text{sgn}(F) - \pi^3*b^4*c^4*\log(\text{abs}(F)) + \pi*b^4*c^4*\log(\text{abs}(F))^3)^2) + 4*(\pi^3*b^4*c^4*\log(\text{abs}(F))*\text{sgn}(F) - \pi*b^4*c^4*\log(\text{abs}(F))^3*\text{sgn}(F) - \pi^3*b^4*c^4*\log(\text{abs}(F)) + \pi*b^4*c^4*\log(\text{abs}(F))^3) * (3*\pi^2*b^3*c^3*x^3*\log(\text{abs}(F))*\text{sgn}(F) - 3*\pi^2*b^3*c^3*x^3*\log(\text{abs}(F)) + 2*b^3*c^3*x^3*\log(\text{abs}(F))^3 - 3*\pi^2*b^2*c^2*x^2*\text{sgn}(F) + 3*\pi^2*b^2*c^2*x^2 - 6*b^2*c^2*x^2*\log(\text{abs}(F))^2 + 12*b*c*x*\log(\text{abs}(F)) - 12) / ((\pi^4*b^4*c^4*\text{sgn}(F) - 6*\pi^2*b^4*c^4*\log(\text{abs}(F))^2*\text{sgn}(F) - \pi^4*b^4*c^4 + 6*\pi^2*b^4*c^4*\log(\text{abs}(F))^2 - 2*b^4*c^4*\log(\text{abs}(F))^4)^2 + 16*(\pi^3*b^4*c^4*\log(\text{abs}(F))*\text{sgn}(F) - \pi*b^4*c^4*\log(\text{abs}(F))^3*\text{sgn}(F) - \pi^3*b^4*c^4*\log(\text{abs}(F)) + \pi*b^4*c^4*\log(\text{abs}(F))^3)^2) * \sin(-1/2*\pi*b*c*x*\text{sgn}(F) + 1/2*\pi*b*c*x - 1/2*\pi*a*c*\text{sgn}(F) + 1/2*\pi*a*c)) * e^(b*c*x*\log(\text{abs}(F)) + a*c*\log(\text{abs}(F)) + 3) - 1/2*I*((8*\pi^3*b^3*c^3*x^3*\text{sgn}(F) + 24*I*\pi^2*b^3*c^3*x^3*\log(\text{abs}(F))*\text{sgn}(F) - 24*\pi*b^3*c^3*x^3*\log(\text{abs}(F))^2*\text{sgn}(F) - 8*\pi^3*b^3*c^3*x^3 - 24*I*\pi^2*b^3*c^3*x^3*\log(\text{abs}(F)) + 24*\pi*b^3*c^3*x^3*\log(\text{abs}(F))^2 + 16*I*b^3*c^3*x^3*\log(\text{abs}(F))^3 - 24*I*\pi^2*b^2*c^2*x^2*\text{sgn}(F) + 48*\pi*b^2*c^2*x^2*\log(\text{abs}(F))*\text{sgn}(F) + 24*I*\pi^2*b^2*c^2*x^2 - 48*\pi*b^2*c^2*x^2*\log(\text{abs}(F)) - 48*I*b^2*c^2*x^2*\log(\text{abs}(F))^2 - 48*\pi*b*c*x*\text{sgn}(F) + 48*\pi*b*c*x + 96*I*b*c*x*\log(\text{abs}(F)) - 96*I)*e^(1/2*I*\pi*b*c*x*\text{sgn}(F) - 1/2*I*\pi*b*c*x + 1/2*I*\pi*a*c*\text{sgn}(F) - 1/2*I*\pi*a*c) / (8*\pi^4*b^4*c^4*\text{sgn}(F) + 32*I*\pi^3*b^4*c^4*\log(\text{abs}(F))*\text{sgn}(F) - 48*\pi^2*b^4*c^4*\log(\text{abs}(F))^2*\text{sgn}(F) - 32*I*\pi*b^4*c^4*\log(\text{abs}(F))^3*\text{sgn}(F) - 8*\pi^4*b^4*c^4 - 32*I*\pi^3*b^4*c^4*\log(\text{abs}(F)) + 48*\pi^2*b^4*c^4*\log(\text{abs}(F))^2 + 32*I*\pi*b^4*c^4*\log(\text{abs}(F))^3 - 16*b^4*c^4*\log(\text{abs}(F))^4) + (8*\pi^3*b^3*c^3*x^3*\text{sgn}(F) - 24*I*\pi^2*b^3*c^3*x^3*\log(\text{abs}(F))*\text{sgn}(F) - 24*\pi*b^3*c^3*x^3*\log(\text{abs}(F))^2*\text{sgn}(F) - 8*\pi^3*b^3*c^3*x^3 + 24*I*\pi^2*b^3*c^3*x^3*\log(\text{abs}(F)) + 24*\pi*b^3*c^3*x^3*\log(\text{abs}(F))^2 - 16*I*b^3*c^3*x^3*\log(\text{abs}(F))^3 + 24*I*\pi^2*b^2*c^2*x^2*\text{sgn}(F) + 48*\pi*b^2*c^2*x^2*\log(\text{abs}(F))*\text{sgn}(F) - 24*I*\pi^2*b^2*c^2*x^2 - 48*\pi*b^2*c^2*x^2*\log(\text{abs}(F)) + 48*I*b^2*c^2*x^2*\log(\text{abs}(F))^2 - 48*\pi*b*c*x*\text{sgn}(F) + 48*\pi*b*c*x - 96*I*b*c*x*\log(\text{abs}(F)) + 96*I)*e^(-1/2*I*\pi*b*c*x*\text{sgn}(F) + 1/2*I*\pi*b*c*x - 1/2*I*\pi*a*c*\text{sgn}(F) + 1/2*I*\pi*a*c) / (8*\pi^4*b^4*c^4*\text{sgn}(F) - 32*I*\pi^3*b^4*c^4*\log(\text{abs}(F))*\text{sgn}(F) - 48*\pi^2*b^4*c^4*\log(\text{abs}(F))^2*\text{sgn}(F) + 32*I*\pi*b^4*c^4*\log(\text{abs}(F))^3*\text{sgn}(F) - 8*\pi^4*b^4*c^4 + 32*I*\pi^3*b^4*c^4*\log(\text{abs}(F)) + 48*\pi^2*b^4*c^4*\log(\text{abs}(F))^2 - 32*I*\pi*b^4*c^4*\log(\text{abs}(F))^3 - 16*b^4*c^4*\log(\text{abs}(F))^4) * e^(b*c*x*\log(\text{abs}(F)) + a*c*\log(\text{abs}(F)) + 3) + 3*((\pi^2*b^2*c^2*d*x^2*\text{sgn}(F) - \pi^2*b^2*c^2*d*x^2 + 2*b^2*c^2*d*x^2*\log(\text{abs}(F))^2
\end{aligned}$$

$$\begin{aligned}
& - 4*b*c*d*x*\log(\text{abs}(F)) + 4*d*(3*\pi^2*b^3*c^3*\log(\text{abs}(F))*\text{sgn}(F) - 3*\pi^2* \\
& b^3*c^3*\log(\text{abs}(F)) + 2*b^3*c^3*\log(\text{abs}(F))^3)/((\pi^3*b^3*c^3*\text{sgn}(F) - 3*\pi \\
& *b^3*c^3*\log(\text{abs}(F))^2*\text{sgn}(F) - \pi^3*b^3*c^3 + 3*\pi*b^3*c^3*\log(\text{abs}(F))^2)^2 \\
& + (3*\pi^2*b^3*c^3*\log(\text{abs}(F))*\text{sgn}(F) - 3*\pi^2*b^3*c^3*\log(\text{abs}(F)) + 2*b^3 \\
& *c^3*\log(\text{abs}(F))^3)^2) - 2*(\pi^3*b^3*c^3*\text{sgn}(F) - 3*\pi*b^3*c^3*\log(\text{abs}(F))^2 \\
& *\text{sgn}(F) - \pi^3*b^3*c^3 + 3*\pi*b^3*c^3*\log(\text{abs}(F))^2)*(pi*b^2*c^2*d*x^2*\log \\
& (\text{abs}(F))*\text{sgn}(F) - pi*b^2*c^2*d*x^2*\log(\text{abs}(F)) - pi*b*c*d*x*\text{sgn}(F) + pi*b*c \\
& *d*x)/((\pi^3*b^3*c^3*\text{sgn}(F) - 3*\pi*b^3*c^3*\log(\text{abs}(F))^2*\text{sgn}(F) - \pi^3*b^3*c^3 \\
& + 3*\pi*b^3*c^3*\log(\text{abs}(F))^2)^2 + (3*\pi^2*b^3*c^3*\log(\text{abs}(F))*\text{sgn}(F) - \\
& 3*\pi^2*b^3*c^3*\log(\text{abs}(F)) + 2*b^3*c^3*\log(\text{abs}(F))^3)^2))*\cos(-1/2*\pi*b*c*x \\
& *\text{sgn}(F) + 1/2*\pi*b*c*x - 1/2*\pi*a*c*\text{sgn}(F) + 1/2*\pi*a*c) + ((\pi^3*b^3*c^3*\text{sgn} \\
& (\text{abs}(F)) - 3*\pi*b^3*c^3*\log(\text{abs}(F))^2*\text{sgn}(F) - \pi^3*b^3*c^3 + 3*\pi*b^3*c^3*\log \\
& (\text{abs}(F))^2)*(pi^2*b^2*c^2*d*x^2*\text{sgn}(F) - pi^2*b^2*c^2*d*x^2 + 2*b^2*c^2*d*x \\
& ^2*\log(\text{abs}(F))^2 - 4*b*c*d*x*\log(\text{abs}(F)) + 4*d)/((\pi^3*b^3*c^3*\text{sgn}(F) - 3*\pi \\
& *b^3*c^3*\log(\text{abs}(F))^2*\text{sgn}(F) - \pi^3*b^3*c^3 + 3*\pi*b^3*c^3*\log(\text{abs}(F))^2) \\
& ^2 + (3*\pi^2*b^3*c^3*\log(\text{abs}(F))*\text{sgn}(F) - 3*\pi^2*b^3*c^3*\log(\text{abs}(F)) + 2*b^3 \\
& *c^3*\log(\text{abs}(F))^3)^2) + 2*(3*\pi^2*b^3*c^3*\log(\text{abs}(F))*\text{sgn}(F) - 3*\pi^2*b^3 \\
& *c^3*\log(\text{abs}(F)) + 2*b^3*c^3*\log(\text{abs}(F))^3)*(pi*b^2*c^2*d*x^2*\log(\text{abs}(F))*\text{sgn} \\
& (\text{abs}(F)) - pi*b^2*c^2*d*x^2*\log(\text{abs}(F)) - pi*b*c*d*x*\text{sgn}(F) + pi*b*c*d*x)/((\pi \\
& ^3*b^3*c^3*\text{sgn}(F) - 3*\pi*b^3*c^3*\log(\text{abs}(F))^2*\text{sgn}(F) - \pi^3*b^3*c^3 + 3*\pi \\
& *b^3*c^3*\log(\text{abs}(F))^2)^2 + (3*\pi^2*b^3*c^3*\log(\text{abs}(F))*\text{sgn}(F) - 3*\pi^2*b^3 \\
& *c^3*\log(\text{abs}(F)) + 2*b^3*c^3*\log(\text{abs}(F))^3)^2))*\sin(-1/2*\pi*b*c*x*\text{sgn}(F) + \\
& 1/2*\pi*b*c*x - 1/2*\pi*a*c*\text{sgn}(F) + 1/2*\pi*a*c))*e^{(b*c*x*\log(\text{abs}(F)) + a*c* \\
& \log(\text{abs}(F)) + 2) + 1/2*I*((12*I*\pi^2*b^2*c^2*d*x^2*\text{sgn}(F) - 24*\pi*b^2*c^2*d \\
& *x^2*\log(\text{abs}(F))*\text{sgn}(F) - 12*I*\pi^2*b^2*c^2*d*x^2 + 24*\pi*b^2*c^2*d*x^2*\log \\
& (\text{abs}(F)) + 24*I*b^2*c^2*d*x^2*\log(\text{abs}(F))^2 + 24*\pi*b*c*d*x*\text{sgn}(F) - 24*\pi* \\
& b*c*d*x - 48*I*b*c*d*x*\log(\text{abs}(F)) + 48*I*d)*e^{(1/2*I*\pi*b*c*x*\text{sgn}(F) - 1/2 \\
& *I*\pi*b*c*x + 1/2*I*\pi*a*c*\text{sgn}(F) - 1/2*I*\pi*a*c)/(-4*I*\pi^3*b^3*c^3*\text{sgn}(F) \\
& + 12*\pi^2*b^3*c^3*\log(\text{abs}(F))*\text{sgn}(F) + 12*I*\pi*b^3*c^3*\log(\text{abs}(F))^2*\text{sgn}(F) \\
& ) + 4*I*\pi^3*b^3*c^3 - 12*\pi^2*b^3*c^3*\log(\text{abs}(F)) - 12*I*\pi*b^3*c^3*\log(\text{abs} \\
& (\text{abs}(F))^2 + 8*b^3*c^3*\log(\text{abs}(F))^3) - (12*I*\pi^2*b^2*c^2*d*x^2*\text{sgn}(F) + 24*\pi \\
& *b^2*c^2*d*x^2*\log(\text{abs}(F))*\text{sgn}(F) - 12*I*\pi^2*b^2*c^2*d*x^2 - 24*\pi*b^2*c^2 \\
& *d*x^2*\log(\text{abs}(F)) + 24*I*b^2*c^2*d*x^2*\log(\text{abs}(F))^2 - 24*\pi*b*c*d*x*\text{sgn}( \\
& F) + 24*\pi*b*c*d*x - 48*I*b*c*d*x*\log(\text{abs}(F)) + 48*I*d)*e^{(-1/2*I*\pi*b*c*x* \\
& \text{sgn}(F) + 1/2*I*\pi*b*c*x - 1/2*I*\pi*a*c*\text{sgn}(F) + 1/2*I*\pi*a*c)/(4*I*\pi^3*b^3 \\
& *c^3*\text{sgn}(F) + 12*\pi^2*b^3*c^3*\log(\text{abs}(F))*\text{sgn}(F) - 12*I*\pi*b^3*c^3*\log(\text{abs} \\
& (F))^2*\text{sgn}(F) - 4*I*\pi^3*b^3*c^3 - 12*\pi^2*b^3*c^3*\log(\text{abs}(F)) + 12*I*\pi*b^3 \\
& *c^3*\log(\text{abs}(F))^2 + 8*b^3*c^3*\log(\text{abs}(F))^3))*e^{(b*c*x*\log(\text{abs}(F)) + a*c* \\
& \log(\text{abs}(F)) + 2) + 3*(2*((\pi*b*c*d^2*x*\text{sgn}(F) - \pi*b*c*d^2*x)*(pi*b^2*c^2*\log \\
& (\text{abs}(F))*\text{sgn}(F) - pi*b^2*c^2*\log(\text{abs}(F)))/((\pi^2*b^2*c^2*\text{sgn}(F) - \pi^2*b^2 \\
& *c^2 + 2*b^2*c^2*\log(\text{abs}(F))^2)^2 + 4*(\pi*b^2*c^2*\log(\text{abs}(F))*\text{sgn}(F) - \pi*b \\
& ^2*c^2*\log(\text{abs}(F)))^2) + (\pi^2*b^2*c^2*\text{sgn}(F) - \pi^2*b^2*c^2 + 2*b^2*c^2*\log \\
& (\text{abs}(F))^2)*(b*c*d^2*x*\log(\text{abs}(F)) - d^2)/((\pi^2*b^2*c^2*\text{sgn}(F) - \pi^2*b^2 \\
& *c^2 + 2*b^2*c^2*\log(\text{abs}(F))^2)^2 + 4*(\pi*b^2*c^2*\log(\text{abs}(F))*\text{sgn}(F) - \pi*b \\
& ^2*c^2*\log(\text{abs}(F)))^2))*\cos(-1/2*\pi*b*c*x*\text{sgn}(F) + 1/2*\pi*b*c*x - 1/2*\pi*a*
\end{aligned}$$

$$\begin{aligned}
& c \operatorname{sgn}(F) + 1/2 \pi a c) + ((\pi^2 b^2 c^2 \operatorname{sgn}(F) - \pi^2 b^2 c^2 + 2 b^2 c^2 \log(\operatorname{abs}(F))^2) * (\pi b c d^2 x \operatorname{sgn}(F) - \pi b c d^2 x) / ((\pi^2 b^2 c^2 \operatorname{sgn}(F) - \pi^2 b^2 c^2 + 2 b^2 c^2 \log(\operatorname{abs}(F))^2)^2 + 4 (\pi b^2 c^2 \log(\operatorname{abs}(F)) \operatorname{sgn}(F) - \pi b^2 c^2 \log(\operatorname{abs}(F)))^2) - 4 (\pi b^2 c^2 \log(\operatorname{abs}(F)) \operatorname{sgn}(F) - \pi b^2 c^2 \log(\operatorname{abs}(F))) * (b c d^2 x \log(\operatorname{abs}(F)) - d^2) / ((\pi^2 b^2 c^2 \operatorname{sgn}(F) - \pi^2 b^2 c^2 + 2 b^2 c^2 \log(\operatorname{abs}(F))^2)^2 + 4 (\pi b^2 c^2 \log(\operatorname{abs}(F)) \operatorname{sgn}(F) - \pi b^2 c^2 \log(\operatorname{abs}(F)))^2) * \sin(-1/2 \pi b c x \operatorname{sgn}(F) + 1/2 \pi b c x - 1/2 \pi a c \operatorname{sgn}(F) + 1/2 \pi a c)) * e^{(b c x \log(\operatorname{abs}(F)) + a c \log(\operatorname{abs}(F)) + 1) - 1/2 I * ((6 \pi b c d^2 x \operatorname{sgn}(F) - 6 \pi b c d^2 x - 12 I b c d^2 x \log(\operatorname{abs}(F)) + 12 I d^2) * e^{(1/2 I \pi b c x \operatorname{sgn}(F) - 1/2 I \pi b c x + 1/2 I \pi a c \operatorname{sgn}(F) - 1/2 I \pi a c) / (2 \pi^2 b^2 c^2 \operatorname{sgn}(F) + 4 I \pi b^2 c^2 \log(\operatorname{abs}(F)) \operatorname{sgn}(F) - 2 \pi^2 b^2 c^2 - 4 I \pi b^2 c^2 \log(\operatorname{abs}(F)) + 4 b^2 c^2 \log(\operatorname{abs}(F))^2) + (6 \pi b c d^2 x \operatorname{sgn}(F) - 6 \pi b c d^2 x + 12 I b c d^2 x \log(\operatorname{abs}(F)) - 12 I d^2) * e^{(-1/2 I \pi b c x \operatorname{sgn}(F) + 1/2 I \pi b c x - 1/2 I \pi a c \operatorname{sgn}(F) + 1/2 I \pi a c) / (2 \pi^2 b^2 c^2 \operatorname{sgn}(F) - 4 I \pi b^2 c^2 \log(\operatorname{abs}(F)) \operatorname{sgn}(F) - 2 \pi^2 b^2 c^2 + 4 I \pi b^2 c^2 \log(\operatorname{abs}(F)) + 4 b^2 c^2 \log(\operatorname{abs}(F))^2) * e^{(b c x \log(\operatorname{abs}(F)) + a c \log(\operatorname{abs}(F)) + 1) + 2 * (2 b c d^3 \cos(-1/2 \pi b c x \operatorname{sgn}(F) + 1/2 \pi b c x - 1/2 \pi a c \operatorname{sgn}(F) + 1/2 \pi a c) * \log(\operatorname{abs}(F)) / (4 b^2 c^2 \log(\operatorname{abs}(F))^2 + (\pi b c \operatorname{sgn}(F) - \pi b c)^2) - (\pi b c \operatorname{sgn}(F) - \pi b c) * d^3 \sin(-1/2 \pi b c x \operatorname{sgn}(F) + 1/2 \pi b c x - 1/2 \pi a c \operatorname{sgn}(F) + 1/2 \pi a c) / (4 b^2 c^2 \log(\operatorname{abs}(F))^2 + (\pi b c \operatorname{sgn}(F) - \pi b c)^2) * e^{(b c x \log(\operatorname{abs}(F)) + a c \log(\operatorname{abs}(F))) - 1/2 I * (-2 I d^3 e^{(1/2 I \pi b c x \operatorname{sgn}(F) - 1/2 I \pi b c x + 1/2 I \pi a c \operatorname{sgn}(F) - 1/2 I \pi a c) / (I \pi b c \operatorname{sgn}(F) - I \pi b c + 2 b c \log(\operatorname{abs}(F))) + 2 I d^3 e^{(-1/2 I \pi b c x \operatorname{sgn}(F) + 1/2 I \pi b c x - 1/2 I \pi a c \operatorname{sgn}(F) + 1/2 I \pi a c) / (-I \pi b c \operatorname{sgn}(F) + I \pi b c + 2 b c \log(\operatorname{abs}(F))) * e^{(b c x \log(\operatorname{abs}(F)) + a c \log(\operatorname{abs}(F)))}
\end{aligned}$$

### 3.14 $\int F^{c(a+bx)} (d^2 + 2dex + e^2x^2) dx$

**Optimal.** Leaf size=79

$$-\frac{2e(d+ex)F^{c(a+bx)}}{b^2c^2\log^2(F)} + \frac{2e^2F^{c(a+bx)}}{b^3c^3\log^3(F)} + \frac{(d+ex)^2F^{c(a+bx)}}{bc\log(F)}$$

[Out]  $(2e^2F^{c(a+bx)})/(b^3c^3\log[F]^3) - (2eF^{c(a+bx)})(d+ex)/(b^2c^2\log[F]^2) + (F^{c(a+bx)})(d+ex)^2/(bc\log[F])$

**Rubi [A]** time = 0.0454934, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$ , Rules used = {27, 2176, 2194}

$$-\frac{2e(d+ex)F^{c(a+bx)}}{b^2c^2\log^2(F)} + \frac{2e^2F^{c(a+bx)}}{b^3c^3\log^3(F)} + \frac{(d+ex)^2F^{c(a+bx)}}{bc\log(F)}$$

Antiderivative was successfully verified.

[In] Int[F^(c\*(a + b\*x))\*(d^2 + 2\*d\*e\*x + e^2\*x^2), x]

[Out]  $(2e^2F^{c(a+bx)})/(b^3c^3\log[F]^3) - (2eF^{c(a+bx)})(d+ex)/(b^2c^2\log[F]^2) + (F^{c(a+bx)})(d+ex)^2/(bc\log[F])$

#### Rule 27

Int[(u\_)\*((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(p\_), x\_Symbol] :> Int[u\*Cancel[(b/2 + c\*x)^(2\*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4\*a\*c, 0] && IntegerQ[p]

#### Rule 2176

Int[((b\_)\*(F\_)^((g\_)\*((e\_) + (f\_)\*(x\_))))^(n\_)\*((c\_) + (d\_)\*(x\_))^(m\_), x\_Symbol] :> Simp[((c + d\*x)^m\*(bF^(g\*(e + f\*x)))^n)/(f\*g\*n\*Log[F]), x] - Dist[(d\*m)/(f\*g\*n\*Log[F]), Int[(c + d\*x)^(m-1)\*(bF^(g\*(e + f\*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2\*m] && !\$UseGamma == True

#### Rule 2194

`Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] := Simp[(F^(c*(a + b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]`

### Rubi steps

$$\begin{aligned} \int F^{c(a+bx)} (d^2 + 2dex + e^2x^2) dx &= \int F^{c(a+bx)} (d+ex)^2 dx \\ &= \frac{F^{c(a+bx)}(d+ex)^2}{bc \log(F)} - \frac{(2e) \int F^{c(a+bx)}(d+ex) dx}{bc \log(F)} \\ &= -\frac{2eF^{c(a+bx)}(d+ex)}{b^2c^2 \log^2(F)} + \frac{F^{c(a+bx)}(d+ex)^2}{bc \log(F)} + \frac{(2e^2) \int F^{c(a+bx)} dx}{b^2c^2 \log^2(F)} \\ &= \frac{2e^2F^{c(a+bx)}}{b^3c^3 \log^3(F)} - \frac{2eF^{c(a+bx)}(d+ex)}{b^2c^2 \log^2(F)} + \frac{F^{c(a+bx)}(d+ex)^2}{bc \log(F)} \end{aligned}$$

**Mathematica [A]** time = 0.0385979, size = 56, normalized size = 0.71

$$\frac{F^{c(a+bx)} (b^2c^2 \log^2(F)(d+ex)^2 - 2bce \log(F)(d+ex) + 2e^2)}{b^3c^3 \log^3(F)}$$

Antiderivative was successfully verified.

[In] Integrate[F^(c\*(a + b\*x))\*(d^2 + 2\*d\*e\*x + e^2\*x^2), x]

[Out] (F^(c\*(a + b\*x))\*(2\*e^2 - 2\*b\*c\*e\*(d + e\*x)\*Log[F] + b^2\*c^2\*(d + e\*x)^2\*Log[F]^2))/(b^3\*c^3\*Log[F]^3)

**Maple [A]** time = 0.004, size = 91, normalized size = 1.2

$$\frac{(e^2x^2b^2c^2 (\ln(F))^2 + 2 (\ln(F))^2 b^2c^2dex + b^2c^2 (\ln(F))^2 d^2 - 2 \ln(F) bce^2x - 2 \ln(F) bced + 2e^2) F^{c(bx+a)}}{b^3c^3 (\ln(F))^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(c\*(b\*x+a))\*(e^2\*x^2+2\*d\*e\*x+d^2), x)

[Out] (e^2\*x^2\*b^2\*c^2\*ln(F)^2+2\*ln(F)^2\*b^2\*c^2\*d\*e\*x+b^2\*c^2\*ln(F)^2\*d^2-2\*ln(F)\*b\*c\*e^2\*x-2\*ln(F)\*b\*c\*e\*d+2\*e^2)\*F^(c\*(b\*x+a))/b^3/c^3/ln(F)^3

---

**Maxima [A]** time = 1.15033, size = 166, normalized size = 2.1

$$\frac{F^{bcx+ac} d^2}{bc \log(F)} + \frac{2(F^{ac}bcx \log(F) - F^{ac})F^{bcx} de}{b^2 c^2 \log(F)^2} + \frac{(F^{ac} b^2 c^2 x^2 \log(F)^2 - 2 F^{ac} bcx \log(F) + 2 F^{ac}) F^{bcx} e^2}{b^3 c^3 \log(F)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(c\*(b\*x+a))\*(e^2\*x^2+2\*d\*e\*x+d^2),x, algorithm="maxima")

[Out] F^(b\*c\*x + a\*c)\*d^2/(b\*c\*log(F)) + 2\*(F^(a\*c)\*b\*c\*x\*log(F) - F^(a\*c))\*F^(b\*c\*x)\*d\*e/(b^2\*c^2\*log(F)^2) + (F^(a\*c)\*b^2\*c^2\*x^2\*log(F)^2 - 2\*F^(a\*c)\*b\*c\*x\*log(F) + 2\*F^(a\*c))\*F^(b\*c\*x)\*e^2/(b^3\*c^3\*log(F)^3)

---

**Fricas [A]** time = 1.52873, size = 186, normalized size = 2.35

$$\frac{((b^2 c^2 e^2 x^2 + 2 b^2 c^2 d e x + b^2 c^2 d^2) \log(F)^2 + 2 e^2 - 2 (b c e^2 x + b c d e) \log(F)) F^{bcx+ac}}{b^3 c^3 \log(F)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(c\*(b\*x+a))\*(e^2\*x^2+2\*d\*e\*x+d^2),x, algorithm="fricas")

[Out] ((b^2\*c^2\*e^2\*x^2 + 2\*b^2\*c^2\*d\*e\*x + b^2\*c^2\*d^2)\*log(F)^2 + 2\*e^2 - 2\*(b\*c\*e^2\*x + b\*c\*d\*e)\*log(F))\*F^(b\*c\*x + a\*c)/(b^3\*c^3\*log(F)^3)

---

**Sympy [A]** time = 0.274006, size = 133, normalized size = 1.68

$$\begin{cases} \frac{F^{c(a+bx)}(b^2 c^2 d^2 \log(F)^2 + 2 b^2 c^2 d e x \log(F) + b^2 c^2 e^2 x^2 \log(F)^2 - 2 b c d e \log(F) - 2 b c e^2 x \log(F) + 2 e^2)}{b^3 c^3 \log(F)^3} & \text{for } b^3 c^3 \log(F)^3 \neq 0 \\ d^2 x + d e x^2 + \frac{e^2 x^3}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(c\*(b\*x+a))\*(e\*\*2\*x\*\*2+2\*d\*e\*x+d\*\*2),x)

[Out] Piecewise((F\*\*(c\*(a + b\*x))\*(b\*\*2\*c\*\*2\*d\*\*2\*log(F)\*\*2 + 2\*b\*\*2\*c\*\*2\*d\*e\*x\*log(F)\*\*2 + b\*\*2\*c\*\*2\*e\*\*2\*x\*\*2\*log(F)\*\*2 - 2\*b\*c\*d\*e\*log(F) - 2\*b\*c\*e\*\*2\*x\*

```
log(F) + 2*e**2)/(b**3*c**3*log(F)**3), Ne(b**3*c**3*log(F)**3, 0)), (d**2*x + d*e*x**2 + e**2*x**3/3, True))
```

**Giac [C]** time = 1.28433, size = 3367, normalized size = 42.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*(e^2*x^2+2*d*e*x+d^2),x, algorithm="giac")
```

```
[Out] (((3*pi^2*b^3*c^3*log(abs(F))*sgn(F) - 3*pi^2*b^3*c^3*log(abs(F)) + 2*b^3*c^3*log(abs(F))^3)*(pi^2*b^2*c^2*x^2*sgn(F) - pi^2*b^2*c^2*x^2 + 2*b^2*c^2*x^2*log(abs(F))^2 - 4*b*c*x*log(abs(F)) + 4)/((pi^3*b^3*c^3*sgn(F) - 3*pi*b^3*c^3*log(abs(F))^2*sgn(F) - pi^3*b^3*c^3 + 3*pi*b^3*c^3*log(abs(F))^2 + (3*pi^2*b^3*c^3*log(abs(F))*sgn(F) - 3*pi^2*b^3*c^3*log(abs(F)) + 2*b^3*c^3*log(abs(F))^3)^2) - 2*(pi^3*b^3*c^3*sgn(F) - 3*pi*b^3*c^3*log(abs(F))^2*sgn(F) - pi^3*b^3*c^3 + 3*pi*b^3*c^3*log(abs(F))^2)*(pi*b^2*c^2*x^2*log(abs(F))*sgn(F) - pi*b^2*c^2*x^2*log(abs(F)) - pi*b*c*x*sgn(F) + pi*b*c*x)/((pi^3*b^3*c^3*sgn(F) - 3*pi*b^3*c^3*log(abs(F))^2*sgn(F) - pi^3*b^3*c^3 + 3*pi*b^3*c^3*log(abs(F))^2)^2 + (3*pi^2*b^3*c^3*log(abs(F))*sgn(F) - 3*pi^2*b^3*c^3*log(abs(F)) + 2*b^3*c^3*log(abs(F))^3)^2))*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c) + ((pi^3*b^3*c^3*sgn(F) - 3*pi*b^3*c^3*log(abs(F))^2*sgn(F) - pi^3*b^3*c^3 + 3*pi*b^3*c^3*log(abs(F))^2)*(pi^2*b^2*c^2*x^2*sgn(F) - pi^2*b^2*c^2*x^2 + 2*b^2*c^2*x^2*log(abs(F))^2 - 4*b*c*x*log(abs(F)) + 4)/((pi^3*b^3*c^3*sgn(F) - 3*pi*b^3*c^3*log(abs(F))^2*sgn(F) - pi^3*b^3*c^3 + 3*pi*b^3*c^3*log(abs(F))^2)^2 + (3*pi^2*b^3*c^3*log(abs(F))*sgn(F) - 3*pi^2*b^3*c^3*log(abs(F)) + 2*b^3*c^3*log(abs(F))^3)^2) + 2*(3*pi^2*b^3*c^3*log(abs(F))*sgn(F) - 3*pi^2*b^3*c^3*log(abs(F)) + 2*b^3*c^3*log(abs(F))^3)*(pi*b^2*c^2*x^2*log(abs(F))*sgn(F) - pi*b^2*c^2*x^2*log(abs(F)) - pi*b*c*x*sgn(F) + pi*b*c*x)/((pi^3*b^3*c^3*sgn(F) - 3*pi*b^3*c^3*log(abs(F))^2*sgn(F) - pi^3*b^3*c^3 + 3*pi*b^3*c^3*log(abs(F))^2)^2 + (3*pi^2*b^3*c^3*log(abs(F))*sgn(F) - 3*pi^2*b^3*c^3*log(abs(F)) + 2*b^3*c^3*log(abs(F))^3)^2))*sin(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F)) + 2) + 1/2*I*((4*I*pi^2*b^2*c^2*x^2*sgn(F) - 8*pi*b^2*c^2*x^2*log(abs(F))*sgn(F) - 4*I*pi^2*b^2*c^2*x^2 + 8*pi*b^2*c^2*x^2*log(abs(F)) + 8*I*b^2*c^2*x^2*log(abs(F))^2 + 8*pi*b*c*x*sgn(F) - 8*pi*b*c*x - 16*I*b*c*x*log(abs(F)) + 16*I)*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*a*c)/(-4*I*pi^3*b^3*c^3*sgn(F) + 12*pi^2*b^3*c^3*log(abs(F))*sgn(F) + 12*I*pi*b^3*c^3*log(abs(F))^2*sgn(F) + 4*I*pi^3*b^3*c^3 - 12*pi^2*b^3*c^3*log(abs(F)) - 12*I*pi*b^3*c^3*log(abs(F))^2 + 8*b^3*c^3*log(abs(F))^3) - (4*I*pi^2*b^2*c^2
```

$$\begin{aligned}
& *x^2*\text{sgn}(F) + 8*\pi*b^2*c^2*x^2*\log(\text{abs}(F))*\text{sgn}(F) - 4*I*\pi^2*b^2*c^2*x^2 - \\
& 8*\pi*b^2*c^2*x^2*\log(\text{abs}(F)) + 8*I*b^2*c^2*x^2*\log(\text{abs}(F))^2 - 8*\pi*b*c*x*s \\
& \text{gn}(F) + 8*\pi*b*c*x - 16*I*b*c*x*\log(\text{abs}(F)) + 16*I)*e^{(-1/2*I*\pi*b*c*x*\text{sgn}( \\
& F) + 1/2*I*\pi*b*c*x - 1/2*I*\pi*a*c*\text{sgn}(F) + 1/2*I*\pi*a*c)/(4*I*\pi^3*b^3*c^3 \\
& *\text{sgn}(F) + 12*\pi^2*b^3*c^3*\log(\text{abs}(F))*\text{sgn}(F) - 12*I*\pi*b^3*c^3*\log(\text{abs}(F))^ \\
& 2*\text{sgn}(F) - 4*I*\pi^3*b^3*c^3 - 12*\pi^2*b^3*c^3*\log(\text{abs}(F)) + 12*I*\pi*b^3*c^3 \\
& *\log(\text{abs}(F))^2 + 8*b^3*c^3*\log(\text{abs}(F))^3)}*e^{(b*c*x*\log(\text{abs}(F)) + a*c*\log(a \\
& bs(F)) + 2) + 2*(2*((\pi*b^2*c^2*\log(\text{abs}(F))*\text{sgn}(F) - \pi*b^2*c^2*\log(\text{abs}(F)) \\
& )*(\pi*b*c*d*x*\text{sgn}(F) - \pi*b*c*d*x)/((\pi^2*b^2*c^2*\text{sgn}(F) - \pi^2*b^2*c^2 + 2 \\
& *b^2*c^2*\log(\text{abs}(F))^2)^2 + 4*(\pi*b^2*c^2*\log(\text{abs}(F))*\text{sgn}(F) - \pi*b^2*c^2*l \\
& og(\text{abs}(F)))^2) + (\pi^2*b^2*c^2*\text{sgn}(F) - \pi^2*b^2*c^2 + 2*b^2*c^2*\log(\text{abs}(F) \\
& )^2)*(b*c*d*x*\log(\text{abs}(F)) - d)/((\pi^2*b^2*c^2*\text{sgn}(F) - \pi^2*b^2*c^2 + 2*b^2 \\
& *c^2*\log(\text{abs}(F))^2)^2 + 4*(\pi*b^2*c^2*\log(\text{abs}(F))*\text{sgn}(F) - \pi*b^2*c^2*\log(a \\
& bs(F)))^2))*\cos(-1/2*\pi*b*c*x*\text{sgn}(F) + 1/2*\pi*b*c*x - 1/2*\pi*a*c*\text{sgn}(F) + 1 \\
& /2*\pi*a*c) + ((\pi^2*b^2*c^2*\text{sgn}(F) - \pi^2*b^2*c^2 + 2*b^2*c^2*\log(\text{abs}(F))^2 \\
& )*(\pi*b*c*d*x*\text{sgn}(F) - \pi*b*c*d*x)/((\pi^2*b^2*c^2*\text{sgn}(F) - \pi^2*b^2*c^2 + 2 \\
& *b^2*c^2*\log(\text{abs}(F))^2)^2 + 4*(\pi*b^2*c^2*\log(\text{abs}(F))*\text{sgn}(F) - \pi*b^2*c^2*l \\
& og(\text{abs}(F)))^2) - 4*(\pi*b^2*c^2*\log(\text{abs}(F))*\text{sgn}(F) - \pi*b^2*c^2*\log(\text{abs}(F))) \\
& *(b*c*d*x*\log(\text{abs}(F)) - d)/((\pi^2*b^2*c^2*\text{sgn}(F) - \pi^2*b^2*c^2 + 2*b^2*c^2 \\
& *\log(\text{abs}(F))^2)^2 + 4*(\pi*b^2*c^2*\log(\text{abs}(F))*\text{sgn}(F) - \pi*b^2*c^2*\log(\text{abs}(F) \\
& )))^2))*\sin(-1/2*\pi*b*c*x*\text{sgn}(F) + 1/2*\pi*b*c*x - 1/2*\pi*a*c*\text{sgn}(F) + 1/2*\pi \\
& i*a*c))*e^{(b*c*x*\log(\text{abs}(F)) + a*c*\log(\text{abs}(F)) + 1) - 1/2*I*((4*\pi*b*c*d*x* \\
& \text{sgn}(F) - 4*\pi*b*c*d*x - 8*I*b*c*d*x*\log(\text{abs}(F)) + 8*I*d)*e^{(1/2*I*\pi*b*c*x* \\
& \text{sgn}(F) - 1/2*I*\pi*b*c*x + 1/2*I*\pi*a*c*\text{sgn}(F) - 1/2*I*\pi*a*c)/(2*\pi^2*b^2*c \\
& ^2*\text{sgn}(F) + 4*I*\pi*b^2*c^2*\log(\text{abs}(F))*\text{sgn}(F) - 2*\pi^2*b^2*c^2 - 4*I*\pi*b^2 \\
& *c^2*\log(\text{abs}(F)) + 4*b^2*c^2*\log(\text{abs}(F))^2) + (4*\pi*b*c*d*x*\text{sgn}(F) - 4*\pi*b \\
& *c*d*x + 8*I*b*c*d*x*\log(\text{abs}(F)) - 8*I*d)*e^{(-1/2*I*\pi*b*c*x*\text{sgn}(F) + 1/2*I \\
& *\pi*b*c*x - 1/2*I*\pi*a*c*\text{sgn}(F) + 1/2*I*\pi*a*c)/(2*\pi^2*b^2*c^2*\text{sgn}(F) - 4* \\
& I*\pi*b^2*c^2*\log(\text{abs}(F))*\text{sgn}(F) - 2*\pi^2*b^2*c^2 + 4*I*\pi*b^2*c^2*\log(\text{abs}(F) \\
& )) + 4*b^2*c^2*\log(\text{abs}(F))^2)}*e^{(b*c*x*\log(\text{abs}(F)) + a*c*\log(\text{abs}(F)) + 1) \\
& + 2*(2*b*c*d^2*\cos(-1/2*\pi*b*c*x*\text{sgn}(F) + 1/2*\pi*b*c*x - 1/2*\pi*a*c*\text{sgn}(F) \\
& + 1/2*\pi*a*c)*\log(\text{abs}(F))/(4*b^2*c^2*\log(\text{abs}(F))^2 + (\pi*b*c*\text{sgn}(F) - \pi*b*c \\
& )^2) - (\pi*b*c*\text{sgn}(F) - \pi*b*c)*d^2*\sin(-1/2*\pi*b*c*x*\text{sgn}(F) + 1/2*\pi*b*c*x \\
& - 1/2*\pi*a*c*\text{sgn}(F) + 1/2*\pi*a*c)/(4*b^2*c^2*\log(\text{abs}(F))^2 + (\pi*b*c*\text{sgn}( \\
& F) - \pi*b*c)^2))*e^{(b*c*x*\log(\text{abs}(F)) + a*c*\log(\text{abs}(F)))} - 1/2*I*(-2*I*d^2* \\
& e^{(1/2*I*\pi*b*c*x*\text{sgn}(F) - 1/2*I*\pi*b*c*x + 1/2*I*\pi*a*c*\text{sgn}(F) - 1/2*I*\pi* \\
& a*c)/(I*\pi*b*c*\text{sgn}(F) - I*\pi*b*c + 2*b*c*\log(\text{abs}(F)))} + 2*I*d^2*e^{(-1/2*I*\pi \\
& i*b*c*x*\text{sgn}(F) + 1/2*I*\pi*b*c*x - 1/2*I*\pi*a*c*\text{sgn}(F) + 1/2*I*\pi*a*c)/(-I*\pi \\
& i*b*c*\text{sgn}(F) + I*\pi*b*c + 2*b*c*\log(\text{abs}(F)))})*e^{(b*c*x*\log(\text{abs}(F)) + a*c*\log \\
& (\text{abs}(F)))}
\end{aligned}$$



$$3.15 \quad \int \frac{F^{c(a+bx)}}{d^2+2dex+e^2x^2} dx$$

**Optimal.** Leaf size=57

$$\frac{bc \log(F) F^{c\left(a-\frac{bd}{e}\right)} \text{Ei}\left(\frac{bc(d+ex)\log(F)}{e}\right)}{e^2} - \frac{F^{c(a+bx)}}{e(d+ex)}$$

[Out]  $-(F^{c(a+bx)})/(e(d+ex)) + (b*c*F^{c(a-(b*d)/e)})*\text{ExpIntegralEi}[(b*c*(d+e*x)*\text{Log}[F])/e]*\text{Log}[F])/e^2$

**Rubi [A]** time = 0.0426247, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 28,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$ , Rules used = {27, 2177, 2178}

$$\frac{bc \log(F) F^{c\left(a-\frac{bd}{e}\right)} \text{Ei}\left(\frac{bc(d+ex)\log(F)}{e}\right)}{e^2} - \frac{F^{c(a+bx)}}{e(d+ex)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[F^{c(a+bx)}]/(d^2+2*d*e*x+e^2*x^2), x]$

[Out]  $-(F^{c(a+bx)})/(e(d+ex)) + (b*c*F^{c(a-(b*d)/e)})*\text{ExpIntegralEi}[(b*c*(d+e*x)*\text{Log}[F])/e]*\text{Log}[F])/e^2$

### Rule 27

$\text{Int}[(u_.)*((a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2)^{(p_.)}, x\_Symbol] \rightarrow \text{Int}[u*\text{Cancel}[(b/2 + c*x)^{(2*p)}/c^p], x] /; \text{FreeQ}[\{a, b, c\}, x] \ \&\& \ \text{EqQ}[b^2 - 4*a*c, 0] \ \&\& \ \text{IntegerQ}[p]$

### Rule 2177

$\text{Int}[(b_.)*(F_.)^{((g_.)*((e_.) + (f_.)*(x_)))^{(n_.)*((c_.) + (d_.)*(x_))^{(m_.)}}, x\_Symbol] \rightarrow \text{Simp}[(c + d*x)^{(m+1)}*(b*F^{(g*(e+f*x))})^n]/(d*(m+1)), x] - \text{Dist}[(f*g*n*\text{Log}[F])/d*(m+1), \text{Int}[(c + d*x)^{(m+1)}*(b*F^{(g*(e+f*x))})^n, x], x] /; \text{FreeQ}[\{F, b, c, d, e, f, g, n\}, x] \ \&\& \ \text{LtQ}[m, -1] \ \&\& \ \text{IntegerQ}[2*m] \ \&\& \ !\$UseGamma == True$

### Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] := Si
mp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; F
reeQ[{F, c, d, e, f, g}, x] && !$UseGamma == True
```

### Rubi steps

$$\begin{aligned} \int \frac{F^{c(a+bx)}}{d^2 + 2dex + e^2x^2} dx &= \int \frac{F^{c(a+bx)}}{(d+ex)^2} dx \\ &= -\frac{F^{c(a+bx)}}{e(d+ex)} + \frac{(bc \log(F)) \int \frac{F^{c(a+bx)}}{d+ex} dx}{e} \\ &= -\frac{F^{c(a+bx)}}{e(d+ex)} + \frac{bcF^{c\left(a-\frac{bd}{e}\right)} \text{Ei}\left(\frac{bc(d+ex)\log(F)}{e}\right) \log(F)}{e^2} \end{aligned}$$

**Mathematica [A]** time = 0.0598772, size = 55, normalized size = 0.96

$$\frac{F^{ac} \left( bc \log(F) F^{-\frac{bcd}{e}} \text{Ei}\left(\frac{bc(d+ex)\log(F)}{e}\right) - \frac{eF^{bcx}}{d+ex} \right)}{e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[F^(c*(a + b*x))/(d^2 + 2*d*e*x + e^2*x^2), x]
```

```
[Out] (F^(a*c)*(-((e*F^(b*c*x))/(d + e*x)) + (b*c*ExpIntegralEi[(b*c*(d + e*x)*Lo
g[F])/e]*Log[F])/F^((b*c*d)/e)))/e^2
```

**Maple [A]** time = 0.043, size = 99, normalized size = 1.7

$$-\frac{bc \ln(F) F^{bcx} F^{ac}}{e^2} \left( bcx \ln(F) + \frac{\ln(F) bcd}{e} \right)^{-1} - \frac{bc \ln(F)}{e^2} F^{\frac{c(ae-bd)}{e}} \text{Ei}\left(1, -bcx \ln(F) - ac \ln(F) - \frac{-\ln(F) ace + \ln(F) bcd}{e}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(b*x+a))/(e^2*x^2+2*d*e*x+d^2), x)
```

```
[Out] -b*c*ln(F)/e^2*F^(b*c*x)*F^(a*c)/(b*c*x*ln(F)+1/e*ln(F)*b*c*d)-b*c*ln(F)/e^
2*F^(c*(a*e-b*d)/e)*Ei(1, -b*c*x*ln(F)-a*c*ln(F)-(-ln(F)*a*c*e+ln(F)*b*c*d)/
```

e)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{(bx+a)c}}{e^2x^2 + 2dex + d^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(c\*(b\*x+a))/(e^2\*x^2+2\*d\*e\*x+d^2),x, algorithm="maxima")

[Out] integrate(F^((b\*x + a)\*c)/(e^2\*x^2 + 2\*d\*e\*x + d^2), x)

---

**Fricas [A]** time = 1.53621, size = 159, normalized size = 2.79

$$\frac{F^{bcx+ac} e - \frac{(bcex+bcd) \operatorname{Ei}\left(\frac{(bcex+bcd) \log(F)}{e}\right) \log(F)}{\frac{bcd-ace}{F^e}}}{e^3x + de^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(c\*(b\*x+a))/(e^2\*x^2+2\*d\*e\*x+d^2),x, algorithm="fricas")

[Out]  $-(F^{(b*c*x + a*c)}*e - (b*c*e*x + b*c*d)*\operatorname{Ei}((b*c*e*x + b*c*d)*\log(F)/e)*\log(F))/F^{(b*c*d - a*c*e)/e}/(e^3*x + d*e^2)$

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{c(a+bx)}}{(d+ex)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(c\*(b\*x+a))/(e\*\*2\*x\*\*2+2\*d\*e\*x+d\*\*2),x)

[Out] Integral(F\*\*(c\*(a + b\*x))/(d + e\*x)\*\*2, x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{(bx+a)c}}{e^{2x^2 + 2dex + d^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))/(e^2*x^2+2*d*e*x+d^2),x, algorithm="giac")
```

```
[Out] integrate(F^((b*x + a)*c)/(e^2*x^2 + 2*d*e*x + d^2), x)
```

$$3.16 \quad \int \frac{F^{c(a+bx)}}{d^3 + 3d^2ex + 3de^2x^2 + e^3x^3} dx$$

**Optimal.** Leaf size=95

$$\frac{b^2c^2 \log^2(F) F^{c\left(a-\frac{bd}{e}\right)} \text{Ei}\left(\frac{bc(d+ex)\log(F)}{e}\right)}{2e^3} - \frac{bc \log(F) F^{c(a+bx)}}{2e^2(d+ex)} - \frac{F^{c(a+bx)}}{2e(d+ex)^2}$$

[Out]  $-F^{c(a+bx)}/(2e(d+ex)^2) - (bcF^{c(a+bx)}\text{Log}[F])/(2e^2(d+ex)) + (b^2c^2F^{c(a-(bd)/e)}\text{ExpIntegralEi}[(bc(d+ex)\text{Log}[F])/e]\text{Log}[F]^2)/(2e^3)$

**Rubi [A]** time = 0.097557, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 39,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$ , Rules used = {2187, 2177, 2178}

$$\frac{b^2c^2 \log^2(F) F^{c\left(a-\frac{bd}{e}\right)} \text{Ei}\left(\frac{bc(d+ex)\log(F)}{e}\right)}{2e^3} - \frac{bc \log(F) F^{c(a+bx)}}{2e^2(d+ex)} - \frac{F^{c(a+bx)}}{2e(d+ex)^2}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[F^{c(a+bx)}/(d^3 + 3d^2ex + 3de^2x^2 + e^3x^3), x]$

[Out]  $-F^{c(a+bx)}/(2e(d+ex)^2) - (bcF^{c(a+bx)}\text{Log}[F])/(2e^2(d+ex)) + (b^2c^2F^{c(a-(bd)/e)}\text{ExpIntegralEi}[(bc(d+ex)\text{Log}[F])/e]\text{Log}[F]^2)/(2e^3)$

### Rule 2187

$\text{Int}[(a_. + (b_.)*(F_.)^((g_.)*(v_.)))^{(n_.)}^{(p_.)}*(u_.)^{(m_.)}, x\_Symbol] :$   
 $> \text{Int}[\text{NormalizePowerOfLinear}[u, x]^{m*(a + b*(F^{g*\text{ExpandToSum}[v, x]})^n)^p},$   
 $x] /; \text{FreeQ}\{F, a, b, g, n, p\}, x] \&\& \text{LinearQ}[v, x] \&\& \text{PowerOfLinearQ}[u, x]$   
 $] \&\& !(\text{LinearMatchQ}[v, x] \&\& \text{PowerOfLinearMatchQ}[u, x]) \&\& \text{IntegerQ}[m]$

### Rule 2177

$\text{Int}[(b_.)*(F_.)^((g_.)*((e_.) + (f_.)*(x_.)))^{(n_.)}*((c_.) + (d_.)*(x_.))^{(m_.)}, x\_Symbol] :$   
 $> \text{Simp}[(c + dx)^{(m+1)}*(bF^{g*(e+fx)})^n/(d*(m+1))$   
 $, x] - \text{Dist}[(f*g*n*\text{Log}[F])/(d*(m+1)), \text{Int}[(c + dx)^{(m+1)}*(bF^{g*(e+fx)})^n, x]$   
 $] /; \text{FreeQ}\{F, b, c, d, e, f, g, n\}, x] \&\& \text{LtQ}[m, -1] \&\& \text{Int}$

egerQ[2\*m] && !\$UseGamma === True

### Rule 2178

Int[(F\_)^((g\_)\*(e\_) + (f\_)\*(x\_)))/((c\_) + (d\_)\*(x\_)), x\_Symbol] := Simp[(F^(g\*(e - (c\*f)/d))\*ExpIntegralEi[(f\*g\*(c + d\*x)\*Log[F])/d])/d, x] /; FreeQ[{F, c, d, e, f, g}, x] && !\$UseGamma === True

### Rubi steps

$$\begin{aligned} \int \frac{F^{c(ax)} dx}{d^3 + 3d^2ex + 3de^2x^2 + e^3x^3} &= \int \frac{F^{c(ax)}}{(d + ex)^3} dx \\ &= -\frac{F^{c(ax)}}{2e(d + ex)^2} + \frac{(bc \log(F)) \int \frac{F^{c(ax)}}{(d+ex)^2} dx}{2e} \\ &= -\frac{F^{c(ax)}}{2e(d + ex)^2} - \frac{bcF^{c(ax)} \log(F)}{2e^2(d + ex)} + \frac{(b^2c^2 \log^2(F)) \int \frac{F^{c(ax)}}{d+ex} dx}{2e^2} \\ &= -\frac{F^{c(ax)}}{2e(d + ex)^2} - \frac{bcF^{c(ax)} \log(F)}{2e^2(d + ex)} + \frac{b^2c^2F^{c\left(a-\frac{bd}{e}\right)} \text{Ei}\left(\frac{bc(d+ex)\log(F)}{e}\right) \log^2(F)}{2e^3} \end{aligned}$$

**Mathematica [A]** time = 0.0505127, size = 88, normalized size = 0.93

$$\frac{F^{c\left(a-\frac{bd}{e}\right)} \left( b^2c^2 \log^2(F)(d + ex)^2 \text{Ei}\left(\frac{bc(d+ex)\log(F)}{e}\right) - eF^{\frac{bc(d+ex)}{e}} (bc \log(F)(d + ex) + e) \right)}{2e^3(d + ex)^2}$$

Antiderivative was successfully verified.

[In] Integrate[F^(c\*(a + b\*x))/(d^3 + 3\*d^2\*e\*x + 3\*d\*e^2\*x^2 + e^3\*x^3), x]

[Out] (F^(c\*(a - (b\*d)/e))\*(b^2\*c^2\*(d + e\*x)^2\*ExpIntegralEi[(b\*c\*(d + e\*x)\*Log[F])/e]\*Log[F]^2 - e\*F^((b\*c\*(d + e\*x))/e)\*(e + b\*c\*(d + e\*x)\*Log[F]))/(2\*e^3\*(d + e\*x)^2)

**Maple [A]** time = 0.059, size = 155, normalized size = 1.6

$$-\frac{b^2c^2(\ln(F))^2 F^{bcx} F^{ac}}{2e^3} \left( bcx \ln(F) + \frac{\ln(F) bcd}{e} \right)^{-2} - \frac{b^2c^2(\ln(F))^2 F^{bcx} F^{ac}}{2e^3} \left( bcx \ln(F) + \frac{\ln(F) bcd}{e} \right)^{-1} - \frac{b^2c^2(\ln(F))^2 F^{\frac{c(ae-bd)}{e}}}{2e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(F^(c*(b*x+a))/(e^3*x^3+3*d*e^2*x^2+3*d^2*e*x+d^3),x)`

[Out] 
$$-1/2*b^2*c^2*\ln(F)^2/e^3*F^(b*c*x)*F^(a*c)/(b*c*x*\ln(F)+1/e*\ln(F)*b*c*d)^2 - 1/2*b^2*c^2*\ln(F)^2/e^3*F^(b*c*x)*F^(a*c)/(b*c*x*\ln(F)+1/e*\ln(F)*b*c*d) - 1/2*b^2*c^2*\ln(F)^2/e^3*F^(c*(a*e-b*d)/e)*Ei(1,-b*c*x*\ln(F)-a*c*\ln(F)-(-\ln(F)*a*c*e+\ln(F)*b*c*d)/e)$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{(bx+a)c}}{e^3x^3 + 3de^2x^2 + 3d^2ex + d^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(c*(b*x+a))/(e^3*x^3+3*d*e^2*x^2+3*d^2*e*x+d^3),x, algorithm="maxima")`

[Out] `integrate(F^((b*x + a)*c)/(e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3), x)`

**Fricas [A]** time = 1.49918, size = 278, normalized size = 2.93

$$\frac{\frac{(b^2c^2e^2x^2+2b^2c^2dex+b^2c^2d^2)Ei\left(\frac{(bcex+bcd)\log(F)}{e}\right)\log(F)^2}{F^{\frac{bcd-ace}{e}}}}{2(e^5x^2+2de^4x+d^2e^3)} - (e^2 + (bce^2x + bcde)\log(F))F^{bcx+ac}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(c*(b*x+a))/(e^3*x^3+3*d*e^2*x^2+3*d^2*e*x+d^3),x, algorithm="fricas")`

[Out] 
$$1/2*((b^2*c^2*e^2*x^2 + 2*b^2*c^2*d*e*x + b^2*c^2*d^2)*Ei((b*c*e*x + b*c*d)*\log(F)/e)*\log(F)^2/F^((b*c*d - a*c*e)/e) - (e^2 + (b*c*e^2*x + b*c*d*e)*\log(F))*F^(b*c*x + a*c))/(e^5*x^2 + 2*d*e^4*x + d^2*e^3)$$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{c(a+bx)}}{(d+ex)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(c\*(b\*x+a))/(e\*\*3\*x\*\*3+3\*d\*e\*\*2\*x\*\*2+3\*d\*\*2\*e\*x+d\*\*3), x)

[Out] Integral(F\*\*(c\*(a + b\*x))/(d + e\*x)\*\*3, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{(bx+a)c}}{e^3x^3 + 3de^2x^2 + 3d^2ex + d^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(c\*(b\*x+a))/(e^3\*x^3+3\*d\*e^2\*x^2+3\*d^2\*e\*x+d^3), x, algorithm="giac")

[Out] integrate(F^((b\*x + a)\*c)/(e^3\*x^3 + 3\*d\*e^2\*x^2 + 3\*d^2\*e\*x + d^3), x)



$$3.17 \quad \int \frac{F^{c(a+bx)}}{d^4 + 4d^3ex + 6d^2e^2x^2 + 4de^3x^3 + e^4x^4} dx$$

**Optimal.** Leaf size=128

$$\frac{b^3c^3 \log^3(F) F^{c\left(a-\frac{bd}{e}\right)} \text{Ei}\left(\frac{bc(d+ex)\log(F)}{e}\right)}{6e^4} - \frac{b^2c^2 \log^2(F) F^{c(a+bx)}}{6e^3(d+ex)} - \frac{bc \log(F) F^{c(a+bx)}}{6e^2(d+ex)^2} - \frac{F^{c(a+bx)}}{3e(d+ex)^3}$$

[Out]  $-F^{c(a+bx)}/(3e^3(d+ex)^3) - (b^3c^3 \log^3(F) F^{c(a-\frac{bd}{e})} \text{Ei}(\frac{bc(d+ex)\log(F)}{e}))/6e^4 - (b^2c^2 \log^2(F) F^{c(a+bx)})/(6e^3(d+ex)) - (bc \log(F) F^{c(a+bx)})/(6e^2(d+ex)^2) - F^{c(a+bx)}/(3e(d+ex)^3)$

**Rubi [A]** time = 0.133982, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 50,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.06$ , Rules used = {2187, 2177, 2178}

$$\frac{b^3c^3 \log^3(F) F^{c\left(a-\frac{bd}{e}\right)} \text{Ei}\left(\frac{bc(d+ex)\log(F)}{e}\right)}{6e^4} - \frac{b^2c^2 \log^2(F) F^{c(a+bx)}}{6e^3(d+ex)} - \frac{bc \log(F) F^{c(a+bx)}}{6e^2(d+ex)^2} - \frac{F^{c(a+bx)}}{3e(d+ex)^3}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[F^{c(a+bx)}/(d^4 + 4d^3ex + 6d^2e^2x^2 + 4de^3x^3 + e^4x^4), x]$

[Out]  $-F^{c(a+bx)}/(3e^3(d+ex)^3) - (b^3c^3 \log^3(F) F^{c(a-\frac{bd}{e})} \text{Ei}(\frac{bc(d+ex)\log(F)}{e}))/6e^4 - (b^2c^2 \log^2(F) F^{c(a+bx)})/(6e^3(d+ex)) + (bc \log(F) F^{c(a+bx)})/(6e^2(d+ex)^2) - F^{c(a+bx)}/(3e(d+ex)^3)$

### Rule 2187

$\text{Int}[\left((a_.) + (b_.) \cdot ((F_.)^{(g_.) \cdot (v_.)})^{(n_.)})^{(p_.)} \cdot (u_.)^{(m_.)}, x\_Symbol] :> \text{Int}[\text{NormalizePowerOfLinear}[u, x]^m \cdot (a + b \cdot (F^{g \cdot \text{ExpandToSum}[v, x]}))^n]^p, x] /; \text{FreeQ}\{F, a, b, g, n, p\}, x] \ \&\& \ \text{LinearQ}[v, x] \ \&\& \ \text{PowerOfLinearQ}[u, x] \ \&\& \ !(\text{LinearMatchQ}[v, x] \ \&\& \ \text{PowerOfLinearMatchQ}[u, x]) \ \&\& \ \text{IntegerQ}[m]$

### Rule 2177

$\text{Int}[\left((b_.) \cdot (F_.)^{(g_.) \cdot ((e_.) + (f_.) \cdot (x_.)})^{(n_.)} \cdot ((c_.) + (d_.) \cdot (x_.)\right)^{(m_.)}, x\_Symbol] :> \text{Simp}[\left((c + d \cdot x)^{(m+1)} \cdot (b \cdot F^{g \cdot (e + f \cdot x)})^n\right) / (d \cdot (m+1))$

```
, x] - Dist[(f*g*n*Log[F])/(d*(m + 1)), Int[(c + d*x)^(m + 1)*(b*F^(g*(e + f*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && LtQ[m, -1] && IntegerQ[2*m] && !$UseGamma == True
```

### Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; FreeQ[{F, c, d, e, f, g}, x] && !$UseGamma == True
```

### Rubi steps

$$\begin{aligned}
 \int \frac{F^{c(ax)} d^4 + 4d^3 ex + 6d^2 e^2 x^2 + 4de^3 x^3 + e^4 x^4}{(d + ex)^4} dx &= \int \frac{F^{c(ax)}}{(d + ex)^4} dx \\
 &= -\frac{F^{c(ax)}}{3e(d + ex)^3} + \frac{(bc \log(F)) \int \frac{F^{c(ax)}}{(d + ex)^3} dx}{3e} \\
 &= -\frac{F^{c(ax)}}{3e(d + ex)^3} - \frac{bc F^{c(ax)} \log(F)}{6e^2(d + ex)^2} + \frac{(b^2 c^2 \log^2(F)) \int \frac{F^{c(ax)}}{(d + ex)^2} dx}{6e^2} \\
 &= -\frac{F^{c(ax)}}{3e(d + ex)^3} - \frac{bc F^{c(ax)} \log(F)}{6e^2(d + ex)^2} - \frac{b^2 c^2 F^{c(ax)} \log^2(F)}{6e^3(d + ex)} + \frac{(b^3 c^3 \log^3(F)) \int F^{c(ax)} dx}{6e^3} \\
 &= -\frac{F^{c(ax)}}{3e(d + ex)^3} - \frac{bc F^{c(ax)} \log(F)}{6e^2(d + ex)^2} - \frac{b^2 c^2 F^{c(ax)} \log^2(F)}{6e^3(d + ex)} + \frac{b^3 c^3 F^{c(a - \frac{bd}{e})} \text{Ei}\left(\frac{bc(d + ex) \log(F)}{e}\right)}{6e^4}
 \end{aligned}$$

**Mathematica [A]** time = 0.0800948, size = 99, normalized size = 0.77

$$\frac{F^{ac} \left( b^3 c^3 \log^3(F) F^{-\frac{bcd}{e}} \text{Ei}\left(\frac{bc(d+ex)\log(F)}{e}\right) - \frac{e F^{bcx} (b^2 c^2 \log^2(F)(d+ex)^2 + bce \log(F)(d+ex) + 2e^2)}{(d+ex)^3} \right)}{6e^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[F^(c*(a + b*x))/(d^4 + 4*d^3*e*x + 6*d^2*e^2*x^2 + 4*d*e^3*x^3 + e^4*x^4), x]
```

```
[Out] (F^(a*c)*((b^3*c^3*ExpIntegralEi[(b*c*(d + e*x)*Log[F])/e]*Log[F]^3)/F^((b*c*d)/e) - (e*F^(b*c*x)*(2*e^2 + b*c*e*(d + e*x)*Log[F] + b^2*c^2*(d + e*x)^2*Log[F]^2))/(d + e*x)^3))/(6*e^4)
```

---

**Maple [A]** time = 0.076, size = 199, normalized size = 1.6

$$-\frac{b^3 c^3 (\ln(F))^3 F^{bcx} F^{ac}}{3 e^4} \left( bcx \ln(F) + \frac{\ln(F) bcd}{e} \right)^{-3} - \frac{b^3 c^3 (\ln(F))^3 F^{bcx} F^{ac}}{6 e^4} \left( bcx \ln(F) + \frac{\ln(F) bcd}{e} \right)^{-2} - \frac{b^3 c^3 (\ln(F))^3 F^{bcx} F^{ac}}{6 e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(c\*(b\*x+a)))/(e^4\*x^4+4\*d\*e^3\*x^3+6\*d^2\*e^2\*x^2+4\*d^3\*e\*x+d^4), x)

[Out] 
$$-1/3*b^3*c^3*\ln(F)^3/e^4*F^{(b*c*x)*F^{(a*c)}}/(b*c*x*\ln(F)+1/e*\ln(F)*b*c*d)^3-1/6*b^3*c^3*\ln(F)^3/e^4*F^{(b*c*x)*F^{(a*c)}}/(b*c*x*\ln(F)+1/e*\ln(F)*b*c*d)^2-1/6*b^3*c^3*\ln(F)^3/e^4*F^{(b*c*x)*F^{(a*c)}}/(b*c*x*\ln(F)+1/e*\ln(F)*b*c*d)-1/6*b^3*c^3*\ln(F)^3/e^4*F^{(c*(a*e-b*d)/e)*Ei(1,-b*c*x*\ln(F)-a*c*\ln(F)-(-\ln(F)*a*c*e+\ln(F)*b*c*d)/e)}$$

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{(bx+a)c}}{e^4 x^4 + 4 d e^3 x^3 + 6 d^2 e^2 x^2 + 4 d^3 e x + d^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(c\*(b\*x+a)))/(e^4\*x^4+4\*d\*e^3\*x^3+6\*d^2\*e^2\*x^2+4\*d^3\*e\*x+d^4), x, algorithm="maxima")

[Out] integrate(F^((b\*x + a)\*c))/(e^4\*x^4 + 4\*d\*e^3\*x^3 + 6\*d^2\*e^2\*x^2 + 4\*d^3\*e\*x + d^4), x)

---

**Fricas [A]** time = 1.57511, size = 425, normalized size = 3.32

$$\frac{(b^3 c^3 e^3 x^3 + 3 b^3 c^3 d e^2 x^2 + 3 b^3 c^3 d^2 e x + b^3 c^3 d^3) \operatorname{Ei}\left(\frac{(bcx+bcd) \log(F)}{e}\right) \log(F)^3}{\frac{bcd-ace}{F} e} - \left( 2 e^3 + (b^2 c^2 e^3 x^2 + 2 b^2 c^2 d e^2 x + b^2 c^2 d^2 e) \log(F)^2 + (b c e^3 x + b^2 c^2 d e) \log(F) + b^2 c^2 d^2 e \right) \log(F) + b^2 c^2 d^2 e$$

$$6 (e^7 x^3 + 3 d e^6 x^2 + 3 d^2 e^5 x + d^3 e^4)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(c\*(b\*x+a)))/(e^4\*x^4+4\*d\*e^3\*x^3+6\*d^2\*e^2\*x^2+4\*d^3\*e\*x+d^4), x, algorithm="fricas")

```
[Out] 1/6*((b^3*c^3*e^3*x^3 + 3*b^3*c^3*d*e^2*x^2 + 3*b^3*c^3*d^2*e*x + b^3*c^3*d^3)*Ei((b*c*e*x + b*c*d)*log(F)/e)*log(F)^3/F^((b*c*d - a*c*e)/e) - (2*e^3 + (b^2*c^2*e^3*x^2 + 2*b^2*c^2*d*e^2*x + b^2*c^2*d^2*e)*log(F)^2 + (b*c*e^3*x + b*c*d*e^2)*log(F))*F^(b*c*x + a*c))/(e^7*x^3 + 3*d*e^6*x^2 + 3*d^2*e^5*x + d^3*e^4)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(c*(b*x+a))/(e**4*x**4+4*d*e**3*x**3+6*d**2*e**2*x**2+4*d**3*e*x+d**4), x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{(bx+a)c}}{e^4x^4 + 4de^3x^3 + 6d^2e^2x^2 + 4d^3ex + d^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))/(e^4*x^4+4*d*e^3*x^3+6*d^2*e^2*x^2+4*d^3*e*x+d^4), x, algorithm="giac")
```

```
[Out] integrate(F^((b*x + a)*c)/(e^4*x^4 + 4*d*e^3*x^3 + 6*d^2*e^2*x^2 + 4*d^3*e*x + d^4), x)
```

$$3.18 \quad \int \frac{F^{c(a+bx)}}{d^5 + 5d^4ex + 10d^3e^2x^2 + 10d^2e^3x^3 + 5de^4x^4 + e^5x^5} dx$$

**Optimal.** Leaf size=161

$$\frac{b^4c^4 \log^4(F) F^{c\left(a-\frac{bd}{e}\right)} \operatorname{Ei}\left(\frac{bc(d+ex)\log(F)}{e}\right)}{24e^5} - \frac{b^3c^3 \log^3(F) F^{c(a+bx)}}{24e^4(d+ex)} - \frac{b^2c^2 \log^2(F) F^{c(a+bx)}}{24e^3(d+ex)^2} - \frac{bc \log(F) F^{c(a+bx)}}{12e^2(d+ex)^3} - \frac{F^{c(a+bx)}}{4e(d+ex)^4}$$

[Out]  $-F^{c(a+bx)}/(4e*(d+ex)^4) - (b*c*F^{c(a+bx)}*Log[F])/(12*e^2*(d+ex)^3) - (b^2*c^2*F^{c(a+bx)}*Log[F]^2)/(24*e^3*(d+ex)^2) - (b^3*c^3*F^{c(a+bx)}*Log[F]^3)/(24*e^4*(d+ex)) + (b^4*c^4*F^{c(a+bx)}*ExpIntegralEi[(b*c*(d+ex)*Log[F])/e]*Log[F]^4)/(24*e^5)$

**Rubi [A]** time = 0.180379, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 61,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.049$ , Rules used = {2187, 2177, 2178}

$$\frac{b^4c^4 \log^4(F) F^{c\left(a-\frac{bd}{e}\right)} \operatorname{Ei}\left(\frac{bc(d+ex)\log(F)}{e}\right)}{24e^5} - \frac{b^3c^3 \log^3(F) F^{c(a+bx)}}{24e^4(d+ex)} - \frac{b^2c^2 \log^2(F) F^{c(a+bx)}}{24e^3(d+ex)^2} - \frac{bc \log(F) F^{c(a+bx)}}{12e^2(d+ex)^3} - \frac{F^{c(a+bx)}}{4e(d+ex)^4}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[F^{c(a+bx)}/(d^5 + 5d^4ex + 10d^3e^2x^2 + 10d^2e^3x^3 + 5de^4x^4 + e^5x^5), x]$

[Out]  $-F^{c(a+bx)}/(4e*(d+ex)^4) - (b*c*F^{c(a+bx)}*Log[F])/(12*e^2*(d+ex)^3) - (b^2*c^2*F^{c(a+bx)}*Log[F]^2)/(24*e^3*(d+ex)^2) - (b^3*c^3*F^{c(a+bx)}*Log[F]^3)/(24*e^4*(d+ex)) + (b^4*c^4*F^{c(a+bx)}*ExpIntegralEi[(b*c*(d+ex)*Log[F])/e]*Log[F]^4)/(24*e^5)$

### Rule 2187

```
Int[((a_.) + (b_.)*((F_)^((g_.)*(v_)))^(n_.))^(p_.)*(u_)^(m_.), x_Symbol] :
> Int[NormalizePowerOfLinear[u, x]^m*(a + b*(F^(g*ExpandToSum[v, x]))^n)^p,
x] /; FreeQ[{F, a, b, g, n, p}, x] && LinearQ[v, x] && PowerOfLinearQ[u, x]
&& !(LinearMatchQ[v, x] && PowerOfLinearMatchQ[u, x]) && IntegerQ[m]
```

### Rule 2177

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[((c + d*x)^(m + 1)*(b*F^(g*(e + f*x)))^n)/(d*(m + 1))
```

```
, x] - Dist[(f*g*n*Log[F])/(d*(m + 1)), Int[(c + d*x)^(m + 1)*(b*F^(g*(e + f*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && LtQ[m, -1] && IntegerQ[2*m] && !$UseGamma === True
```

### Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; FreeQ[{F, c, d, e, f, g}, x] && !$UseGamma === True
```

### Rubi steps

$$\begin{aligned} \int \frac{F^{c(ax)} d^5 + 5d^4 ex + 10d^3 e^2 x^2 + 10d^2 e^3 x^3 + 5de^4 x^4 + e^5 x^5}{(d + ex)^5} dx &= \int \frac{F^{c(ax)}}{(d + ex)^5} dx \\ &= -\frac{F^{c(ax)}}{4e(d + ex)^4} + \frac{(bc \log(F)) \int \frac{F^{c(ax)}}{(d + ex)^4} dx}{4e} \\ &= -\frac{F^{c(ax)}}{4e(d + ex)^4} - \frac{bc F^{c(ax)} \log(F)}{12e^2(d + ex)^3} + \frac{(b^2 c^2 \log^2(F)) \int \frac{F^{c(ax)}}{(d + ex)^3} dx}{12e^2} \\ &= -\frac{F^{c(ax)}}{4e(d + ex)^4} - \frac{bc F^{c(ax)} \log(F)}{12e^2(d + ex)^3} - \frac{b^2 c^2 F^{c(ax)} \log^2(F)}{24e^3(d + ex)^2} + \frac{(b^3 c^3 \log^3(F)) \int \frac{F^{c(ax)}}{(d + ex)^2} dx}{24e^3} \\ &= -\frac{F^{c(ax)}}{4e(d + ex)^4} - \frac{bc F^{c(ax)} \log(F)}{12e^2(d + ex)^3} - \frac{b^2 c^2 F^{c(ax)} \log^2(F)}{24e^3(d + ex)^2} - \frac{b^3 c^3 F^{c(ax)} \log^3(F)}{24e^3(d + ex)^2} \\ &= -\frac{F^{c(ax)}}{4e(d + ex)^4} - \frac{bc F^{c(ax)} \log(F)}{12e^2(d + ex)^3} - \frac{b^2 c^2 F^{c(ax)} \log^2(F)}{24e^3(d + ex)^2} - \frac{b^3 c^3 F^{c(ax)} \log^3(F)}{24e^3(d + ex)^2} \end{aligned}$$

**Mathematica [A]** time = 0.0342022, size = 121, normalized size = 0.75

$$\frac{F^{ac} \left( b^4 c^4 \log^4(F) F^{-\frac{bcd}{e}} \operatorname{Ei} \left( \frac{bc(d+ex) \log(F)}{e} \right) - \frac{e F^{bcx} (b^2 c^2 e \log^2(F)(d+ex)^2 + b^3 c^3 \log^3(F)(d+ex)^3 + 2bce^2 \log(F)(d+ex) + 6e^3)}{(d+ex)^4} \right)}{24e^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[F^(c*(a + b*x))/(d^5 + 5*d^4*e*x + 10*d^3*e^2*x^2 + 10*d^2*e^3*x^3 + 5*d*e^4*x^4 + e^5*x^5), x]
```

```
[Out] (F^(a*c)*((b^4*c^4*ExpIntegralEi[(b*c*(d + e*x)*Log[F])/e]*Log[F]^4)/F^((b*c*d)/e) - (e*F^(b*c*x)*(6*e^3 + 2*b*c*e^2*(d + e*x)*Log[F] + b^2*c^2*e*(d +
```

$$e*x)^2*\text{Log}[F]^2 + b^3*c^3*(d + e*x)^3*\text{Log}[F]^3)/(d + e*x)^4)/(24*e^5)$$

**Maple [A]** time = 0.096, size = 243, normalized size = 1.5

$$-\frac{b^4c^4(\ln(F))^4 F^{bcx}F^{ac}}{4e^5} \left( bcx \ln(F) + \frac{\ln(F) bcd}{e} \right)^{-4} - \frac{b^4c^4(\ln(F))^4 F^{bcx}F^{ac}}{12e^5} \left( bcx \ln(F) + \frac{\ln(F) bcd}{e} \right)^{-3} - \frac{b^4c^4(\ln(F))^4 F^{bcx}F^{ac}}{24e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(c\*(b\*x+a)))/(e^5\*x^5+5\*d\*e^4\*x^4+10\*d^2\*e^3\*x^3+10\*d^3\*e^2\*x^2+5\*d^4\*e\*x+d^5), x)

[Out]  $-\frac{1}{4}b^4c^4*\ln(F)^4/e^5*F^{(b*c*x)*F^{(a*c)/(b*c*x*\ln(F)+1/e*\ln(F)*b*c*d)}-1/12*b^4c^4*\ln(F)^4/e^5*F^{(b*c*x)*F^{(a*c)/(b*c*x*\ln(F)+1/e*\ln(F)*b*c*d)}-1/24*b^4c^4*\ln(F)^4/e^5*F^{(b*c*x)*F^{(a*c)/(b*c*x*\ln(F)+1/e*\ln(F)*b*c*d)}-1/24*b^4c^4*\ln(F)^4/e^5*F^{(b*c*x)*F^{(a*c)/(b*c*x*\ln(F)+1/e*\ln(F)*b*c*d)}-1/24*b^4c^4*\ln(F)^4/e^5*F^{(c*(a*e-b*d)/e)*Ei(1,-b*c*x*\ln(F)-a*c*\ln(F)-(-\ln(F))*a*c*e+\ln(F)*b*c*d)/e}$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{(bx+a)c}}{e^5x^5 + 5de^4x^4 + 10d^2e^3x^3 + 10d^3e^2x^2 + 5d^4ex + d^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(c\*(b\*x+a)))/(e^5\*x^5+5\*d\*e^4\*x^4+10\*d^2\*e^3\*x^3+10\*d^3\*e^2\*x^2+5\*d^4\*e\*x+d^5), x, algorithm="maxima")

[Out] integrate(F^((b\*x + a)\*c)/(e^5\*x^5 + 5\*d\*e^4\*x^4 + 10\*d^2\*e^3\*x^3 + 10\*d^3\*e^2\*x^2 + 5\*d^4\*e\*x + d^5), x)

**Fricas [A]** time = 1.59425, size = 606, normalized size = 3.76

$$\frac{(b^4c^4e^4x^4+4b^4c^4de^3x^3+6b^4c^4d^2e^2x^2+4b^4c^4d^3ex+b^4c^4d^4)Ei\left(\frac{(bcex+bcd)\log(F)}{e}\right)\log(F)^4}{F^{\frac{bcd-ace}{e}}} - \left(6e^4 + (b^3c^3e^4x^3 + 3b^3c^3de^3x^2 + 3b^3c^3d^2e^2x + b^3c^3d^3)\right)$$

$$24(e^9x^4 + 4de^8x^3 + 6d^2e^7x^2 + 4d^3e^6x + d^4e^5)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))/(e^5*x^5+5*d*e^4*x^4+10*d^2*e^3*x^3+10*d^3*e^2*x^2+5*d^4*e*x+d^5),x, algorithm="fricas")
```

```
[Out] 1/24*((b^4*c^4*e^4*x^4 + 4*b^4*c^4*d*e^3*x^3 + 6*b^4*c^4*d^2*e^2*x^2 + 4*b^4*c^4*d^3*e*x + b^4*c^4*d^4)*Ei((b*c*e*x + b*c*d)*log(F)/e)*log(F)^4/F^((b*c*d - a*c*e)/e) - (6*e^4 + (b^3*c^3*e^4*x^3 + 3*b^3*c^3*d*e^3*x^2 + 3*b^3*c^3*d^2*e^2*x + b^3*c^3*d^3*e)*log(F)^3 + (b^2*c^2*e^4*x^2 + 2*b^2*c^2*d*e^3*x + b^2*c^2*d^2*e^2)*log(F)^2 + 2*(b*c*e^4*x + b*c*d*e^3)*log(F))*F^(b*c*x + a*c))/(e^9*x^4 + 4*d*e^8*x^3 + 6*d^2*e^7*x^2 + 4*d^3*e^6*x + d^4*e^5)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(c*(b*x+a))/(e**5*x**5+5*d*e**4*x**4+10*d**2*e**3*x**3+10*d**3*e**2*x**2+5*d**4*e*x+d**5),x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{(bx+a)c}}{e^5x^5 + 5de^4x^4 + 10d^2e^3x^3 + 10d^3e^2x^2 + 5d^4ex + d^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))/(e^5*x^5+5*d*e^4*x^4+10*d^2*e^3*x^3+10*d^3*e^2*x^2+5*d^4*e*x+d^5),x, algorithm="giac")
```

```
[Out] integrate(F^((b*x + a)*c)/(e^5*x^5 + 5*d*e^4*x^4 + 10*d^2*e^3*x^3 + 10*d^3*e^2*x^2 + 5*d^4*e*x + d^5), x)
```



### 3.19 $\int F^{c(a+bx)} ((d+ex)^n)^m dx$

**Optimal.** Leaf size=72

$$\frac{((d+ex)^n)^m F^{c\left(a-\frac{bd}{e}\right)} \left(-\frac{bc \log(F)(d+ex)}{e}\right)^{-mn} \text{Gamma}\left(mn+1, -\frac{bc \log(F)(d+ex)}{e}\right)}{bc \log(F)}$$

[Out] (F^(c\*(a - (b\*d)/e))\*((d + e\*x)^n)^m\*Gamma[1 + m\*n, -((b\*c\*(d + e\*x)\*Log[F])/e)]/(b\*c\*Log[F]\*(-(b\*c\*(d + e\*x)\*Log[F])/e))^(m\*n))

**Rubi [A]** time = 0.0536632, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$ , Rules used = {2188, 2181}

$$\frac{((d+ex)^n)^m F^{c\left(a-\frac{bd}{e}\right)} \left(-\frac{bc \log(F)(d+ex)}{e}\right)^{-mn} \text{Gamma}\left(mn+1, -\frac{bc \log(F)(d+ex)}{e}\right)}{bc \log(F)}$$

Antiderivative was successfully verified.

[In] Int[F^(c\*(a + b\*x))\*((d + e\*x)^n)^m,x]

[Out] (F^(c\*(a - (b\*d)/e))\*((d + e\*x)^n)^m\*Gamma[1 + m\*n, -((b\*c\*(d + e\*x)\*Log[F])/e)]/(b\*c\*Log[F]\*(-(b\*c\*(d + e\*x)\*Log[F])/e))^(m\*n))

#### Rule 2188

```
Int[((a_.) + (b_.)*((F_)^((g_.)*(v_)))^(n_.))^(p_.)*(u_)^(m_.), x_Symbol] :
> Module[{uu = NormalizePowerOfLinear[u, x], z}, Simp[z = If[PowerQ[uu] &&
FreeQ[uu[[2]], x], uu[[1]]^(m*uu[[2]]), uu^m]; (uu^m*Int[z*(a + b*(F^(g*Exp
andToSum[v, x]))^n)^p, x])/z, x] /; FreeQ[{F, a, b, g, m, n, p}, x] && Lin
earQ[v, x] && PowerOfLinearQ[u, x] && !(LinearMatchQ[v, x] && PowerOfLinea
rMatchQ[u, x]) && !IntegerQ[m]
```

#### Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d)*(c + d*x])]/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-(f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !I
```

ntegerQ [m]

### Rubi steps

$$\int F^{c(a+bx)} ((d+ex)^n)^m dx = (d+ex)^{-mn} ((d+ex)^n)^m \int F^{ac+bcx} (d+ex)^{mn} dx$$

$$= \frac{F^{c\left(a-\frac{bd}{e}\right)} ((d+ex)^n)^m \Gamma\left(1+mn, -\frac{bc(d+ex)\log(F)}{e}\right) \left(-\frac{bc(d+ex)\log(F)}{e}\right)^{-mn}}{bc \log(F)}$$

**Mathematica [A]** time = 0.0130439, size = 72, normalized size = 1.

$$\frac{((d+ex)^n)^m F^{c\left(a-\frac{bd}{e}\right)} \left(-\frac{bc \log(F)(d+ex)}{e}\right)^{-mn} \text{Gamma}\left(mn+1, -\frac{bc \log(F)(d+ex)}{e}\right)}{bc \log(F)}$$

Antiderivative was successfully verified.

[In] Integrate[F^(c\*(a + b\*x))\*((d + e\*x)^n)^m, x]

[Out] (F^(c\*(a - (b\*d)/e))\*((d + e\*x)^n)^m\*Gamma[1 + m\*n, -((b\*c\*(d + e\*x)\*Log[F])/e)]/(b\*c\*Log[F]\*(-(b\*c\*(d + e\*x)\*Log[F])/e))^(m\*n))

**Maple [F]** time = 0.043, size = 0, normalized size = 0.

$$\int F^{c(bx+a)} ((ex+d)^n)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(c\*(b\*x+a))\*((e\*x+d)^n)^m, x)

[Out] int(F^(c\*(b\*x+a))\*((e\*x+d)^n)^m, x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int ((ex+d)^n)^m F^{(bx+a)c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(c\*(b\*x+a))\*((e\*x+d)^n)^m,x, algorithm="maxima")

[Out] integrate(((e\*x + d)^n)^m\*F^((b\*x + a)\*c), x)

**Fricas [A]** time = 1.77738, size = 158, normalized size = 2.19

$$\frac{e^{\left(-\frac{emn \log\left(-\frac{bc \log(F)}{e}\right) + (bcd - ace) \log(F)}{e}\right)} \Gamma\left(mn + 1, -\frac{(bcex + bcd) \log(F)}{e}\right)}{bc \log(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(c\*(b\*x+a))\*((e\*x+d)^n)^m,x, algorithm="fricas")

[Out] e^(-(e\*m\*n\*log(-b\*c\*log(F)/e) + (b\*c\*d - a\*c\*e)\*log(F))/e)\*gamma(m\*n + 1, -(b\*c\*e\*x + b\*c\*d)\*log(F)/e)/(b\*c\*log(F))

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(c\*(b\*x+a))\*((e\*x+d)\*\*n)\*\*m,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (ex + d)^n F^{(bx+a)c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*((e*x+d)^n)^m,x, algorithm="giac")
```

```
[Out] integrate(((e*x + d)^n)^m*F^((b*x + a)*c), x)
```

$$3.20 \quad \int F^{c(a+bx)} \left( d^4 + 4d^3ex + 6d^2e^2x^2 + 4de^3x^3 + e^4x^4 \right)^m dx$$

**Optimal.** Leaf size=71

$$\frac{\left( (d+ex)^4 \right)^m F^{c\left(a-\frac{bd}{e}\right)} \left( -\frac{bc \log(F)(d+ex)}{e} \right)^{-4m} \text{Gamma}\left( 4m+1, -\frac{bc \log(F)(d+ex)}{e} \right)}{bc \log(F)}$$

[Out] (F^(c\*(a - (b\*d)/e))\*((d + e\*x)^4)^m\*Gamma[1 + 4\*m, -((b\*c\*(d + e\*x)\*Log[F])/e)]/(b\*c\*Log[F]\*(-(b\*c\*(d + e\*x)\*Log[F])/e))^(4\*m))

**Rubi [A]** time = 0.0699975, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 50,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.04$ , Rules used = {2188, 2181}

$$\frac{\left( (d+ex)^4 \right)^m F^{c\left(a-\frac{bd}{e}\right)} \left( -\frac{bc \log(F)(d+ex)}{e} \right)^{-4m} \text{Gamma}\left( 4m+1, -\frac{bc \log(F)(d+ex)}{e} \right)}{bc \log(F)}$$

Antiderivative was successfully verified.

[In] Int[F^(c\*(a + b\*x))\*(d^4 + 4\*d^3\*e\*x + 6\*d^2\*e^2\*x^2 + 4\*d\*e^3\*x^3 + e^4\*x^4)^m,x]

[Out] (F^(c\*(a - (b\*d)/e))\*((d + e\*x)^4)^m\*Gamma[1 + 4\*m, -((b\*c\*(d + e\*x)\*Log[F])/e)]/(b\*c\*Log[F]\*(-(b\*c\*(d + e\*x)\*Log[F])/e))^(4\*m))

### Rule 2188

```
Int[((a_.) + (b_.)*((F_)^((g_.)*(v_)))^(n_.))^(p_.)*(u_)^(m_.), x_Symbol] :
> Module[{uu = NormalizePowerOfLinear[u, x], z}, Simp[z = If[PowerQ[uu] &&
FreeQ[uu[[2]], x], uu[[1]]^(m*uu[[2]]), uu^m]; (uu^m*Int[z*(a + b*(F^(g*Exp
andToSum[v, x]))^n)^p, x])/z, x] /; FreeQ[{F, a, b, g, m, n, p}, x] && Lin
earQ[v, x] && PowerOfLinearQ[u, x] && !(LinearMatchQ[v, x] && PowerOfLinea
rMatchQ[u, x]) && !IntegerQ[m]
```

### Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/((d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-(f*g*Log[F]
)*(c + d*x)/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !I
```

ntegerQ [m]

### Rubi steps

$$\int F^{c(a+bx)} (d^4 + 4d^3ex + 6d^2e^2x^2 + 4de^3x^3 + e^4x^4)^m dx = (d + ex)^{-4m} ((d + ex)^4)^m \int F^{c(a+bx)} (d + ex)^{4m} dx$$

$$= \frac{F^{c\left(a-\frac{bd}{e}\right)} ((d + ex)^4)^m \Gamma\left(1 + 4m, -\frac{bc(d+ex)\log(F)}{e}\right) \left(-\frac{bc(d+ex)\log(F)}{e}\right)}{bc \log(F)}$$

**Mathematica [A]** time = 0.0147353, size = 71, normalized size = 1.

$$\frac{((d + ex)^4)^m F^{c\left(a-\frac{bd}{e}\right)} \left(-\frac{bc \log(F)(d+ex)}{e}\right)^{-4m} \text{Gamma}\left(4m + 1, -\frac{bc \log(F)(d+ex)}{e}\right)}{bc \log(F)}$$

Antiderivative was successfully verified.

[In] Integrate[F^(c\*(a + b\*x))\*(d^4 + 4\*d^3\*e\*x + 6\*d^2\*e^2\*x^2 + 4\*d\*e^3\*x^3 + e^4\*x^4)^m, x]

[Out] (F^(c\*(a - (b\*d)/e))\*((d + e\*x)^4)^m\*Gamma[1 + 4\*m, -((b\*c\*(d + e\*x)\*Log[F])/e)]/(b\*c\*Log[F]\*(-((b\*c\*(d + e\*x)\*Log[F])/e))^(4\*m))

**Maple [F]** time = 0.158, size = 0, normalized size = 0.

$$\int F^{c(bx+a)} (e^4x^4 + 4de^3x^3 + 6d^2e^2x^2 + 4d^3ex + d^4)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(c\*(b\*x+a))\*(e^4\*x^4+4\*d\*e^3\*x^3+6\*d^2\*e^2\*x^2+4\*d^3\*e\*x+d^4)^m, x)

[Out] int(F^(c\*(b\*x+a))\*(e^4\*x^4+4\*d\*e^3\*x^3+6\*d^2\*e^2\*x^2+4\*d^3\*e\*x+d^4)^m, x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (e^4x^4 + 4de^3x^3 + 6d^2e^2x^2 + 4d^3ex + d^4)^m F^{(bx+a)c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*(e^4*x^4+4*d*e^3*x^3+6*d^2*e^2*x^2+4*d^3*e*x+d^4)^m
,x, algorithm="maxima")
```

```
[Out] integrate((e^4*x^4 + 4*d*e^3*x^3 + 6*d^2*e^2*x^2 + 4*d^3*e*x + d^4)^m*F^(b
*x + a)*c), x)
```

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(e^4x^4 + 4de^3x^3 + 6d^2e^2x^2 + 4d^3ex + d^4\right)^m F^{bcx+ac}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*(e^4*x^4+4*d*e^3*x^3+6*d^2*e^2*x^2+4*d^3*e*x+d^4)^m
,x, algorithm="fricas")
```

```
[Out] integral((e^4*x^4 + 4*d*e^3*x^3 + 6*d^2*e^2*x^2 + 4*d^3*e*x + d^4)^m*F^(b*c
*x + a*c), x)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(c*(b*x+a))*(e**4*x**4+4*d*e**3*x**3+6*d**2*e**2*x**2+4*d**3*e
*x+d**4)**m,x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \left(e^4x^4 + 4de^3x^3 + 6d^2e^2x^2 + 4d^3ex + d^4\right)^m F^{(bx+a)c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*(e^4*x^4+4*d*e^3*x^3+6*d^2*e^2*x^2+4*d^3*e*x+d^4)^m
,x, algorithm="giac")
```

```
[Out] integrate((e^4*x^4 + 4*d*e^3*x^3 + 6*d^2*e^2*x^2 + 4*d^3*e*x + d^4)^m*F^(b
*x + a)*c), x)
```



$$3.21 \quad \int F^{c(a+bx)} \left( d^3 + 3d^2ex + 3de^2x^2 + e^3x^3 \right)^m dx$$

**Optimal.** Leaf size=71

$$\frac{\left( (d+ex)^3 \right)^m F^{c\left(a-\frac{bd}{e}\right)} \left( -\frac{bc \log(F)(d+ex)}{e} \right)^{-3m} \text{Gamma}\left( 3m+1, -\frac{bc \log(F)(d+ex)}{e} \right)}{bc \log(F)}$$

[Out] (F^(c\*(a - (b\*d)/e))\*((d + e\*x)^3)^m\*Gamma[1 + 3\*m, -((b\*c\*(d + e\*x)\*Log[F])/e)]/(b\*c\*Log[F]\*(-(b\*c\*(d + e\*x)\*Log[F])/e))^(3\*m))

**Rubi [A]** time = 0.0604419, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 39,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.051$ , Rules used = {2188, 2181}

$$\frac{\left( (d+ex)^3 \right)^m F^{c\left(a-\frac{bd}{e}\right)} \left( -\frac{bc \log(F)(d+ex)}{e} \right)^{-3m} \text{Gamma}\left( 3m+1, -\frac{bc \log(F)(d+ex)}{e} \right)}{bc \log(F)}$$

Antiderivative was successfully verified.

[In] Int[F^(c\*(a + b\*x))\*(d^3 + 3\*d^2\*e\*x + 3\*d\*e^2\*x^2 + e^3\*x^3)^m,x]

[Out] (F^(c\*(a - (b\*d)/e))\*((d + e\*x)^3)^m\*Gamma[1 + 3\*m, -((b\*c\*(d + e\*x)\*Log[F])/e)]/(b\*c\*Log[F]\*(-(b\*c\*(d + e\*x)\*Log[F])/e))^(3\*m))

### Rule 2188

```
Int[((a_.) + (b_.)*((F_)^((g_.)*(v_)))^(n_.))^(p_.)*(u_)^(m_.), x_Symbol] :
> Module[{uu = NormalizePowerOfLinear[u, x], z}, Simp[z = If[PowerQ[uu] &&
FreeQ[uu[[2]], x], uu[[1]]^(m*uu[[2]]), uu^m]; (uu^m*Int[z*(a + b*(F^(g*Exp
andToSum[v, x]))^n)^p, x])/z, x]] /; FreeQ[{F, a, b, g, m, n, p}, x] && Lin
earQ[v, x] && PowerOfLinearQ[u, x] && !(LinearMatchQ[v, x] && PowerOfLinea
rMatchQ[u, x]) && !IntegerQ[m]
```

### Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x])]/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F]
)*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !I
```

ntegerQ [m]

### Rubi steps

$$\int F^{c(a+bx)} (d^3 + 3d^2ex + 3de^2x^2 + e^3x^3)^m dx = (d + ex)^{-3m} (d + ex)^3)^m \int F^{c(a+bx)} (d + ex)^{3m} dx$$

$$= \frac{F^{c\left(a-\frac{bd}{e}\right)} ((d + ex)^3)^m \Gamma\left(1 + 3m, -\frac{bc(d+ex)\log(F)}{e}\right) \left(-\frac{bc(d+ex)\log(F)}{e}\right)^{-3m}}{bc \log(F)}$$

**Mathematica [A]** time = 0.013664, size = 71, normalized size = 1.

$$\frac{((d + ex)^3)^m F^{c\left(a-\frac{bd}{e}\right)} \left(-\frac{bc \log(F)(d+ex)}{e}\right)^{-3m} \text{Gamma}\left(3m + 1, -\frac{bc \log(F)(d+ex)}{e}\right)}{bc \log(F)}$$

Antiderivative was successfully verified.

[In] Integrate[F^(c\*(a + b\*x))\*(d^3 + 3\*d^2\*e\*x + 3\*d\*e^2\*x^2 + e^3\*x^3)^m,x]

[Out] (F^(c\*(a - (b\*d)/e))\*((d + e\*x)^3)^m\*Gamma[1 + 3\*m, -((b\*c\*(d + e\*x)\*Log[F])/e)]/(b\*c\*Log[F]\*(-(b\*c\*(d + e\*x)\*Log[F])/e))^(3\*m))

**Maple [F]** time = 0.077, size = 0, normalized size = 0.

$$\int F^{c(bx+a)} (e^3x^3 + 3de^2x^2 + 3d^2ex + d^3)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(c\*(b\*x+a))\*(e^3\*x^3+3\*d\*e^2\*x^2+3\*d^2\*e\*x+d^3)^m,x)

[Out] int(F^(c\*(b\*x+a))\*(e^3\*x^3+3\*d\*e^2\*x^2+3\*d^2\*e\*x+d^3)^m,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (e^3x^3 + 3de^2x^2 + 3d^2ex + d^3)^m F^{(bx+a)c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*(e^3*x^3+3*d*e^2*x^2+3*d^2*e*x+d^3)^m,x, algorithm="maxima")
```

```
[Out] integrate((e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3)^m*F^((b*x + a)*c), x)
```

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(e^3x^3 + 3de^2x^2 + 3d^2ex + d^3\right)^m F^{bcx+ac}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*(e^3*x^3+3*d*e^2*x^2+3*d^2*e*x+d^3)^m,x, algorithm="fricas")
```

```
[Out] integral((e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3)^m*F^(b*c*x + a*c), x)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(c*(b*x+a))*(e**3*x**3+3*d*e**2*x**2+3*d**2*e*x+d**3)**m,x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \left(e^3x^3 + 3de^2x^2 + 3d^2ex + d^3\right)^m F^{(bx+a)c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*(e^3*x^3+3*d*e^2*x^2+3*d^2*e*x+d^3)^m,x, algorithm="giac")
```

```
[Out] integrate((e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3)^m*F^((b*x + a)*c), x)
```

$$3.22 \quad \int F^{c(a+bx)} (d^2 + 2dex + e^2x^2)^m dx$$

**Optimal.** Leaf size=71

$$\frac{(d+ex)^2 F^{c\left(a-\frac{bd}{e}\right)} \left(-\frac{bc \log(F)(d+ex)}{e}\right)^{-2m} \Gamma\left(2m+1, -\frac{bc \log(F)(d+ex)}{e}\right)}{bc \log(F)}$$

[Out] (F^(c\*(a - (b\*d)/e))\*((d + e\*x)^2)^m\*Gamma[1 + 2\*m, -((b\*c\*(d + e\*x)\*Log[F])/e)]/(b\*c\*Log[F]\*(-(b\*c\*(d + e\*x)\*Log[F])/e)^(2\*m))

**Rubi [A]** time = 0.0569066, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 28,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$ , Rules used = {2188, 2181}

$$\frac{(d+ex)^2 F^{c\left(a-\frac{bd}{e}\right)} \left(-\frac{bc \log(F)(d+ex)}{e}\right)^{-2m} \Gamma\left(2m+1, -\frac{bc \log(F)(d+ex)}{e}\right)}{bc \log(F)}$$

Antiderivative was successfully verified.

[In] Int[F^(c\*(a + b\*x))\*(d^2 + 2\*d\*e\*x + e^2\*x^2)^m,x]

[Out] (F^(c\*(a - (b\*d)/e))\*((d + e\*x)^2)^m\*Gamma[1 + 2\*m, -((b\*c\*(d + e\*x)\*Log[F])/e)]/(b\*c\*Log[F]\*(-(b\*c\*(d + e\*x)\*Log[F])/e)^(2\*m))

### Rule 2188

```
Int[((a_.) + (b_.)*((F_)^((g_.)*(v_)))^(n_.))^(p_.)*(u_)^(m_.), x_Symbol] :
> Module[{uu = NormalizePowerOfLinear[u, x], z}, Simp[z = If[PowerQ[uu] &&
FreeQ[uu[[2]], x], uu[[1]]^(m*uu[[2]]), uu^m]; (uu^m*Int[z*(a + b*(F^(g*Exp
andToSum[v, x]))^n)^p, x])/z, x]] /; FreeQ[{F, a, b, g, m, n, p}, x] && Lin
earQ[v, x] && PowerOfLinearQ[u, x] && !(LinearMatchQ[v, x] && PowerOfLinea
rMatchQ[u, x]) && !IntegerQ[m]
```

### Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x])]/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F]
)*(c + d*x))/d)^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !I
```

ntegerQ [m]

### Rubi steps

$$\int F^{c(a+bx)} (d^2 + 2dex + e^2x^2)^m dx = (d + ex)^{-2m} ((d + ex)^2)^m \int F^{c(a+bx)} (d + ex)^{2m} dx$$

$$= \frac{F^{c\left(a-\frac{bd}{e}\right)} ((d + ex)^2)^m \Gamma\left(1 + 2m, -\frac{bc(d+ex)\log(F)}{e}\right) \left(-\frac{bc(d+ex)\log(F)}{e}\right)^{-2m}}{bc \log(F)}$$

**Mathematica [A]** time = 0.0140685, size = 71, normalized size = 1.

$$\frac{((d + ex)^2)^m F^{c\left(a-\frac{bd}{e}\right)} \left(-\frac{bc \log(F)(d+ex)}{e}\right)^{-2m} \text{Gamma}\left(2m + 1, -\frac{bc \log(F)(d+ex)}{e}\right)}{bc \log(F)}$$

Antiderivative was successfully verified.

[In] Integrate[F^(c\*(a + b\*x))\*(d^2 + 2\*d\*e\*x + e^2\*x^2)^m, x]

[Out] (F^(c\*(a - (b\*d)/e))\*((d + e\*x)^2)^m\*Gamma[1 + 2\*m, -((b\*c\*(d + e\*x)\*Log[F])/e)])/ (b\*c\*Log[F]\*(-(b\*c\*(d + e\*x)\*Log[F])/e))^(2\*m))

**Maple [F]** time = 0.103, size = 0, normalized size = 0.

$$\int F^{c(bx+a)} (e^2x^2 + 2dex + d^2)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(c\*(b\*x+a))\*(e^2\*x^2+2\*d\*e\*x+d^2)^m, x)

[Out] int(F^(c\*(b\*x+a))\*(e^2\*x^2+2\*d\*e\*x+d^2)^m, x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (e^2x^2 + 2dex + d^2)^m F^{(bx+a)c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(c*(b*x+a))*(e^2*x^2+2*d*e*x+d^2)^m,x, algorithm="maxima")`

[Out] `integrate((e^2*x^2 + 2*d*e*x + d^2)^m*F^((b*x + a)*c), x)`

**Fricas** [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(e^2x^2 + 2dex + d^2\right)^m F^{bcx+ac}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(c*(b*x+a))*(e^2*x^2+2*d*e*x+d^2)^m,x, algorithm="fricas")`

[Out] `integral((e^2*x^2 + 2*d*e*x + d^2)^m*F^(b*c*x + a*c), x)`

**Sympy** [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F**(c*(b*x+a))*(e**2*x**2+2*d*e*x+d**2)**m,x)`

[Out] Timed out

**Giac** [F] time = 0., size = 0, normalized size = 0.

$$\int \left(e^2x^2 + 2dex + d^2\right)^m F^{(bx+a)c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(c*(b*x+a))*(e^2*x^2+2*d*e*x+d^2)^m,x, algorithm="giac")`

[Out] `integrate((e^2*x^2 + 2*d*e*x + d^2)^m*F^((b*x + a)*c), x)`

### 3.23 $\int F^{c(a+bx)}(d+ex)^m dx$

**Optimal.** Leaf size=67

$$\frac{(d+ex)^m F^{c\left(a-\frac{bd}{e}\right)} \left(-\frac{bc \log(F)(d+ex)}{e}\right)^{-m} \text{Gamma}\left(m+1, -\frac{bc \log(F)(d+ex)}{e}\right)}{bc \log(F)}$$

[Out]  $(F^{(c*(a - (b*d)/e)})*(d + e*x)^m*\text{Gamma}[1 + m, -((b*c*(d + e*x)*\text{Log}[F])/e)]) / (b*c*\text{Log}[F]*(-((b*c*(d + e*x)*\text{Log}[F])/e))^m)$

**Rubi [A]** time = 0.0216785, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$ , Rules used = {2181}

$$\frac{(d+ex)^m F^{c\left(a-\frac{bd}{e}\right)} \left(-\frac{bc \log(F)(d+ex)}{e}\right)^{-m} \text{Gamma}\left(m+1, -\frac{bc \log(F)(d+ex)}{e}\right)}{bc \log(F)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[F^{(c*(a + b*x))}*(d + e*x)^m, x]$

[Out]  $(F^{(c*(a - (b*d)/e)})*(d + e*x)^m*\text{Gamma}[1 + m, -((b*c*(d + e*x)*\text{Log}[F])/e)]) / (b*c*\text{Log}[F]*(-((b*c*(d + e*x)*\text{Log}[F])/e))^m)$

#### Rule 2181

$\text{Int}[(F_)^{((g_.)*((e_.) + (f_.)*(x_)))}*((c_.) + (d_.)*(x_))^{(m_)}, x\_Symbol]$   
 $\text{:> -Simp}[(F^{(g*(e - (c*f)/d)})*(c + d*x)^{\text{FracPart}[m]}*\text{Gamma}[m + 1, (-((f*g*\text{Log}[F])/d))*(c + d*x])]/(d*(-((f*g*\text{Log}[F])/d))^{\text{IntPart}[m] + 1}*(-((f*g*\text{Log}[F])*(c + d*x)/d))^{\text{FracPart}[m]}), x] /; \text{FreeQ}\{F, c, d, e, f, g, m\}, x] \&\& \text{!IntegerQ}[m]$

#### Rubi steps

$$\int F^{c(a+bx)}(d+ex)^m dx = \frac{F^{c\left(a-\frac{bd}{e}\right)}(d+ex)^m \Gamma\left(1+m, -\frac{bc(d+ex)\log(F)}{e}\right) \left(-\frac{bc(d+ex)\log(F)}{e}\right)^{-m}}{bc \log(F)}$$



**Mathematica [A]** time = 0.0122916, size = 67, normalized size = 1.

$$\frac{(d + ex)^m F^{c\left(a - \frac{bd}{e}\right)} \left(-\frac{bc \log(F)(d+ex)}{e}\right)^{-m} \Gamma\left(m + 1, -\frac{bc \log(F)(d+ex)}{e}\right)}{bc \log(F)}$$

Antiderivative was successfully verified.

[In] Integrate[F^(c\*(a + b\*x))\*(d + e\*x)^m, x]

[Out] (F^(c\*(a - (b\*d)/e))\*(d + e\*x)^m\*Gamma[1 + m, -((b\*c\*(d + e\*x)\*Log[F])/e)]) / (b\*c\*Log[F]\*(-(b\*c\*(d + e\*x)\*Log[F])/e))^m

**Maple [F]** time = 0., size = 0, normalized size = 0.

$$\int F^{c(bx+a)} (ex + d)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(c\*(b\*x+a))\*(e\*x+d)^m, x)

[Out] int(F^(c\*(b\*x+a))\*(e\*x+d)^m, x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (ex + d)^m F^{(bx+a)c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(c\*(b\*x+a))\*(e\*x+d)^m, x, algorithm="maxima")

[Out] integrate((e\*x + d)^m \* F^((b\*x + a)\*c), x)

**Fricas [A]** time = 1.53953, size = 153, normalized size = 2.28

$$\frac{e^{\left(-\frac{em \log\left(-\frac{bc \log(F)}{e}\right) + (bcd - ace) \log(F)}{e}\right)} \Gamma\left(m + 1, -\frac{(bcex + bcd) \log(F)}{e}\right)}{bc \log(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*(e*x+d)^m,x, algorithm="fricas")
```

```
[Out] e^(-(e*m*log(-b*c*log(F)/e) + (b*c*d - a*c*e)*log(F))/e)*gamma(m + 1, -(b*c
*e*x + b*c*d)*log(F)/e)/(b*c*log(F))
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(c*(b*x+a))*(e*x+d)**m,x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (ex + d)^m F^{(bx+a)c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*(e*x+d)^m,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^m * F^((b*x + a)*c), x)
```

### 3.24 $\int F^{c(a+bx)}(d+ex)^{-m} dx$

**Optimal.** Leaf size=69

$$\frac{(d+ex)^{-m} F^{c\left(a-\frac{bd}{e}\right)} \left(-\frac{bc \log(F)(d+ex)}{e}\right)^m \Gamma\left(1-m, -\frac{bc \log(F)(d+ex)}{e}\right)}{bc \log(F)}$$

[Out]  $(F^{(c*(a - (b*d)/e)})*\Gamma[1 - m, -((b*c*(d + e*x)*\text{Log}[F])/e)]*(-((b*c*(d + e*x)*\text{Log}[F])/e))^m)/(b*c*(d + e*x)^m*\text{Log}[F])$

**Rubi [A]** time = 0.024389, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$ , Rules used = {2181}

$$\frac{(d+ex)^{-m} F^{c\left(a-\frac{bd}{e}\right)} \left(-\frac{bc \log(F)(d+ex)}{e}\right)^m \Gamma\left(1-m, -\frac{bc \log(F)(d+ex)}{e}\right)}{bc \log(F)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[F^{(c*(a + b*x))}/(d + e*x)^m, x]$

[Out]  $(F^{(c*(a - (b*d)/e)})*\Gamma[1 - m, -((b*c*(d + e*x)*\text{Log}[F])/e)]*(-((b*c*(d + e*x)*\text{Log}[F])/e))^m)/(b*c*(d + e*x)^m*\text{Log}[F])$

#### Rule 2181

```
Int[(F_)^((g_)*(e_) + (f_)*(x_))*((c_) + (d_)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Log[F])/d))*(c + d*x])]/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F])*(c + d*x))/d)^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !IntegerQ[m]
```

#### Rubi steps

$$\int F^{c(a+bx)}(d+ex)^{-m} dx = \frac{F^{c\left(a-\frac{bd}{e}\right)}(d+ex)^{-m} \Gamma\left(1-m, -\frac{bc(d+ex) \log(F)}{e}\right) \left(-\frac{bc(d+ex) \log(F)}{e}\right)^m}{bc \log(F)}$$

**Mathematica [A]** time = 0.0148788, size = 69, normalized size = 1.

$$\frac{(d + ex)^{-m} F^{c\left(a - \frac{bd}{e}\right)} \left(-\frac{bc \log(F)(d+ex)}{e}\right)^m \text{Gamma}\left(1 - m, -\frac{bc \log(F)(d+ex)}{e}\right)}{bc \log(F)}$$

Antiderivative was successfully verified.

[In] Integrate[F^(c\*(a + b\*x))/(d + e\*x)^m, x]

[Out] (F^(c\*(a - (b\*d)/e))\*Gamma[1 - m, -((b\*c\*(d + e\*x)\*Log[F])/e)]\*(-((b\*c\*(d + e\*x)\*Log[F])/e))^m)/(b\*c\*(d + e\*x)^m\*Log[F])

**Maple [F]** time = 0.045, size = 0, normalized size = 0.

$$\int \frac{F^{c(bx+a)}}{(ex + d)^m} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(c\*(b\*x+a))/((e\*x+d)^m), x)

[Out] int(F^(c\*(b\*x+a))/((e\*x+d)^m), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{(bx+a)c}}{(ex + d)^m} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(c\*(b\*x+a))/((e\*x+d)^m), x, algorithm="maxima")

[Out] integrate(F^((b\*x + a)\*c)/(e\*x + d)^m, x)

**Fricas [A]** time = 1.56557, size = 153, normalized size = 2.22

$$\frac{e^{\left(\frac{em \log\left(-\frac{bc \log(F)}{e}\right) - (bcd - ace) \log(F)}{e}\right)} \Gamma\left(-m + 1, -\frac{(bcex + bcd) \log(F)}{e}\right)}{bc \log(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(c\*(b\*x+a))/((e\*x+d)^m),x, algorithm="fricas")

[Out] e^((e\*m\*log(-b\*c\*log(F)/e) - (b\*c\*d - a\*c\*e)\*log(F))/e)\*gamma(-m + 1, -(b\*c\*e\*x + b\*c\*d)\*log(F)/e)/(b\*c\*log(F))

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(c\*(b\*x+a))/((e\*x+d)\*\*m),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{(bx+a)c}}{(ex+d)^m} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(c\*(b\*x+a))/((e\*x+d)^m),x, algorithm="giac")

[Out] integrate(F^((b\*x + a)\*c)/(e\*x + d)^m, x)

$$3.25 \quad \int F^{c(a+bx)} (d^2 + 2dex + e^2x^2)^{-m} dx$$

**Optimal.** Leaf size=73

$$\frac{(d+ex)^{-m} F^{c\left(a-\frac{bd}{e}\right)} \left(-\frac{bc \log(F)(d+ex)}{e}\right)^{2m} \text{Gamma}\left(1-2m, -\frac{bc \log(F)(d+ex)}{e}\right)}{bc \log(F)}$$

[Out] (F^(c\*(a - (b\*d)/e))\*Gamma[1 - 2\*m, -((b\*c\*(d + e\*x)\*Log[F])/e)]\*(-((b\*c\*(d + e\*x)\*Log[F])/e)^(2\*m)))/(b\*c\*((d + e\*x)^2)^m\*Log[F])

**Rubi [A]** time = 0.0566247, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 30,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$ , Rules used = {2188, 2181}

$$\frac{(d+ex)^{-m} F^{c\left(a-\frac{bd}{e}\right)} \left(-\frac{bc \log(F)(d+ex)}{e}\right)^{2m} \text{Gamma}\left(1-2m, -\frac{bc \log(F)(d+ex)}{e}\right)}{bc \log(F)}$$

Antiderivative was successfully verified.

[In] Int[F^(c\*(a + b\*x))/(d^2 + 2\*d\*e\*x + e^2\*x^2)^m, x]

[Out] (F^(c\*(a - (b\*d)/e))\*Gamma[1 - 2\*m, -((b\*c\*(d + e\*x)\*Log[F])/e)]\*(-((b\*c\*(d + e\*x)\*Log[F])/e)^(2\*m)))/(b\*c\*((d + e\*x)^2)^m\*Log[F])

### Rule 2188

```
Int[((a_.) + (b_.)*((F_)^((g_.)*(v_)))^(n_.))^(p_.)*(u_)^(m_.), x_Symbol] :
> Module[{uu = NormalizePowerOfLinear[u, x], z}, Simp[z = If[PowerQ[uu] &&
FreeQ[uu[[2]], x], uu[[1]]^(m*uu[[2]]), uu^m]; (uu^m*Int[z*(a + b*(F^(g*Exp
andToSum[v, x]))^n)^p, x])/z, x] /; FreeQ[{F, a, b, g, m, n, p}, x] && Lin
earQ[v, x] && PowerOfLinearQ[u, x] && !(LinearMatchQ[v, x] && PowerOfLinea
rMatchQ[u, x]) && !IntegerQ[m]
```

### Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d)*(c + d*x))]/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !I
```

ntegerQ [m]

### Rubi steps

$$\int F^{c(a+bx)} (d^2 + 2dex + e^2x^2)^{-m} dx = (d + ex)^{2m} ((d + ex)^2)^{-m} \int F^{c(a+bx)} (d + ex)^{-2m} dx$$

$$= \frac{F^{c\left(a-\frac{bd}{e}\right)} ((d + ex)^2)^{-m} \Gamma\left(1 - 2m, -\frac{bc(d+ex)\log(F)}{e}\right) \left(-\frac{bc(d+ex)\log(F)}{e}\right)^{2m}}{bc \log(F)}$$

**Mathematica [A]** time = 0.0107577, size = 73, normalized size = 1.

$$\frac{((d + ex)^2)^{-m} F^{c\left(a-\frac{bd}{e}\right)} \left(-\frac{bc \log(F)(d+ex)}{e}\right)^{2m} \text{Gamma}\left(1 - 2m, -\frac{bc \log(F)(d+ex)}{e}\right)}{bc \log(F)}$$

Antiderivative was successfully verified.

[In] Integrate[F^(c\*(a + b\*x))/(d^2 + 2\*d\*e\*x + e^2\*x^2)^m, x]

[Out] (F^(c\*(a - (b\*d)/e))\*Gamma[1 - 2\*m, -((b\*c\*(d + e\*x)\*Log[F])/e)]\*(-((b\*c\*(d + e\*x)\*Log[F])/e)^(2\*m)))/(b\*c\*((d + e\*x)^2)^m\*Log[F])

**Maple [F]** time = 0.099, size = 0, normalized size = 0.

$$\int \frac{F^{c(bx+a)}}{(e^2x^2 + 2dex + d^2)^m} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(c\*(b\*x+a))/((e^2\*x^2+2\*d\*e\*x+d^2)^m), x)

[Out] int(F^(c\*(b\*x+a))/((e^2\*x^2+2\*d\*e\*x+d^2)^m), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{(bx+a)c}}{(e^2x^2 + 2dex + d^2)^m} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(c\*(b\*x+a))/((e^2\*x^2+2\*d\*e\*x+d^2)^m),x, algorithm="maxima")

[Out] integrate(F^((b\*x + a)\*c)/(e^2\*x^2 + 2\*d\*e\*x + d^2)^m, x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{F^{bcx+ac}}{(e^2x^2 + 2dex + d^2)^m}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(c\*(b\*x+a))/((e^2\*x^2+2\*d\*e\*x+d^2)^m),x, algorithm="fricas")

[Out] integral(F^(b\*c\*x + a\*c)/(e^2\*x^2 + 2\*d\*e\*x + d^2)^m, x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(c\*(b\*x+a))/((e\*\*2\*x\*\*2+2\*d\*e\*x+d\*\*2)\*\*m),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{(bx+a)c}}{(e^2x^2 + 2dex + d^2)^m} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(c\*(b\*x+a))/((e^2\*x^2+2\*d\*e\*x+d^2)^m),x, algorithm="giac")

[Out] integrate(F^((b\*x + a)\*c)/(e^2\*x^2 + 2\*d\*e\*x + d^2)^m, x)



$$3.26 \quad \int F^{c(a+bx)} \left( d^3 + 3d^2ex + 3de^2x^2 + e^3x^3 \right)^{-m} dx$$

**Optimal.** Leaf size=73

$$\frac{\left( (d+ex)^3 \right)^{-m} F^{c\left(a-\frac{bd}{e}\right)} \left( -\frac{bc \log(F)(d+ex)}{e} \right)^{3m} \text{Gamma}\left(1-3m, -\frac{bc \log(F)(d+ex)}{e}\right)}{bc \log(F)}$$

[Out] (F^(c\*(a - (b\*d)/e))\*Gamma[1 - 3\*m, -((b\*c\*(d + e\*x)\*Log[F])/e)]\*(-((b\*c\*(d + e\*x)\*Log[F])/e))^(3\*m))/(b\*c\*((d + e\*x)^3)^m\*Log[F])

**Rubi [A]** time = 0.0597445, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 41,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.049$ , Rules used = {2188, 2181}

$$\frac{\left( (d+ex)^3 \right)^{-m} F^{c\left(a-\frac{bd}{e}\right)} \left( -\frac{bc \log(F)(d+ex)}{e} \right)^{3m} \text{Gamma}\left(1-3m, -\frac{bc \log(F)(d+ex)}{e}\right)}{bc \log(F)}$$

Antiderivative was successfully verified.

[In] Int[F^(c\*(a + b\*x))/(d^3 + 3\*d^2\*e\*x + 3\*d\*e^2\*x^2 + e^3\*x^3)^m, x]

[Out] (F^(c\*(a - (b\*d)/e))\*Gamma[1 - 3\*m, -((b\*c\*(d + e\*x)\*Log[F])/e)]\*(-((b\*c\*(d + e\*x)\*Log[F])/e))^(3\*m))/(b\*c\*((d + e\*x)^3)^m\*Log[F])

### Rule 2188

```
Int[((a_.) + (b_.)*((F_)^((g_.)*(v_)))^(n_.))^(p_.)*(u_)^(m_.), x_Symbol] :
> Module[{uu = NormalizePowerOfLinear[u, x], z}, Simp[z = If[PowerQ[uu] &&
FreeQ[uu[[2]], x], uu[[1]]^(m*uu[[2]]), uu^m]; (uu^m*Int[z*(a + b*(F^(g*Exp
andToSum[v, x]))^n)^p, x])/z, x]] /; FreeQ[{F, a, b, g, m, n, p}, x] && Lin
earQ[v, x] && PowerOfLinearQ[u, x] && !(LinearMatchQ[v, x] && PowerOfLinea
rMatchQ[u, x]) && !IntegerQ[m]
```

### Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x])]/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F]
)*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !I
```

ntegerQ [m]

### Rubi steps

$$\int F^{c(a+bx)} (d^3 + 3d^2ex + 3de^2x^2 + e^3x^3)^{-m} dx = (d + ex)^{3m} ((d + ex)^3)^{-m} \int F^{c(a+bx)} (d + ex)^{-3m} dx$$

$$= \frac{F^{c\left(a-\frac{bd}{e}\right)} ((d + ex)^3)^{-m} \Gamma\left(1 - 3m, -\frac{bc(d+ex)\log(F)}{e}\right) \left(-\frac{bc(d+ex)\log(F)}{e}\right)^{3m}}{bc \log(F)}$$

**Mathematica [A]** time = 0.0108804, size = 73, normalized size = 1.

$$\frac{((d + ex)^3)^{-m} F^{c\left(a-\frac{bd}{e}\right)} \left(-\frac{bc \log(F)(d+ex)}{e}\right)^{3m} \text{Gamma}\left(1 - 3m, -\frac{bc \log(F)(d+ex)}{e}\right)}{bc \log(F)}$$

Antiderivative was successfully verified.

[In] Integrate[F^(c\*(a + b\*x))/(d^3 + 3\*d^2\*e\*x + 3\*d\*e^2\*x^2 + e^3\*x^3)^m, x]

[Out] (F^(c\*(a - (b\*d)/e))\*Gamma[1 - 3\*m, -((b\*c\*(d + e\*x)\*Log[F])/e)]\*(-((b\*c\*(d + e\*x)\*Log[F])/e))^(3\*m))/(b\*c\*((d + e\*x)^3)^m\*Log[F])

**Maple [F]** time = 0.075, size = 0, normalized size = 0.

$$\int \frac{F^{c(bx+a)}}{(e^3x^3 + 3de^2x^2 + 3d^2ex + d^3)^m} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(c\*(b\*x+a))/((e^3\*x^3+3\*d\*e^2\*x^2+3\*d^2\*e\*x+d^3)^m), x)

[Out] int(F^(c\*(b\*x+a))/((e^3\*x^3+3\*d\*e^2\*x^2+3\*d^2\*e\*x+d^3)^m), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{(bx+a)c}}{(e^3x^3 + 3de^2x^2 + 3d^2ex + d^3)^m} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(c\*(b\*x+a))/((e^3\*x^3+3\*d\*e^2\*x^2+3\*d^2\*e\*x+d^3)^m),x, algorithm="maxima")

[Out] integrate(F^((b\*x + a)\*c)/(e^3\*x^3 + 3\*d\*e^2\*x^2 + 3\*d^2\*e\*x + d^3)^m, x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{F^{bcx+ac}}{(e^3x^3 + 3de^2x^2 + 3d^2ex + d^3)^m}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(c\*(b\*x+a))/((e^3\*x^3+3\*d\*e^2\*x^2+3\*d^2\*e\*x+d^3)^m),x, algorithm="fricas")

[Out] integral(F^(b\*c\*x + a\*c)/(e^3\*x^3 + 3\*d\*e^2\*x^2 + 3\*d^2\*e\*x + d^3)^m, x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(c\*(b\*x+a))/((e\*\*3\*x\*\*3+3\*d\*e\*\*2\*x\*\*2+3\*d\*\*2\*e\*x+d\*\*3)\*\*m),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{(bx+a)c}}{(e^3x^3 + 3de^2x^2 + 3d^2ex + d^3)^m} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))/((e^3*x^3+3*d*e^2*x^2+3*d^2*e*x+d^3)^m),x, algorithm  
m="giac")
```

```
[Out] integrate(F^((b*x + a)*c)/(e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3)^m, x)
```

$$3.27 \quad \int F^{2+5x} dx$$

**Optimal.** Leaf size=15

$$\frac{F^{5x+2}}{5 \log(F)}$$

[Out]  $F^{(2 + 5*x)/(5*\text{Log}[F])}$

**Rubi [A]** time = 0.002795, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 7,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {2194}

$$\frac{F^{5x+2}}{5 \log(F)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[F^{(2 + 5*x)}, x]$

[Out]  $F^{(2 + 5*x)/(5*\text{Log}[F])}$

**Rule 2194**

$\text{Int}[(F_)^{((c_.)*((a_.) + (b_.)*(x_)))^{(n_.)}, x\_Symbol] :> \text{Simp}[(F^{(c*(a + b*x)))^n/(b*c*n*\text{Log}[F]), x] /; \text{FreeQ}\{F, a, b, c, n\}, x]$

**Rubi steps**

$$\int F^{2+5x} dx = \frac{F^{2+5x}}{5 \log(F)}$$

**Mathematica [A]** time = 0.0036337, size = 15, normalized size = 1.

$$\frac{F^{5x+2}}{5 \log(F)}$$

Antiderivative was successfully verified.

[In] Integrate[F^(2 + 5\*x),x]

[Out] F^(2 + 5\*x)/(5\*Log[F])

**Maple [A]** time = 0.004, size = 14, normalized size = 0.9

$$\frac{F^{2+5x}}{5 \ln(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(2+5\*x),x)

[Out] 1/5\*F^(2+5\*x)/ln(F)

**Maxima [A]** time = 1.12231, size = 18, normalized size = 1.2

$$\frac{F^{5x+2}}{5 \log(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(2+5\*x),x, algorithm="maxima")

[Out] 1/5\*F^(5\*x + 2)/log(F)

**Fricas [A]** time = 1.50043, size = 32, normalized size = 2.13

$$\frac{F^{5x+2}}{5 \log(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(2+5\*x),x, algorithm="fricas")

[Out] 1/5\*F^(5\*x + 2)/log(F)

---

**Sympy [A]** time = 0.094902, size = 15, normalized size = 1.

$$\begin{cases} \frac{F^{5x+2}}{5 \log(F)} & \text{for } 5 \log(F) \neq 0 \\ x & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(2+5\*x), x)

[Out] Piecewise((F\*\*(5\*x + 2)/(5\*log(F)), Ne(5\*log(F), 0)), (x, True))

---

**Giac [A]** time = 1.21063, size = 18, normalized size = 1.2

$$\frac{F^{5x+2}}{5 \log(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(2+5\*x), x, algorithm="giac")

[Out] 1/5\*F^(5\*x + 2)/log(F)

### 3.28 $\int F^{a+bx} dx$

**Optimal.** Leaf size=15

$$\frac{F^{a+bx}}{b \log(F)}$$

[Out]  $F^{(a + b*x)/(b*\text{Log}[F])}$

**Rubi [A]** time = 0.0028422, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 7,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {2194}

$$\frac{F^{a+bx}}{b \log(F)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[F^{(a + b*x)}, x]$

[Out]  $F^{(a + b*x)/(b*\text{Log}[F])}$

**Rule 2194**

$\text{Int}[(F_)^{((c_.)*((a_.) + (b_.)*(x_)))^{(n_.)}, x\_Symbol] \rightarrow \text{Simp}[(F^{(c*(a + b*x)))^n/(b*c*n*\text{Log}[F]), x] /; \text{FreeQ}\{F, a, b, c, n\}, x]$

**Rubi steps**

$$\int F^{a+bx} dx = \frac{F^{a+bx}}{b \log(F)}$$

**Mathematica [A]** time = 0.0035217, size = 15, normalized size = 1.

$$\frac{F^{a+bx}}{b \log(F)}$$

Antiderivative was successfully verified.



[In] Integrate[F^(a + b\*x),x]

[Out] F^(a + b\*x)/(b\*Log[F])

**Maple [A]** time = 0.006, size = 16, normalized size = 1.1

$$\frac{F^{bx+a}}{b \ln(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(b\*x+a),x)

[Out] F^(b\*x+a)/b/ln(F)

**Maxima [A]** time = 1.05093, size = 20, normalized size = 1.33

$$\frac{F^{bx+a}}{b \log(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(b\*x+a),x, algorithm="maxima")

[Out] F^(b\*x + a)/(b\*log(F))

**Fricas [A]** time = 1.51011, size = 32, normalized size = 2.13

$$\frac{F^{bx+a}}{b \log(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(b\*x+a),x, algorithm="fricas")

[Out] F^(b\*x + a)/(b\*log(F))

---

**Sympy [A]** time = 0.092784, size = 15, normalized size = 1.

$$\begin{cases} \frac{F^{a+bx}}{b \log(F)} & \text{for } b \log(F) \neq 0 \\ x & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(b\*x+a),x)

[Out] Piecewise((F\*\*(a + b\*x)/(b\*log(F)), Ne(b\*log(F), 0)), (x, True))

---

**Giac [A]** time = 1.19125, size = 20, normalized size = 1.33

$$\frac{F^{bx+a}}{b \log(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(b\*x+a),x, algorithm="giac")

[Out] F^(b\*x + a)/(b\*log(F))

$$3.29 \quad \int 10^{2+5x} dx$$

**Optimal.** Leaf size=19

$$\frac{2^{5x+2}5^{5x+1}}{\log(10)}$$

[Out] (2^(2 + 5\*x)\*5^(1 + 5\*x))/Log[10]

**Rubi [A]** time = 0.0054357, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 7,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {2194}

$$\frac{2^{5x+2}5^{5x+1}}{\log(10)}$$

Antiderivative was successfully verified.

[In] Int[10^(2 + 5\*x), x]

[Out] (2^(2 + 5\*x)\*5^(1 + 5\*x))/Log[10]

**Rule 2194**

Int[((F\_)^((c\_.)\*((a\_.) + (b\_.)\*(x\_))))^(n\_.), x\_Symbol] :> Simp[(F^(c\*(a + b\*x)))^n/(b\*c\*n\*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]

**Rubi steps**

$$\int 10^{2+5x} dx = \frac{2^{2+5x}5^{1+5x}}{\log(10)}$$

**Mathematica [A]** time = 0.0049009, size = 19, normalized size = 1.

$$\frac{2^{5x+2}5^{5x+1}}{\log(10)}$$

Antiderivative was successfully verified.

[In] Integrate[10^(2 + 5\*x),x]

[Out] (2^(2 + 5\*x)\*5^(1 + 5\*x))/Log[10]

**Maple [A]** time = 0.013, size = 14, normalized size = 0.7

$$\frac{10^{2+5x}}{5 \ln(10)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(10^(2+5\*x),x)

[Out] 1/5/ln(10)\*10^(2+5\*x)

**Maxima [A]** time = 1.13901, size = 18, normalized size = 0.95

$$\frac{10^{5x+2}}{5 \log(10)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(10^(2+5\*x),x, algorithm="maxima")

[Out] 1/5\*10^(5\*x + 2)/log(10)

**Fricas [A]** time = 1.49225, size = 35, normalized size = 1.84

$$\frac{10^{5x+2}}{5 \log(10)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(10^(2+5\*x),x, algorithm="fricas")

[Out] 1/5\*10^(5\*x + 2)/log(10)

---

**Sympy [A]** time = 0.089954, size = 10, normalized size = 0.53

$$\frac{10^{5x+2}}{5 \log(10)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(10\*\*(2+5\*x),x)

[Out] 10\*\*(5\*x + 2)/(5\*log(10))

---

**Giac [A]** time = 1.23942, size = 18, normalized size = 0.95

$$\frac{10^{5x+2}}{5 \log(10)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(10^(2+5\*x),x, algorithm="giac")

[Out] 1/5\*10^(5\*x + 2)/log(10)

### 3.30 $\int F^{a+bx} x^{7/2} dx$

**Optimal.** Leaf size=131

$$\frac{105\sqrt{\pi}F^a\operatorname{Erfi}\left(\sqrt{b}\sqrt{x}\sqrt{\log(F)}\right)}{16b^{9/2}\log^2(F)} - \frac{7x^{5/2}F^{a+bx}}{2b^2\log^2(F)} + \frac{35x^{3/2}F^{a+bx}}{4b^3\log^3(F)} - \frac{105\sqrt{x}F^{a+bx}}{8b^4\log^4(F)} + \frac{x^{7/2}F^{a+bx}}{b\log(F)}$$

[Out] (105\*F^a\*Sqrt[Pi]\*Erfi[Sqrt[b]\*Sqrt[x]\*Sqrt[Log[F]]])/(16\*b^(9/2)\*Log[F]^(9/2)) - (105\*F^(a + b\*x)\*Sqrt[x])/(8\*b^4\*Log[F]^4) + (35\*F^(a + b\*x)\*x^(3/2))/(4\*b^3\*Log[F]^3) - (7\*F^(a + b\*x)\*x^(5/2))/(2\*b^2\*Log[F]^2) + (F^(a + b\*x)\*x^(7/2))/(b\*Log[F])

**Rubi [A]** time = 0.147164, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$ , Rules used = {2176, 2180, 2204}

$$\frac{105\sqrt{\pi}F^a\operatorname{Erfi}\left(\sqrt{b}\sqrt{x}\sqrt{\log(F)}\right)}{16b^{9/2}\log^2(F)} - \frac{7x^{5/2}F^{a+bx}}{2b^2\log^2(F)} + \frac{35x^{3/2}F^{a+bx}}{4b^3\log^3(F)} - \frac{105\sqrt{x}F^{a+bx}}{8b^4\log^4(F)} + \frac{x^{7/2}F^{a+bx}}{b\log(F)}$$

Antiderivative was successfully verified.

[In] Int[F^(a + b\*x)\*x^(7/2), x]

[Out] (105\*F^a\*Sqrt[Pi]\*Erfi[Sqrt[b]\*Sqrt[x]\*Sqrt[Log[F]]])/(16\*b^(9/2)\*Log[F]^(9/2)) - (105\*F^(a + b\*x)\*Sqrt[x])/(8\*b^4\*Log[F]^4) + (35\*F^(a + b\*x)\*x^(3/2))/(4\*b^3\*Log[F]^3) - (7\*F^(a + b\*x)\*x^(5/2))/(2\*b^2\*Log[F]^2) + (F^(a + b\*x)\*x^(7/2))/(b\*Log[F])

#### Rule 2176

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[((c + d*x)^m*(b*F^(g*(e + f*x)))^n)/(f*g*n*Log[F]), x] - Dist[(d*m)/(f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m] && !$UseGamma === True
```

#### Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
```

x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !\$UseGamma === True

### Rule 2204

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))<sup>2</sup>), x\_Symbol] := Simp[(F^a\*Sqrt[Pi]\*Erfi[(c + d\*x)\*Rt[b\*Log[F], 2]])/(2\*d\*Rt[b\*Log[F], 2]), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]

### Rubi steps

$$\begin{aligned}
 \int F^{a+bx} x^{7/2} dx &= \frac{F^{a+bx} x^{7/2}}{b \log(F)} - \frac{7 \int F^{a+bx} x^{5/2} dx}{2b \log(F)} \\
 &= -\frac{7F^{a+bx} x^{5/2}}{2b^2 \log^2(F)} + \frac{F^{a+bx} x^{7/2}}{b \log(F)} + \frac{35 \int F^{a+bx} x^{3/2} dx}{4b^2 \log^2(F)} \\
 &= \frac{35F^{a+bx} x^{3/2}}{4b^3 \log^3(F)} - \frac{7F^{a+bx} x^{5/2}}{2b^2 \log^2(F)} + \frac{F^{a+bx} x^{7/2}}{b \log(F)} - \frac{105 \int F^{a+bx} \sqrt{x} dx}{8b^3 \log^3(F)} \\
 &= -\frac{105F^{a+bx} \sqrt{x}}{8b^4 \log^4(F)} + \frac{35F^{a+bx} x^{3/2}}{4b^3 \log^3(F)} - \frac{7F^{a+bx} x^{5/2}}{2b^2 \log^2(F)} + \frac{F^{a+bx} x^{7/2}}{b \log(F)} + \frac{105 \int \frac{F^{a+bx}}{\sqrt{x}} dx}{16b^4 \log^4(F)} \\
 &= -\frac{105F^{a+bx} \sqrt{x}}{8b^4 \log^4(F)} + \frac{35F^{a+bx} x^{3/2}}{4b^3 \log^3(F)} - \frac{7F^{a+bx} x^{5/2}}{2b^2 \log^2(F)} + \frac{F^{a+bx} x^{7/2}}{b \log(F)} + \frac{105 \text{Subst}\left(\int F^{a+bx^2} dx, x, \sqrt{x}\right)}{8b^4 \log^4(F)} \\
 &= \frac{105F^a \sqrt{\pi} \operatorname{erfi}\left(\sqrt{b} \sqrt{x} \sqrt{\log(F)}\right)}{16b^{9/2} \log^2(F)} - \frac{105F^{a+bx} \sqrt{x}}{8b^4 \log^4(F)} + \frac{35F^{a+bx} x^{3/2}}{4b^3 \log^3(F)} - \frac{7F^{a+bx} x^{5/2}}{2b^2 \log^2(F)} + \frac{F^{a+bx} x^{7/2}}{b \log(F)}
 \end{aligned}$$

**Mathematica [A]** time = 0.0062582, size = 36, normalized size = 0.27

$$\frac{F^a \sqrt{-bx \log(F)} \operatorname{Gamma}\left(\frac{9}{2}, -bx \log(F)\right)}{b^5 \sqrt{x} \log^5(F)}$$

Antiderivative was successfully verified.

[In] Integrate[F^(a + b\*x)\*x^(7/2), x]

[Out] (F^a\*Gamma[9/2, -(b\*x\*Log[F])]\*Sqrt[-(b\*x\*Log[F])])/(b^5\*Sqrt[x]\*Log[F]^5)

**Maple [A]** time = 0.022, size = 99, normalized size = 0.8

$$-\frac{F^a}{b} \left( -\frac{(-72 b^3 x^3 (\ln(F))^3 + 252 b^2 x^2 (\ln(F))^2 - 630 b \ln(F) x + 945) e^{b \ln(F) x}}{72 b^4} \sqrt{x} (-b)^{\frac{9}{2}} \sqrt{\ln(F)} + \frac{105 \sqrt{\pi}}{16} (-b)^{\frac{9}{2}} \operatorname{erfi}(\sqrt{b} \ln(F)) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(F^(b*x+a)*x^(7/2),x)`

[Out]  $-F^a/(-b)^{(7/2)}/\ln(F)^{(9/2)}/b*(-1/72*x^{(1/2)}*(-b)^{(9/2)}*\ln(F)^{(1/2)}*(-72*b^3*x^3*\ln(F)^3+252*b^2*x^2*\ln(F)^2-630*b*\ln(F)*x+945)/b^4*\exp(b*\ln(F)*x)+105/16*(-b)^{(9/2)}/b^{(9/2)}*\Pi^{(1/2)}*\operatorname{erfi}(b^{(1/2)}*x^{(1/2)}*\ln(F)^{(1/2)})$

**Maxima [A]** time = 1.22927, size = 32, normalized size = 0.24

$$-\frac{F^a x^{\frac{9}{2}} \Gamma\left(\frac{9}{2}, -bx \log(F)\right)}{(-bx \log(F))^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(b*x+a)*x^(7/2),x, algorithm="maxima")`

[Out]  $-F^a*x^{(9/2)}*\gamma(9/2, -b*x*\log(F))/(-b*x*\log(F))^{(9/2)}$

**Fricas [A]** time = 1.53279, size = 254, normalized size = 1.94

$$\frac{105 \sqrt{\pi} \sqrt{-b \log(F)} F^a \operatorname{erf}(\sqrt{-b \log(F)} \sqrt{x}) - 2(8 b^4 x^3 \log(F)^4 - 28 b^3 x^2 \log(F)^3 + 70 b^2 x \log(F)^2 - 105 b \log(F)) F^{bx+a}}{16 b^5 \log(F)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(b*x+a)*x^(7/2),x, algorithm="fricas")`

[Out]  $-1/16*(105*\sqrt{\pi}*\sqrt{-b*\log(F)}*F^a*\operatorname{erf}(\sqrt{-b*\log(F)}*\sqrt{x}) - 2*(8*b^4*x^3*\log(F)^4 - 28*b^3*x^2*\log(F)^3 + 70*b^2*x*\log(F)^2 - 105*b*\log(F))*F^{(b*x + a)}*\sqrt{x})/(b^5*\log(F)^5)$



---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(b\*x+a)\*x\*\*(7/2),x)

[Out] Timed out

---

**Giac [A]** time = 1.2203, size = 127, normalized size = 0.97

$$-\frac{105\sqrt{\pi}F^a\operatorname{erf}\left(-\sqrt{-b\log(F)}\sqrt{x}\right)}{16\sqrt{-b\log(F)}b^4\log(F)^4} + \frac{\left(8b^3x^{\frac{7}{2}}\log(F)^3 - 28b^2x^{\frac{5}{2}}\log(F)^2 + 70bx^{\frac{3}{2}}\log(F) - 105\sqrt{x}\right)e^{(bx\log(F)+a\log(F))}}{8b^4\log(F)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(b\*x+a)\*x^(7/2),x, algorithm="giac")

[Out]  $-105/16*\sqrt{\pi}*F^a*\operatorname{erf}(-\sqrt{-b*\log(F)}*\sqrt{x})/(\sqrt{-b*\log(F)}*b^4*\log(F)^4) + 1/8*(8*b^3*x^{(7/2)}*\log(F)^3 - 28*b^2*x^{(5/2)}*\log(F)^2 + 70*b*x^{(3/2)}*\log(F) - 105*\sqrt{x})*e^{(b*x*\log(F) + a*\log(F))}/(b^4*\log(F)^4)$

### 3.31 $\int F^{a+bx} x^{5/2} dx$

**Optimal.** Leaf size=108

$$-\frac{15\sqrt{\pi}F^a \operatorname{Erfi}\left(\sqrt{b}\sqrt{x}\sqrt{\log(F)}\right)}{8b^{7/2}\log^{\frac{7}{2}}(F)} - \frac{5x^{3/2}F^{a+bx}}{2b^2\log^2(F)} + \frac{15\sqrt{x}F^{a+bx}}{4b^3\log^3(F)} + \frac{x^{5/2}F^{a+bx}}{b\log(F)}$$

[Out]  $(-15F^a \sqrt{\pi} \operatorname{Erfi}[\sqrt{b}\sqrt{x}\sqrt{\log[F]}]) / (8b^{7/2} \log[F]^{7/2}) + (15F^{a+bx} \sqrt{x}) / (4b^3 \log[F]^3) - (5F^{a+bx} x^{3/2}) / (2b^2 \log[F]^2) + (F^{a+bx} x^{5/2}) / (b \log[F])$

**Rubi [A]** time = 0.0938183, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$ , Rules used = {2176, 2180, 2204}

$$-\frac{15\sqrt{\pi}F^a \operatorname{Erfi}\left(\sqrt{b}\sqrt{x}\sqrt{\log(F)}\right)}{8b^{7/2}\log^{\frac{7}{2}}(F)} - \frac{5x^{3/2}F^{a+bx}}{2b^2\log^2(F)} + \frac{15\sqrt{x}F^{a+bx}}{4b^3\log^3(F)} + \frac{x^{5/2}F^{a+bx}}{b\log(F)}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[F^{a+bx} x^{5/2}, x]$

[Out]  $(-15F^a \sqrt{\pi} \operatorname{Erfi}[\sqrt{b}\sqrt{x}\sqrt{\log[F]}]) / (8b^{7/2} \log[F]^{7/2}) + (15F^{a+bx} \sqrt{x}) / (4b^3 \log[F]^3) - (5F^{a+bx} x^{3/2}) / (2b^2 \log[F]^2) + (F^{a+bx} x^{5/2}) / (b \log[F])$

#### Rule 2176

$\operatorname{Int}[(b \cdot x + c) \cdot (d \cdot x + e)^m \cdot (f \cdot x + g)^n, x] \rightarrow \operatorname{Simp}[(c + d \cdot x)^m \cdot (b \cdot f \cdot (d \cdot x + e)^n) / (f \cdot g \cdot n \cdot \log[F]), x] - \operatorname{Dist}[(d \cdot m) / (f \cdot g \cdot n \cdot \log[F]), \operatorname{Int}[(c + d \cdot x)^{m-1} \cdot (b \cdot f \cdot (d \cdot x + e)^n), x], x] /;$  FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2\*m] && !UseGamma == True

#### Rule 2180

$\operatorname{Int}[F^{a+bx} / \sqrt{c + d \cdot x}, x] \rightarrow \operatorname{Dist}[2/d, \operatorname{Subst}[\operatorname{Int}[F^{a+bx} (g \cdot (e - (c \cdot f)/d) + (f \cdot g \cdot x^2)/d), x], x, \sqrt{c + d \cdot x}], x] /;$  FreeQ[{F, c, d, e, f, g}, x] && !UseGamma == True

Rule 2204

`Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))2), x_Symbol] := Simp[(F^a*Sqrt[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]`

Rubi steps

$$\begin{aligned}
 \int F^{a+bx} x^{5/2} dx &= \frac{F^{a+bx} x^{5/2}}{b \log(F)} - \frac{5 \int F^{a+bx} x^{3/2} dx}{2b \log(F)} \\
 &= -\frac{5F^{a+bx} x^{3/2}}{2b^2 \log^2(F)} + \frac{F^{a+bx} x^{5/2}}{b \log(F)} + \frac{15 \int F^{a+bx} \sqrt{x} dx}{4b^2 \log^2(F)} \\
 &= \frac{15F^{a+bx} \sqrt{x}}{4b^3 \log^3(F)} - \frac{5F^{a+bx} x^{3/2}}{2b^2 \log^2(F)} + \frac{F^{a+bx} x^{5/2}}{b \log(F)} - \frac{15 \int \frac{F^{a+bx}}{\sqrt{x}} dx}{8b^3 \log^3(F)} \\
 &= \frac{15F^{a+bx} \sqrt{x}}{4b^3 \log^3(F)} - \frac{5F^{a+bx} x^{3/2}}{2b^2 \log^2(F)} + \frac{F^{a+bx} x^{5/2}}{b \log(F)} - \frac{15 \text{Subst}\left(\int F^{a+bx^2} dx, x, \sqrt{x}\right)}{4b^3 \log^3(F)} \\
 &= -\frac{15F^a \sqrt{\pi} \operatorname{erfi}\left(\sqrt{b} \sqrt{x} \sqrt{\log(F)}\right)}{8b^{7/2} \log^2(F)} + \frac{15F^{a+bx} \sqrt{x}}{4b^3 \log^3(F)} - \frac{5F^{a+bx} x^{3/2}}{2b^2 \log^2(F)} + \frac{F^{a+bx} x^{5/2}}{b \log(F)}
 \end{aligned}$$

**Mathematica [A]** time = 0.0064971, size = 36, normalized size = 0.33

$$\frac{\sqrt{x} F^a \operatorname{Gamma}\left(\frac{7}{2}, -bx \log(F)\right)}{b^3 \log^3(F) \sqrt{-bx \log(F)}}$$

Antiderivative was successfully verified.

[In] Integrate[F^(a + b\*x)\*x^(5/2), x]

[Out] (F^a\*Sqrt[x]\*Gamma[7/2, -(b\*x\*Log[F])])/(b^3\*Log[F]^3\*Sqrt[-(b\*x\*Log[F])])

**Maple [A]** time = 0.01, size = 87, normalized size = 0.8

$$-\frac{F^a}{b} \left( \frac{(28b^2x^2(\ln(F))^2 - 70b\ln(F)x + 105)e^{b\ln(F)x}}{28b^3} \sqrt{x} (-b)^{\frac{7}{2}} \sqrt{\ln(F)} - \frac{15\sqrt{\pi}}{8} (-b)^{\frac{7}{2}} \operatorname{erfi}\left(\sqrt{b} \sqrt{x} \sqrt{\ln(F)}\right) b^{-\frac{7}{2}} \right) (-b)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(F^(b*x+a)*x^(5/2),x)`

[Out]  $-F^a/(-b)^{(5/2)}/\ln(F)^{(7/2)}/b*(1/28*x^{(1/2)}*(-b)^{(7/2)}*\ln(F)^{(1/2)}*(28*b^2*x^2*\ln(F)^2-70*b*\ln(F)*x+105)/b^3*\exp(b*\ln(F)*x)-15/8*(-b)^{(7/2)}/b^{(7/2)}*\Pi^{(1/2)}*\operatorname{erfi}(b^{(1/2)}*x^{(1/2)}*\ln(F)^{(1/2)})$

**Maxima [A]** time = 1.22414, size = 32, normalized size = 0.3

$$\frac{F^a x^2 \Gamma\left(\frac{7}{2}, -bx \log(F)\right)}{(-bx \log(F))^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(b*x+a)*x^(5/2),x, algorithm="maxima")`

[Out]  $-F^a*x^{(7/2)}*\gamma(7/2, -b*x*\log(F))/(-b*x*\log(F))^{(7/2)}$

**Fricas [A]** time = 1.51819, size = 219, normalized size = 2.03

$$\frac{15 \sqrt{\pi} \sqrt{-b \log(F)} F^a \operatorname{erf}\left(\sqrt{-b \log(F)} \sqrt{x}\right) + 2\left(4 b^3 x^2 \log(F)^3 - 10 b^2 x \log(F)^2 + 15 b \log(F)\right) F^{bx+a} \sqrt{x}}{8 b^4 \log(F)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(b*x+a)*x^(5/2),x, algorithm="fricas")`

[Out]  $1/8*(15*\sqrt{\pi}*\sqrt{-b*\log(F)}*F^a*\operatorname{erf}(\sqrt{-b*\log(F)}*\sqrt{x}) + 2*(4*b^3*x^2*\log(F)^3 - 10*b^2*x*\log(F)^2 + 15*b*\log(F))*F^{(b*x + a)}*\sqrt{x})/(b^4*\log(F)^4)$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(b\*x+a)\*x\*\*(5/2),x)

[Out] Timed out

**Giac [A]** time = 1.25743, size = 111, normalized size = 1.03

$$\frac{15\sqrt{\pi}F^a \operatorname{erf}\left(-\sqrt{-b\log(F)}\sqrt{x}\right)}{8\sqrt{-b\log(F)}b^3\log(F)^3} + \frac{\left(4b^2x^{\frac{5}{2}}\log(F)^2 - 10bx^{\frac{3}{2}}\log(F) + 15\sqrt{x}\right)e^{(bx\log(F)+a\log(F))}}{4b^3\log(F)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(b\*x+a)\*x^(5/2),x, algorithm="giac")

[Out]  $\frac{15}{8}\sqrt{\pi}F^a\operatorname{erf}\left(-\sqrt{-b\log(F)}\sqrt{x}\right)/\left(\sqrt{-b\log(F)}b^3\log(F)^3\right) + \frac{1}{4}\left(4b^2x^{\frac{5}{2}}\log(F)^2 - 10b*x^{\frac{3}{2}}\log(F) + 15\sqrt{x}\right)*e^{(b*x\log(F) + a\log(F))}/(b^3\log(F)^3)$

### 3.32 $\int F^{a+bx} x^{3/2} dx$

**Optimal.** Leaf size=85

$$\frac{3\sqrt{\pi}F^a \operatorname{Erfi}\left(\sqrt{b}\sqrt{x}\sqrt{\log(F)}\right)}{4b^{5/2}\log^2(F)} - \frac{3\sqrt{x}F^{a+bx}}{2b^2\log^2(F)} + \frac{x^{3/2}F^{a+bx}}{b\log(F)}$$

[Out] (3\*F^a\*Sqrt[Pi]\*Erfi[Sqrt[b]\*Sqrt[x]\*Sqrt[Log[F]]])/(4\*b^(5/2)\*Log[F]^(5/2)) - (3\*F^(a + b\*x)\*Sqrt[x])/(2\*b^2\*Log[F]^2) + (F^(a + b\*x)\*x^(3/2))/(b\*Log[F])

**Rubi [A]** time = 0.06581, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$ , Rules used = {2176, 2180, 2204}

$$\frac{3\sqrt{\pi}F^a \operatorname{Erfi}\left(\sqrt{b}\sqrt{x}\sqrt{\log(F)}\right)}{4b^{5/2}\log^2(F)} - \frac{3\sqrt{x}F^{a+bx}}{2b^2\log^2(F)} + \frac{x^{3/2}F^{a+bx}}{b\log(F)}$$

Antiderivative was successfully verified.

[In] Int[F^(a + b\*x)\*x^(3/2), x]

[Out] (3\*F^a\*Sqrt[Pi]\*Erfi[Sqrt[b]\*Sqrt[x]\*Sqrt[Log[F]]])/(4\*b^(5/2)\*Log[F]^(5/2)) - (3\*F^(a + b\*x)\*Sqrt[x])/(2\*b^2\*Log[F]^2) + (F^(a + b\*x)\*x^(3/2))/(b\*Log[F])

#### Rule 2176

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[((c + d*x)^m*(b*F^(g*(e + f*x)))^n)/(f*g*n*Log[F]), x] - Dist[(d*m)/(f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m] && !$UseGamma === True
```

#### Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !$UseGamma === True
```

Rule 2204

$\text{Int}[(F\_)^{\{(a\_.) + (b\_.) * ((c\_.) + (d\_.) * (x\_))^{2}\}}, x\_Symbol] := \text{Simp}[(F^a * \text{Sqrt}[\text{Pi}] * \text{Erfi}[(c + d * x) * \text{Rt}[b * \text{Log}[F], 2]]) / (2 * d * \text{Rt}[b * \text{Log}[F], 2]), x] /;$   $\text{FreeQ}\{F, a, b, c, d, x\} \ \&\& \ \text{PosQ}[b]$

Rubi steps

$$\begin{aligned} \int F^{a+bx} x^{3/2} dx &= \frac{F^{a+bx} x^{3/2}}{b \log(F)} - \frac{3 \int F^{a+bx} \sqrt{x} dx}{2b \log(F)} \\ &= -\frac{3F^{a+bx} \sqrt{x}}{2b^2 \log^2(F)} + \frac{F^{a+bx} x^{3/2}}{b \log(F)} + \frac{3 \int \frac{F^{a+bx}}{\sqrt{x}} dx}{4b^2 \log^2(F)} \\ &= -\frac{3F^{a+bx} \sqrt{x}}{2b^2 \log^2(F)} + \frac{F^{a+bx} x^{3/2}}{b \log(F)} + \frac{3 \text{Subst}\left(\int F^{a+bx^2} dx, x, \sqrt{x}\right)}{2b^2 \log^2(F)} \\ &= \frac{3F^a \sqrt{\pi} \text{erfi}\left(\sqrt{b} \sqrt{x} \sqrt{\log(F)}\right)}{4b^{5/2} \log^2(F)} - \frac{3F^{a+bx} \sqrt{x}}{2b^2 \log^2(F)} + \frac{F^{a+bx} x^{3/2}}{b \log(F)} \end{aligned}$$

**Mathematica [A]** time = 0.0059449, size = 36, normalized size = 0.42

$$\frac{F^a \sqrt{-bx \log(F)} \text{Gamma}\left(\frac{5}{2}, -bx \log(F)\right)}{b^3 \sqrt{x} \log^3(F)}$$

Antiderivative was successfully verified.

[In] Integrate[F^(a + b\*x)\*x^(3/2), x]

[Out] (F^a\*Gamma[5/2, -(b\*x\*Log[F])]\*Sqrt[-(b\*x\*Log[F])])/(b^3\*Sqrt[x]\*Log[F]^3)

**Maple [A]** time = 0.012, size = 75, normalized size = 0.9

$$-\frac{F^a}{b} \left( -\frac{(-10 b \ln(F) x + 15) e^{b \ln(F) x}}{10 b^2} \sqrt{x} (-b)^{\frac{5}{2}} \sqrt{\ln(F)} + \frac{3 \sqrt{\pi}}{4} (-b)^{\frac{5}{2}} \text{erfi}\left(\sqrt{b} \sqrt{x} \sqrt{\ln(F)}\right) b^{-\frac{5}{2}} \right) (-b)^{-\frac{3}{2}} (\ln(F))^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(b\*x+a)\*x^(3/2), x)

[Out]  $-F^a/(-b)^{(3/2)}/\ln(F)^{(5/2)}/b*(-1/10*x^{(1/2)}*(-b)^{(5/2)}*\ln(F)^{(1/2)}*(-10*b*\ln(F)*x+15)/b^2*\exp(b*\ln(F)*x)+3/4*(-b)^{(5/2)}/b^{(5/2)}*\pi^{(1/2)}*\operatorname{erfi}(b^{(1/2)}*x^{(1/2)}*\ln(F)^{(1/2)})$

**Maxima [A]** time = 1.21725, size = 32, normalized size = 0.38

$$\frac{F^a x^2 \Gamma\left(\frac{5}{2}, -bx \log(F)\right)}{(-bx \log(F))^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(b*x+a)*x^(3/2),x, algorithm="maxima")`

[Out]  $-F^a*x^{(5/2)}*\gamma(5/2, -b*x*\log(F))/(-b*x*\log(F))^{(5/2)}$

**Fricas [A]** time = 1.49949, size = 188, normalized size = 2.21

$$\frac{3\sqrt{\pi}\sqrt{-b\log(F)}F^a\operatorname{erf}\left(\sqrt{-b\log(F)}\sqrt{x}\right)-2\left(2b^2x\log(F)^2-3b\log(F)\right)F^{bx+a}\sqrt{x}}{4b^3\log(F)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(b*x+a)*x^(3/2),x, algorithm="fricas")`

[Out]  $-1/4*(3*\sqrt{\pi}*\sqrt{-b*\log(F)}*F^a*\operatorname{erf}(\sqrt{-b*\log(F)}*\sqrt{x})) - 2*(2*b^2*x*\log(F)^2 - 3*b*\log(F))*F^{(b*x + a)}*\sqrt{x})/(b^3*\log(F)^3)$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F**(b*x+a)*x**(3/2),x)`



[Out] Timed out

**Giac [A]** time = 1.24248, size = 95, normalized size = 1.12

$$-\frac{3\sqrt{\pi}F^a \operatorname{erf}\left(-\sqrt{-b\log(F)}\sqrt{x}\right)}{4\sqrt{-b\log(F)}b^2\log(F)^2} + \frac{\left(2bx^{\frac{3}{2}}\log(F) - 3\sqrt{x}\right)e^{(bx\log(F)+a\log(F))}}{2b^2\log(F)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(b*x+a)*x^(3/2),x, algorithm="giac")`

[Out] `-3/4*sqrt(pi)*F^a*erf(-sqrt(-b*log(F))*sqrt(x))/(sqrt(-b*log(F))*b^2*log(F)^2) + 1/2*(2*b*x^(3/2)*log(F) - 3*sqrt(x))*e^(b*x*log(F) + a*log(F))/(b^2*log(F)^2)`

### 3.33 $\int F^{a+bx} \sqrt{x} dx$

**Optimal.** Leaf size=62

$$\frac{\sqrt{x}F^{a+bx}}{b \log(F)} - \frac{\sqrt{\pi}F^a \operatorname{Erfi}(\sqrt{b}\sqrt{x}\sqrt{\log(F)})}{2b^{3/2} \log^{\frac{3}{2}}(F)}$$

[Out]  $-(F^a \sqrt{\pi} \operatorname{Erfi}[\sqrt{b} \sqrt{x} \sqrt{\log(F)}]) / (2b^{3/2} \log[F]^{3/2}) + (F^{a+bx} \sqrt{x}) / (b \log[F])$

**Rubi [A]** time = 0.0443172, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$ , Rules used = {2176, 2180, 2204}

$$\frac{\sqrt{x}F^{a+bx}}{b \log(F)} - \frac{\sqrt{\pi}F^a \operatorname{Erfi}(\sqrt{b}\sqrt{x}\sqrt{\log(F)})}{2b^{3/2} \log^{\frac{3}{2}}(F)}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[F^{a+bx} \sqrt{x}, x]$

[Out]  $-(F^a \sqrt{\pi} \operatorname{Erfi}[\sqrt{b} \sqrt{x} \sqrt{\log(F)}]) / (2b^{3/2} \log[F]^{3/2}) + (F^{a+bx} \sqrt{x}) / (b \log[F])$

#### Rule 2176

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_)))^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[((c + d*x)^m*(b*F^(g*(e + f*x)))^n)/(f*g*n*Log[F]), x] - Dist[(d*m)/(f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m] && !$UseGamma === True
```

#### Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !$UseGamma === True
```

#### Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(2), x_Symbol] := Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

### Rubi steps

$$\begin{aligned}\int F^{a+bx}\sqrt{x} dx &= \frac{F^{a+bx}\sqrt{x}}{b \log(F)} - \frac{\int \frac{F^{a+bx}}{\sqrt{x}} dx}{2b \log(F)} \\ &= \frac{F^{a+bx}\sqrt{x}}{b \log(F)} - \frac{\text{Subst}\left(\int F^{a+bx^2} dx, x, \sqrt{x}\right)}{b \log(F)} \\ &= -\frac{F^a \sqrt{\pi} \operatorname{erfi}\left(\sqrt{b}\sqrt{x}\sqrt{\log(F)}\right)}{2b^{3/2} \log^2(F)} + \frac{F^{a+bx}\sqrt{x}}{b \log(F)}\end{aligned}$$

**Mathematica [A]** time = 0.0076998, size = 30, normalized size = 0.48

$$-\frac{x^{3/2} F^a \operatorname{Gamma}\left(\frac{3}{2}, -bx \log(F)\right)}{(-bx \log(F))^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[F^(a + b\*x)\*Sqrt[x], x]

[Out] -((F^a\*x^(3/2)\*Gamma[3/2, -(b\*x\*Log[F])])/(-(b\*x\*Log[F]))^(3/2))

**Maple [A]** time = 0.008, size = 66, normalized size = 1.1

$$-\frac{F^a}{b} \left( \frac{e^{b \ln(F)x}}{b} \sqrt{x} (-b)^{\frac{3}{2}} \sqrt{\ln(F)} - \frac{\sqrt{\pi}}{2} (-b)^{\frac{3}{2}} \operatorname{erfi}\left(\sqrt{b}\sqrt{x}\sqrt{\ln(F)}\right) b^{-\frac{3}{2}} \right) \frac{1}{\sqrt{-b}} (\ln(F))^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(b\*x+a)\*x^(1/2), x)

[Out] -F^a/(-b)^(1/2)/ln(F)^(3/2)/b\*(x^(1/2)\*(-b)^(3/2)\*ln(F)^(1/2)/b\*exp(b\*ln(F)\*x)-1/2\*(-b)^(3/2)/b^(3/2)\*Pi^(1/2)\*erfi(b^(1/2)\*x^(1/2)\*ln(F)^(1/2))

---

**Maxima [A]** time = 1.24986, size = 32, normalized size = 0.52

$$\frac{F^a x^{\frac{3}{2}} \Gamma\left(\frac{3}{2}, -bx \log(F)\right)}{(-bx \log(F))^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(b\*x+a)\*x^(1/2),x, algorithm="maxima")

[Out] -F^a\*x^(3/2)\*gamma(3/2, -b\*x\*log(F))/(-b\*x\*log(F))^(3/2)

---

**Fricas [A]** time = 1.5187, size = 153, normalized size = 2.47

$$\frac{2F^{bx+a}b\sqrt{x}\log(F) + \sqrt{\pi}\sqrt{-b\log(F)}F^a \operatorname{erf}\left(\sqrt{-b\log(F)}\sqrt{x}\right)}{2b^2\log(F)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(b\*x+a)\*x^(1/2),x, algorithm="fricas")

[Out] 1/2\*(2\*F^(b\*x + a)\*b\*sqrt(x)\*log(F) + sqrt(pi)\*sqrt(-b\*log(F))\*F^a\*erf(sqrt(-b\*log(F))\*sqrt(x)))/(b^2\*log(F)^2)

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int F^{a+bx} \sqrt{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(b\*x+a)\*x\*\*(1/2),x)

[Out] Integral(F\*\*(a + b\*x)\*sqrt(x), x)

---

**Giac [A]** time = 1.22632, size = 78, normalized size = 1.26

$$\frac{\sqrt{\pi}F^a \operatorname{erf}\left(-\sqrt{-b \log(F)}\sqrt{x}\right)}{2\sqrt{-b \log(F)}b \log(F)} + \frac{\sqrt{x}e^{(bx \log(F)+a \log(F))}}{b \log(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(b*x+a)*x^(1/2),x, algorithm="giac")`

[Out] `1/2*sqrt(pi)*F^a*erf(-sqrt(-b*log(F))*sqrt(x))/(sqrt(-b*log(F))*b*log(F)) + sqrt(x)*e^(b*x*log(F) + a*log(F))/(b*log(F))`

$$3.34 \quad \int \frac{F^{a+bx}}{\sqrt{x}} dx$$

**Optimal.** Leaf size=38

$$\frac{\sqrt{\pi} F^a \operatorname{Erfi}(\sqrt{b} \sqrt{x} \sqrt{\log(F)})}{\sqrt{b} \sqrt{\log(F)}}$$

[Out] (F^a\*Sqrt[Pi]\*Erfi[Sqrt[b]\*Sqrt[x]\*Sqrt[Log[F]]])/(Sqrt[b]\*Sqrt[Log[F]])

**Rubi [A]** time = 0.025179, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$ , Rules used = {2180, 2204}

$$\frac{\sqrt{\pi} F^a \operatorname{Erfi}(\sqrt{b} \sqrt{x} \sqrt{\log(F)})}{\sqrt{b} \sqrt{\log(F)}}$$

Antiderivative was successfully verified.

[In] Int[F^(a + b\*x)/Sqrt[x], x]

[Out] (F^a\*Sqrt[Pi]\*Erfi[Sqrt[b]\*Sqrt[x]\*Sqrt[Log[F]]])/(Sqrt[b]\*Sqrt[Log[F]])

#### Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !$UseGamma == True
```

#### Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_)^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

#### Rubi steps

$$\int \frac{F^{a+bx}}{\sqrt{x}} dx = 2 \operatorname{Subst} \left( \int F^{a+bx^2} dx, x, \sqrt{x} \right)$$

$$= \frac{F^a \sqrt{\pi} \operatorname{erfi} \left( \sqrt{b} \sqrt{x} \sqrt{\log(F)} \right)}{\sqrt{b} \sqrt{\log(F)}}$$

**Mathematica [A]** time = 0.0063854, size = 30, normalized size = 0.79

$$-\frac{\sqrt{x} F^a \operatorname{Gamma} \left( \frac{1}{2}, -bx \log(F) \right)}{\sqrt{-bx \log(F)}}$$

Antiderivative was successfully verified.

[In] Integrate[F^(a + b\*x)/Sqrt[x], x]

[Out] -((F^a\*Sqrt[x]\*Gamma[1/2, -(b\*x\*Log[F])])/Sqrt[-(b\*x\*Log[F])])

**Maple [A]** time = 0.008, size = 27, normalized size = 0.7

$$F^a \sqrt{\pi} \operatorname{erfi} \left( \sqrt{b} \sqrt{x} \sqrt{\ln(F)} \right) \frac{1}{\sqrt{b}} \frac{1}{\sqrt{\ln(F)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(b\*x+a)/x^(1/2), x)

[Out] F^a\*erfi(b^(1/2)\*x^(1/2)\*ln(F)^(1/2))\*Pi^(1/2)/b^(1/2)/ln(F)^(1/2)

**Maxima [A]** time = 1.15337, size = 39, normalized size = 1.03

$$\frac{\sqrt{\pi} F^a \sqrt{x} \left( \operatorname{erf} \left( \sqrt{-bx \log(F)} \right) - 1 \right)}{\sqrt{-bx \log(F)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(b\*x+a)/x^(1/2), x, algorithm="maxima")

[Out]  $\sqrt{\pi} F^a \sqrt{x} (\operatorname{erf}(\sqrt{-b x \log(F)}) - 1) / \sqrt{-b x \log(F)}$

---

**Fricas [A]** time = 1.5287, size = 96, normalized size = 2.53

$$-\frac{\sqrt{\pi} \sqrt{-b \log(F)} F^a \operatorname{erf}(\sqrt{-b \log(F)} \sqrt{x})}{b \log(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(b*x+a)/x^(1/2),x, algorithm="fricas")`

[Out]  $-\sqrt{\pi} \sqrt{-b \log(F)} F^a \operatorname{erf}(\sqrt{-b \log(F)} \sqrt{x}) / (b \log(F))$

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{a+bx}}{\sqrt{x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F**(b*x+a)/x**(1/2),x)`

[Out] `Integral(F**(a + b*x)/sqrt(x), x)`

---

**Giac [A]** time = 1.34626, size = 38, normalized size = 1.

$$-\frac{\sqrt{\pi} F^a \operatorname{erf}(-\sqrt{-b \log(F)} \sqrt{x})}{\sqrt{-b \log(F)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(b*x+a)/x^(1/2),x, algorithm="giac")`

[Out]  $-\sqrt{\pi} F^a \operatorname{erf}(-\sqrt{-b \log(F)} \sqrt{x}) / \sqrt{-b \log(F)}$



$$3.35 \quad \int \frac{F^{a+bx}}{x^{3/2}} dx$$

**Optimal.** Leaf size=54

$$2\sqrt{\pi}\sqrt{b}F^a\sqrt{\log(F)}\operatorname{Erfi}\left(\sqrt{b}\sqrt{x}\sqrt{\log(F)}\right) - \frac{2F^{a+bx}}{\sqrt{x}}$$

[Out]  $(-2F^{(a + b*x)})/\operatorname{Sqrt}[x] + 2*\operatorname{Sqrt}[b]*F^a*\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erfi}[\operatorname{Sqrt}[b]*\operatorname{Sqrt}[x]*\operatorname{Sqrt}[\operatorname{Log}[F]]]*\operatorname{Sqrt}[\operatorname{Log}[F]]$

**Rubi [A]** time = 0.0454763, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$ , Rules used = {2177, 2180, 2204}

$$2\sqrt{\pi}\sqrt{b}F^a\sqrt{\log(F)}\operatorname{Erfi}\left(\sqrt{b}\sqrt{x}\sqrt{\log(F)}\right) - \frac{2F^{a+bx}}{\sqrt{x}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[F^{(a + b*x)}/x^{(3/2)}, x]$

[Out]  $(-2F^{(a + b*x)})/\operatorname{Sqrt}[x] + 2*\operatorname{Sqrt}[b]*F^a*\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erfi}[\operatorname{Sqrt}[b]*\operatorname{Sqrt}[x]*\operatorname{Sqrt}[\operatorname{Log}[F]]]*\operatorname{Sqrt}[\operatorname{Log}[F]]$

#### Rule 2177

$\operatorname{Int}[(b_*)*(F_*)^{((g_*)*((e_*) + (f_*)*(x_*)))^n)*((c_*) + (d_*)*(x_*))^{(m_*)}, x\_Symbol] :> \operatorname{Simp}[(c + d*x)^{(m + 1)}*(b*F^{(g*(e + f*x))})^n]/(d*(m + 1)), x] - \operatorname{Dist}[(f*g*n*\operatorname{Log}[F])/d*(m + 1), \operatorname{Int}[(c + d*x)^{(m + 1)}*(b*F^{(g*(e + f*x))})^n, x], x] /;$   $\operatorname{FreeQ}\{F, b, c, d, e, f, g, n\}, x] \&\& \operatorname{LtQ}[m, -1] \&\& \operatorname{IntegerQ}[2*m] \&\& !\$UseGamma == True$

#### Rule 2180

$\operatorname{Int}[(F_*)^{((g_*)*((e_*) + (f_*)*(x_*)))}/\operatorname{Sqrt}[(c_*) + (d_*)*(x_*)], x\_Symbol] :> \operatorname{Dist}[2/d, \operatorname{Subst}[\operatorname{Int}[F^{(g*(e - (c*f)/d) + (f*g*x^2)/d)}, x], x, \operatorname{Sqrt}[c + d*x]], x] /;$   $\operatorname{FreeQ}\{F, c, d, e, f, g\}, x] \&\& !\$UseGamma == True$

#### Rule 2204

$\operatorname{Int}[(F_*)^{((a_*) + (b_*)*((c_*) + (d_*)*(x_*))^2)}, x\_Symbol] :> \operatorname{Simp}[(F^a*\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erfi}[(c + d*x)*\operatorname{Rt}[b*\operatorname{Log}[F], 2]])/(2*d*\operatorname{Rt}[b*\operatorname{Log}[F], 2]), x] /;$   $\operatorname{FreeQ}\{$

$F, a, b, c, d\}, x]$  && PosQ[b]

### Rubi steps

$$\begin{aligned}\int \frac{F^{a+bx}}{x^{3/2}} dx &= -\frac{2F^{a+bx}}{\sqrt{x}} + (2b \log(F)) \int \frac{F^{a+bx}}{\sqrt{x}} dx \\ &= -\frac{2F^{a+bx}}{\sqrt{x}} + (4b \log(F)) \text{Subst} \left( \int F^{a+bx^2} dx, x, \sqrt{x} \right) \\ &= -\frac{2F^{a+bx}}{\sqrt{x}} + 2\sqrt{b}F^a \sqrt{\pi} \operatorname{erfi} \left( \sqrt{b}\sqrt{x}\sqrt{\log(F)} \right) \sqrt{\log(F)}\end{aligned}$$

**Mathematica [A]** time = 0.0151075, size = 38, normalized size = 0.7

$$\frac{2F^a \left( F^{bx} - \sqrt{-bx \log(F)} \operatorname{Gamma} \left( \frac{1}{2}, -bx \log(F) \right) \right)}{\sqrt{x}}$$

Antiderivative was successfully verified.

[In] Integrate[F^(a + b\*x)/x^(3/2), x]

[Out] (-2\*F^a\*(F^(b\*x) - Gamma[1/2, -(b\*x\*Log[F])]\*Sqrt[-(b\*x\*Log[F])]))/Sqrt[x]

**Maple [A]** time = 0.01, size = 64, normalized size = 1.2

$$-\frac{F^a}{b} (-b)^{\frac{3}{2}} \sqrt{\ln(F)} \left( -2 \frac{e^{b \ln(F)x}}{\sqrt{x} \sqrt{-b} \sqrt{\ln(F)}} + 2 \frac{\sqrt{b} \sqrt{\pi} \operatorname{erfi} \left( \sqrt{b} \sqrt{x} \sqrt{\ln(F)} \right)}{\sqrt{-b}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(b\*x+a)/x^(3/2), x)

[Out] -F^a\*(-b)^(3/2)\*ln(F)^(1/2)/b\*(-2/x^(1/2)/(-b)^(1/2)/ln(F)^(1/2)\*exp(b\*ln(F)\*x)+2/(-b)^(1/2)\*b^(1/2)\*Pi^(1/2)\*erfi(b^(1/2)\*x^(1/2)\*ln(F)^(1/2))

**Maxima [A]** time = 1.21438, size = 32, normalized size = 0.59

$$\frac{\sqrt{-bx \log(F)} F^a \Gamma\left(-\frac{1}{2}, -bx \log(F)\right)}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(b\*x+a)/x^(3/2), x, algorithm="maxima")

[Out] -sqrt(-b\*x\*log(F))\*F^a\*gamma(-1/2, -b\*x\*log(F))/sqrt(x)

**Fricas [A]** time = 1.54878, size = 122, normalized size = 2.26

$$\frac{2\left(\sqrt{\pi}\sqrt{-b \log(F)} F^a x \operatorname{erf}\left(\sqrt{-b \log(F)}\sqrt{x}\right) + F^{bx+a}\sqrt{x}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(b\*x+a)/x^(3/2), x, algorithm="fricas")

[Out] -2\*(sqrt(pi)\*sqrt(-b\*log(F))\*F^a\*x\*erf(sqrt(-b\*log(F))\*sqrt(x)) + F^(b\*x + a)\*sqrt(x))/x

**Sympy [A]** time = 6.77587, size = 34, normalized size = 0.63

$$4F^a F^{bx} b \sqrt{x} \log(F) - \frac{2F^a F^{bx}}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(b\*x+a)/x\*\*(3/2), x)

[Out] 4\*F\*\*a\*F\*\*(b\*x)\*b\*sqrt(x)\*log(F) - 2\*F\*\*a\*F\*\*(b\*x)/sqrt(x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{bx+a}}{x^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(b*x+a)/x^(3/2),x, algorithm="giac")
```

```
[Out] integrate(F^(b*x + a)/x^(3/2), x)
```

### 3.36 $\int \frac{F^{a+bx}}{x^{5/2}} dx$

**Optimal.** Leaf size=77

$$\frac{4}{3} \sqrt{\pi} b^{3/2} F^a \log^{\frac{3}{2}}(F) \operatorname{Erfi}\left(\sqrt{b} \sqrt{x} \sqrt{\log(F)}\right) - \frac{2F^{a+bx}}{3x^{3/2}} - \frac{4b \log(F) F^{a+bx}}{3\sqrt{x}}$$

[Out]  $(-2 * F^{(a + b * x)}) / (3 * x^{(3/2)}) - (4 * b * F^{(a + b * x)} * \operatorname{Log}[F]) / (3 * \operatorname{Sqrt}[x]) + (4 * b^{(3/2)} * F^a * \operatorname{Sqrt}[\operatorname{Pi}] * \operatorname{Erfi}[\operatorname{Sqrt}[b] * \operatorname{Sqrt}[x] * \operatorname{Sqrt}[\operatorname{Log}[F]]] * \operatorname{Log}[F]^{(3/2)}) / 3$

**Rubi [A]** time = 0.064935, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$ , Rules used = {2177, 2180, 2204}

$$\frac{4}{3} \sqrt{\pi} b^{3/2} F^a \log^{\frac{3}{2}}(F) \operatorname{Erfi}\left(\sqrt{b} \sqrt{x} \sqrt{\log(F)}\right) - \frac{2F^{a+bx}}{3x^{3/2}} - \frac{4b \log(F) F^{a+bx}}{3\sqrt{x}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[F^{(a + b * x)} / x^{(5/2)}, x]$

[Out]  $(-2 * F^{(a + b * x)}) / (3 * x^{(3/2)}) - (4 * b * F^{(a + b * x)} * \operatorname{Log}[F]) / (3 * \operatorname{Sqrt}[x]) + (4 * b^{(3/2)} * F^a * \operatorname{Sqrt}[\operatorname{Pi}] * \operatorname{Erfi}[\operatorname{Sqrt}[b] * \operatorname{Sqrt}[x] * \operatorname{Sqrt}[\operatorname{Log}[F]]] * \operatorname{Log}[F]^{(3/2)}) / 3$

#### Rule 2177

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_), x_Symbol] :> Simp[((c + d*x)^(m + 1)*(b*F^(g*(e + f*x)))^n)/(d*(m + 1))
, x] - Dist[(f*g*n*Log[F])/(d*(m + 1)), Int[(c + d*x)^(m + 1)*(b*F^(g*(e +
f*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && LtQ[m, -1] && Int
egerQ[2*m] && !$UseGamma == True
```

#### Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !$UseGamma == True
```

#### Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
```

$F, a, b, c, d, x$  && PosQ[b]

### Rubi steps

$$\begin{aligned}
 \int \frac{F^{a+bx}}{x^{5/2}} dx &= -\frac{2F^{a+bx}}{3x^{3/2}} + \frac{1}{3}(2b \log(F)) \int \frac{F^{a+bx}}{x^{3/2}} dx \\
 &= -\frac{2F^{a+bx}}{3x^{3/2}} - \frac{4bF^{a+bx} \log(F)}{3\sqrt{x}} + \frac{1}{3} (4b^2 \log^2(F)) \int \frac{F^{a+bx}}{\sqrt{x}} dx \\
 &= -\frac{2F^{a+bx}}{3x^{3/2}} - \frac{4bF^{a+bx} \log(F)}{3\sqrt{x}} + \frac{1}{3} (8b^2 \log^2(F)) \text{Subst} \left( \int F^{a+bx^2} dx, x, \sqrt{x} \right) \\
 &= -\frac{2F^{a+bx}}{3x^{3/2}} - \frac{4bF^{a+bx} \log(F)}{3\sqrt{x}} + \frac{4}{3} b^{3/2} F^a \sqrt{\pi} \operatorname{erfi} \left( \sqrt{b} \sqrt{x} \sqrt{\log(F)} \right) \log^{\frac{3}{2}}(F)
 \end{aligned}$$

**Mathematica [A]** time = 0.0368925, size = 49, normalized size = 0.64

$$\frac{2F^a \left( 2(-bx \log(F))^{3/2} \operatorname{Gamma} \left( \frac{1}{2}, -bx \log(F) \right) + F^{bx} (2bx \log(F) + 1) \right)}{3x^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[F^(a + b\*x)/x^(5/2), x]

[Out]  $(-2F^a (2 \operatorname{Gamma}[1/2, -(b*x \operatorname{Log}[F])]) * (-(b*x \operatorname{Log}[F]))^{(3/2)} + F^{(b*x)} * (1 + 2 * b*x \operatorname{Log}[F])) / (3*x^{(3/2)})$

**Maple [A]** time = 0.011, size = 72, normalized size = 0.9

$$-\frac{F^a}{b} (-b)^{\frac{5}{2}} (\ln(F))^{\frac{3}{2}} \left( -\frac{(4b \ln(F)x + 2) e^{b \ln(F)x}}{3} x^{-\frac{3}{2}} (-b)^{-\frac{3}{2}} (\ln(F))^{-\frac{3}{2}} + \frac{4\sqrt{\pi}}{3} b^{\frac{3}{2}} \operatorname{erfi} \left( \sqrt{b} \sqrt{x} \sqrt{\ln(F)} \right) (-b)^{-\frac{3}{2}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(b\*x+a)/x^(5/2), x)

[Out]  $-F^a (-b)^{(5/2)} \ln(F)^{(3/2)} / b * (-2/3/x^{(3/2)} / (-b)^{(3/2)} / \ln(F)^{(3/2)} * (2*b*\ln(F)*x+1)*\exp(b*\ln(F)*x)+4/3/(-b)^{(3/2)}*b^{(3/2)}*Pi^{(1/2)}*\operatorname{erfi}(b^{(1/2)}*x^{(1/2)}*\ln(F)^{(1/2)})$

---

**Maxima [A]** time = 1.25236, size = 32, normalized size = 0.42

$$-\frac{(-bx \log(F))^{\frac{3}{2}} F^a \Gamma\left(-\frac{3}{2}, -bx \log(F)\right)}{x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(b\*x+a)/x^(5/2), x, algorithm="maxima")

[Out] -(-b\*x\*log(F))^(3/2)\*F^a\*gamma(-3/2, -b\*x\*log(F))/x^(3/2)

---

**Fricas [A]** time = 1.54017, size = 170, normalized size = 2.21

$$\frac{2\left(2\sqrt{\pi}\sqrt{-b\log(F)}F^a b x^2 \operatorname{erf}\left(\sqrt{-b\log(F)}\sqrt{x}\right)\log(F) + (2bx\log(F) + 1)F^{bx+a}\sqrt{x}\right)}{3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(b\*x+a)/x^(5/2), x, algorithm="fricas")

[Out] -2/3\*(2\*sqrt(pi)\*sqrt(-b\*log(F))\*F^a\*b\*x^2\*erf(sqrt(-b\*log(F))\*sqrt(x))\*log(F) + (2\*b\*x\*log(F) + 1)\*F^(b\*x + a)\*sqrt(x))/x^2

---

**Sympy [A]** time = 175.103, size = 39, normalized size = 0.51

$$\frac{4F^a F^{bx} b \log(F)}{3\sqrt{x}} - \frac{2F^a F^{bx}}{3x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(b\*x+a)/x\*\*(5/2), x)

[Out] -4\*F\*\*a\*F\*\*(b\*x)\*b\*log(F)/(3\*sqrt(x)) - 2\*F\*\*a\*F\*\*(b\*x)/(3\*x\*\*(3/2))

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{bx+a}}{x^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(b\*x+a)/x^(5/2),x, algorithm="giac")

[Out] integrate(F^(b\*x + a)/x^(5/2), x)



### 3.37 $\int \frac{F^{a+bx}}{x^{7/2}} dx$

**Optimal.** Leaf size=100

$$\frac{8}{15} \sqrt{\pi} b^{5/2} F^a \log^2(F) \operatorname{Erfi}\left(\sqrt{b} \sqrt{x} \sqrt{\log(F)}\right) - \frac{8b^2 \log^2(F) F^{a+bx}}{15\sqrt{x}} - \frac{2F^{a+bx}}{5x^{5/2}} - \frac{4b \log(F) F^{a+bx}}{15x^{3/2}}$$

[Out]  $(-2F^{(a + b*x)})/(5*x^{(5/2)}) - (4*b*F^{(a + b*x)*\operatorname{Log}[F]})/(15*x^{(3/2)}) - (8*b^2*F^{(a + b*x)*\operatorname{Log}[F]^2})/(15*\operatorname{Sqrt}[x]) + (8*b^{(5/2)}*F^a*\operatorname{Sqrt}[\pi]*\operatorname{Erfi}[\operatorname{Sqrt}[b]*\operatorname{Sqrt}[x]*\operatorname{Sqrt}[\operatorname{Log}[F]]]*\operatorname{Log}[F]^{(5/2)})/15$

**Rubi [A]** time = 0.0894239, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$ , Rules used = {2177, 2180, 2204}

$$\frac{8}{15} \sqrt{\pi} b^{5/2} F^a \log^2(F) \operatorname{Erfi}\left(\sqrt{b} \sqrt{x} \sqrt{\log(F)}\right) - \frac{8b^2 \log^2(F) F^{a+bx}}{15\sqrt{x}} - \frac{2F^{a+bx}}{5x^{5/2}} - \frac{4b \log(F) F^{a+bx}}{15x^{3/2}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[F^{(a + b*x)}/x^{(7/2)}, x]$

[Out]  $(-2F^{(a + b*x)})/(5*x^{(5/2)}) - (4*b*F^{(a + b*x)*\operatorname{Log}[F]})/(15*x^{(3/2)}) - (8*b^2*F^{(a + b*x)*\operatorname{Log}[F]^2})/(15*\operatorname{Sqrt}[x]) + (8*b^{(5/2)}*F^a*\operatorname{Sqrt}[\pi]*\operatorname{Erfi}[\operatorname{Sqrt}[b]*\operatorname{Sqrt}[x]*\operatorname{Sqrt}[\operatorname{Log}[F]]]*\operatorname{Log}[F]^{(5/2)})/15$

#### Rule 2177

$\operatorname{Int}[(b_*)*(F_*)^{((g_*)*((e_*) + (f_*)*(x_)))}^{(n_*)*((c_*) + (d_*)*(x_))}^{(m_*)}, x\_Symbol] :> \operatorname{Simp}[(c + d*x)^{(m + 1)}*(b*F^{(g*(e + f*x))})^n]/(d*(m + 1)), x] - \operatorname{Dist}[(f*g*n*\operatorname{Log}[F])/(d*(m + 1)), \operatorname{Int}[(c + d*x)^{(m + 1)}*(b*F^{(g*(e + f*x))})^n, x], x] /; \operatorname{FreeQ}\{F, b, c, d, e, f, g, n\}, x] \&\& \operatorname{LtQ}[m, -1] \&\& \operatorname{IntegerQ}[2*m] \&\& !$\operatorname{UseGamma} == \operatorname{True}$

#### Rule 2180

$\operatorname{Int}[(F_*)^{((g_*)*((e_*) + (f_*)*(x_)))}/\operatorname{Sqrt}[(c_*) + (d_*)*(x_)], x\_Symbol] :> \operatorname{Dist}[2/d, \operatorname{Subst}[\operatorname{Int}[F^{(g*(e - (c*f)/d) + (f*g*x^2)/d)}, x], x, \operatorname{Sqrt}[c + d*x]], x] /; \operatorname{FreeQ}\{F, c, d, e, f, g\}, x] \&\& !$\operatorname{UseGamma} == \operatorname{True}$

#### Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_)^2), x_Symbol] := Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

### Rubi steps

$$\begin{aligned}
\int \frac{F^{a+bx}}{x^{7/2}} dx &= -\frac{2F^{a+bx}}{5x^{5/2}} + \frac{1}{5}(2b \log(F)) \int \frac{F^{a+bx}}{x^{5/2}} dx \\
&= -\frac{2F^{a+bx}}{5x^{5/2}} - \frac{4bF^{a+bx} \log(F)}{15x^{3/2}} + \frac{1}{15}(4b^2 \log^2(F)) \int \frac{F^{a+bx}}{x^{3/2}} dx \\
&= -\frac{2F^{a+bx}}{5x^{5/2}} - \frac{4bF^{a+bx} \log(F)}{15x^{3/2}} - \frac{8b^2F^{a+bx} \log^2(F)}{15\sqrt{x}} + \frac{1}{15}(8b^3 \log^3(F)) \int \frac{F^{a+bx}}{\sqrt{x}} dx \\
&= -\frac{2F^{a+bx}}{5x^{5/2}} - \frac{4bF^{a+bx} \log(F)}{15x^{3/2}} - \frac{8b^2F^{a+bx} \log^2(F)}{15\sqrt{x}} + \frac{1}{15}(16b^3 \log^3(F)) \text{Subst}\left(\int F^{a+bx^2} dx, x, \sqrt{x}\right) \\
&= -\frac{2F^{a+bx}}{5x^{5/2}} - \frac{4bF^{a+bx} \log(F)}{15x^{3/2}} - \frac{8b^2F^{a+bx} \log^2(F)}{15\sqrt{x}} + \frac{8}{15}b^{5/2}F^a\sqrt{\pi}\text{erfi}\left(\sqrt{b}\sqrt{x}\sqrt{\log(F)}\right)\log^5(F)
\end{aligned}$$

**Mathematica [A]** time = 0.0500716, size = 61, normalized size = 0.61

$$\frac{2F^a \left( F^{bx} \left( 4b^2x^2 \log^2(F) + 2bx \log(F) + 3 \right) - 4(-bx \log(F))^{5/2} \text{Gamma}\left(\frac{1}{2}, -bx \log(F)\right) \right)}{15x^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[F^(a + b\*x)/x^(7/2), x]

[Out]  $(-2F^a(-4\text{Gamma}[1/2, -(b*x*\text{Log}[F])])*(-(b*x*\text{Log}[F]))^{5/2} + F^{(b*x)}*(3 + 2*b*x*\text{Log}[F] + 4*b^2*x^2*\text{Log}[F]^2))/(15*x^{5/2})$

**Maple [A]** time = 0.012, size = 84, normalized size = 0.8

$$-\frac{F^a}{b}(-b)^{\frac{7}{2}}(\ln(F))^{\frac{5}{2}}\left(-\frac{2e^{b\ln(F)x}}{5}\left(\frac{4b^2x^2(\ln(F))^2}{3} + \frac{2b\ln(F)x}{3} + 1\right)x^{-\frac{5}{2}}(-b)^{-\frac{5}{2}}(\ln(F))^{-\frac{5}{2}} + \frac{8\sqrt{\pi}}{15}b^{\frac{5}{2}}\text{erfi}\left(\sqrt{b}\sqrt{x}\sqrt{\ln(F)}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(b\*x+a)/x^(7/2), x)

[Out]  $-F^a(-b)^{7/2} \ln(F)^{5/2} / b(-2/5/x^{5/2} / (-b)^{5/2} / \ln(F)^{5/2} * (4/3*b^2 * x^2 * \ln(F)^2 + 2/3*b*\ln(F)*x+1) * \exp(b*\ln(F)*x) + 8/15 / (-b)^{5/2} * b^{5/2} * \text{Pi}^{1/2} * \text{erfi}(b^{1/2} * x^{1/2} * \ln(F)^{1/2}))$

**Maxima [A]** time = 1.25492, size = 32, normalized size = 0.32

$$\frac{(-bx \log(F))^{\frac{5}{2}} F^a \Gamma\left(-\frac{5}{2}, -bx \log(F)\right)}{x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(b*x+a)/x^(7/2),x, algorithm="maxima")`

[Out]  $-(-b*x*\log(F))^{5/2} * F^a * \text{gamma}(-5/2, -b*x*\log(F)) / x^{5/2}$

**Fricas [A]** time = 1.52355, size = 205, normalized size = 2.05

$$\frac{2\left(4\sqrt{\pi}\sqrt{-b\log(F)}F^ab^2x^3\text{erf}\left(\sqrt{-b\log(F)}\sqrt{x}\right)\log(F)^2 + \left(4b^2x^2\log(F)^2 + 2bx\log(F) + 3\right)F^{bx+a}\sqrt{x}\right)}{15x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(b*x+a)/x^(7/2),x, algorithm="fricas")`

[Out]  $-2/15 * (4*\text{sqrt}(\text{pi})*\text{sqrt}(-b*\log(F))*F^a*b^2*x^3*\text{erf}(\text{sqrt}(-b*\log(F))*\text{sqrt}(x))*\log(F)^2 + (4*b^2*x^2*\log(F)^2 + 2*b*x*\log(F) + 3)*F^{(b*x + a)}*\text{sqrt}(x))/x^3$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F**(b*x+a)/x**(7/2),x)`

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{bx+a}}{x^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(b\*x+a)/x^(7/2),x, algorithm="giac")

[Out] integrate(F^(b\*x + a)/x^(7/2), x)

### 3.38 $\int \frac{F^{a+bx}}{x^{9/2}} dx$

**Optimal.** Leaf size=123

$$\frac{16}{105} \sqrt{\pi} b^{7/2} F^a \log^2(F) \operatorname{Erfi}\left(\sqrt{b} \sqrt{x} \sqrt{\log(F)}\right) - \frac{8b^2 \log^2(F) F^{a+bx}}{105x^{3/2}} - \frac{16b^3 \log^3(F) F^{a+bx}}{105\sqrt{x}} - \frac{2F^{a+bx}}{7x^{7/2}} - \frac{4b \log(F) F^{a+bx}}{35x^{5/2}}$$

[Out]  $(-2F^{(a + b*x)})/(7*x^{(7/2)}) - (4*b*F^{(a + b*x)*\operatorname{Log}[F]})/(35*x^{(5/2)}) - (8*b^2*F^{(a + b*x)*\operatorname{Log}[F]^2})/(105*x^{(3/2)}) - (16*b^3*F^{(a + b*x)*\operatorname{Log}[F]^3})/(105*\operatorname{Sqrt}[x]) + (16*b^{(7/2)}*F^a*\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erfi}[\operatorname{Sqrt}[b]*\operatorname{Sqrt}[x]*\operatorname{Sqrt}[\operatorname{Log}[F]]])*\operatorname{Log}[F]^{(7/2)}/105$

**Rubi [A]** time = 0.1082, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$ , Rules used = {2177, 2180, 2204}

$$\frac{16}{105} \sqrt{\pi} b^{7/2} F^a \log^2(F) \operatorname{Erfi}\left(\sqrt{b} \sqrt{x} \sqrt{\log(F)}\right) - \frac{8b^2 \log^2(F) F^{a+bx}}{105x^{3/2}} - \frac{16b^3 \log^3(F) F^{a+bx}}{105\sqrt{x}} - \frac{2F^{a+bx}}{7x^{7/2}} - \frac{4b \log(F) F^{a+bx}}{35x^{5/2}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[F^{(a + b*x)}/x^{(9/2)}, x]$

[Out]  $(-2F^{(a + b*x)})/(7*x^{(7/2)}) - (4*b*F^{(a + b*x)*\operatorname{Log}[F]})/(35*x^{(5/2)}) - (8*b^2*F^{(a + b*x)*\operatorname{Log}[F]^2})/(105*x^{(3/2)}) - (16*b^3*F^{(a + b*x)*\operatorname{Log}[F]^3})/(105*\operatorname{Sqrt}[x]) + (16*b^{(7/2)}*F^a*\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erfi}[\operatorname{Sqrt}[b]*\operatorname{Sqrt}[x]*\operatorname{Sqrt}[\operatorname{Log}[F]]])*\operatorname{Log}[F]^{(7/2)}/105$

#### Rule 2177

$\operatorname{Int}[(b_*)*(F_*)^{((g_*)*((e_*) + (f_*)*(x_*)))^{(n_*)*((c_*) + (d_*)*(x_*))^{(m_*)}}, x\_Symbol] :> \operatorname{Simp}[(c + d*x)^{(m + 1)}*(b*F^{(g*(e + f*x))})^n]/(d*(m + 1)), x] - \operatorname{Dist}[(f*g*n*\operatorname{Log}[F])/(d*(m + 1)), \operatorname{Int}[(c + d*x)^{(m + 1)}*(b*F^{(g*(e + f*x))})^n, x], x] /; \operatorname{FreeQ}\{F, b, c, d, e, f, g, n\}, x] \&\& \operatorname{LtQ}[m, -1] \&\& \operatorname{IntegerQ}[2*m] \&\& !$\operatorname{UseGamma} == \operatorname{True}$

#### Rule 2180

$\operatorname{Int}[(F_*)^{((g_*)*((e_*) + (f_*)*(x_*)))}/\operatorname{Sqrt}[(c_*) + (d_*)*(x_*)], x\_Symbol] :> \operatorname{Dist}[2/d, \operatorname{Subst}[\operatorname{Int}[F^{(g*(e - (c*f)/d) + (f*g*x^2)/d)}, x], x, \operatorname{Sqrt}[c + d*$

x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !\$UseGamma == True

### Rule 2204

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_)^2), x\_Symbol] := Simp[(F^a\*Sqrt[Pi]\*Erfi[(c + d\*x)\*Rt[b\*Log[F], 2]])/(2\*d\*Rt[b\*Log[F], 2]), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]

### Rubi steps

$$\begin{aligned}
 \int \frac{F^{a+bx}}{x^{9/2}} dx &= -\frac{2F^{a+bx}}{7x^{7/2}} + \frac{1}{7}(2b \log(F)) \int \frac{F^{a+bx}}{x^{7/2}} dx \\
 &= -\frac{2F^{a+bx}}{7x^{7/2}} - \frac{4bF^{a+bx} \log(F)}{35x^{5/2}} + \frac{1}{35} (4b^2 \log^2(F)) \int \frac{F^{a+bx}}{x^{5/2}} dx \\
 &= -\frac{2F^{a+bx}}{7x^{7/2}} - \frac{4bF^{a+bx} \log(F)}{35x^{5/2}} - \frac{8b^2F^{a+bx} \log^2(F)}{105x^{3/2}} + \frac{1}{105} (8b^3 \log^3(F)) \int \frac{F^{a+bx}}{x^{3/2}} dx \\
 &= -\frac{2F^{a+bx}}{7x^{7/2}} - \frac{4bF^{a+bx} \log(F)}{35x^{5/2}} - \frac{8b^2F^{a+bx} \log^2(F)}{105x^{3/2}} - \frac{16b^3F^{a+bx} \log^3(F)}{105\sqrt{x}} + \frac{1}{105} (16b^4 \log^4(F)) \int \frac{F^{a+bx}}{\sqrt{x}} dx \\
 &= -\frac{2F^{a+bx}}{7x^{7/2}} - \frac{4bF^{a+bx} \log(F)}{35x^{5/2}} - \frac{8b^2F^{a+bx} \log^2(F)}{105x^{3/2}} - \frac{16b^3F^{a+bx} \log^3(F)}{105\sqrt{x}} + \frac{1}{105} (32b^4 \log^4(F)) \text{Subst} \left( \int F^{a+bx^2} dx \right) \\
 &= -\frac{2F^{a+bx}}{7x^{7/2}} - \frac{4bF^{a+bx} \log(F)}{35x^{5/2}} - \frac{8b^2F^{a+bx} \log^2(F)}{105x^{3/2}} - \frac{16b^3F^{a+bx} \log^3(F)}{105\sqrt{x}} + \frac{16}{105} b^{7/2} F^a \sqrt{\pi} \text{erfi} \left( \sqrt{b} \sqrt{x} \sqrt{\log(F)} \right)
 \end{aligned}$$

**Mathematica [A]** time = 0.0620532, size = 73, normalized size = 0.59

$$\frac{2F^a \left( 8(-bx \log(F))^{7/2} \text{Gamma} \left( \frac{1}{2}, -bx \log(F) \right) + F^{bx} (8b^3 x^3 \log^3(F) + 4b^2 x^2 \log^2(F) + 6bx \log(F) + 15) \right)}{105x^{7/2}}$$

Antiderivative was successfully verified.

[In] Integrate[F^(a + b\*x)/x^(9/2), x]

[Out] (-2\*F^a\*(8\*Gamma[1/2, -(b\*x\*Log[F])]\*(-(b\*x\*Log[F]))^(7/2) + F^(b\*x)\*(15 + 6\*b\*x\*Log[F] + 4\*b^2\*x^2\*Log[F]^2 + 8\*b^3\*x^3\*Log[F]^3)))/(105\*x^(7/2))

**Maple [A]** time = 0.013, size = 96, normalized size = 0.8

$$-\frac{F^a}{b} (-b)^{\frac{9}{2}} (\ln(F))^{\frac{7}{2}} \left( -\frac{2e^{b \ln(F)x}}{7} \left( \frac{8b^3 x^3 (\ln(F))^3}{15} + \frac{4b^2 x^2 (\ln(F))^2}{15} + \frac{2b \ln(F)x}{5} + 1 \right) x^{-\frac{7}{2}} (-b)^{-\frac{7}{2}} (\ln(F))^{-\frac{7}{2}} + \frac{16\sqrt{\pi}}{105} b^{\frac{7}{2}} e^{\dots} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(F^(b*x+a)/x^(9/2),x)`

[Out]  $-F^a(-b)^{(9/2)} \ln(F)^{(7/2)} / b * (-2/7/x^{(7/2)} / (-b)^{(7/2)} / \ln(F)^{(7/2)} * (8/15 * b^3 * x^3 * \ln(F)^3 + 4/15 * b^2 * x^2 * \ln(F)^2 + 2/5 * b * \ln(F) * x + 1) * \exp(b * \ln(F) * x) + 16/105 / (-b)^{(7/2)} * b^{(7/2)} * \text{Pi}^{(1/2)} * \text{erfi}(b^{(1/2)} * x^{(1/2)} * \ln(F)^{(1/2)})$

**Maxima [A]** time = 1.29661, size = 32, normalized size = 0.26

$$-\frac{(-bx \log(F))^{\frac{7}{2}} F^a \Gamma\left(-\frac{7}{2}, -bx \log(F)\right)}{x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(b*x+a)/x^(9/2),x, algorithm="maxima")`

[Out]  $-(-b*x*\log(F))^{(7/2)} * F^a * \text{gamma}(-7/2, -b*x*\log(F)) / x^{(7/2)}$

**Fricas [A]** time = 1.48007, size = 236, normalized size = 1.92

$$\frac{2 \left( 8 \sqrt{\pi} \sqrt{-b \log(F)} F^a b^3 x^4 \operatorname{erf}\left(\sqrt{-b \log(F)} \sqrt{x}\right) \log(F)^3 + \left( 8 b^3 x^3 \log(F)^3 + 4 b^2 x^2 \log(F)^2 + 6 b x \log(F) + 15 \right) F^{bx+a} \right)}{105 x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(b*x+a)/x^(9/2),x, algorithm="fricas")`

[Out]  $-2/105 * (8 * \text{sqrt}(\text{pi}) * \text{sqrt}(-b * \log(F)) * F^a * b^3 * x^4 * \text{erf}(\text{sqrt}(-b * \log(F)) * \text{sqrt}(x)) * \log(F)^3 + (8 * b^3 * x^3 * \log(F)^3 + 4 * b^2 * x^2 * \log(F)^2 + 6 * b * x * \log(F) + 15) * F^{(b*x + a)} * \text{sqrt}(x)) / x^4$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(b*x+a)/x**(9/2),x)
```

```
[Out] Timed out
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{bx+a}}{x^{\frac{9}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(b*x+a)/x^(9/2),x, algorithm="giac")
```

```
[Out] integrate(F^(b*x + a)/x^(9/2), x)
```



### 3.39 $\int F^{c(a+bx)}(d+ex)^{7/2} dx$

**Optimal.** Leaf size=208

$$\frac{105\sqrt{\pi}e^{7/2}F^{c\left(\frac{a-bd}{e}\right)}\operatorname{Erfi}\left(\frac{\sqrt{b}\sqrt{c}\sqrt{\log(F)}\sqrt{d+ex}}{\sqrt{e}}\right)}{16b^{9/2}c^{9/2}\log^2(F)} + \frac{35e^2(d+ex)^{3/2}F^{c(a+bx)}}{4b^3c^3\log^3(F)} - \frac{105e^3\sqrt{d+ex}F^{c(a+bx)}}{8b^4c^4\log^4(F)} - \frac{7e(d+ex)^{5/2}F^{c(a+bx)}}{2b^2c^2\log^2(F)} + \frac{(d+ex)^{7/2}F^{c(a+bx)}}{b^2c^2\log^2(F)}$$

```
[Out] (105*e^(7/2)*F^(c*(a - (b*d)/e))*Sqrt[Pi]*Erfi[(Sqrt[b]*Sqrt[c]*Sqrt[d + e*x]*Sqrt[Log[F]])/Sqrt[e]]/(16*b^(9/2)*c^(9/2)*Log[F]^(9/2)) - (105*e^3*F^(c*(a + b*x))*Sqrt[d + e*x])/(8*b^4*c^4*Log[F]^4) + (35*e^2*F^(c*(a + b*x))*(d + e*x)^(3/2))/(4*b^3*c^3*Log[F]^3) - (7*e*F^(c*(a + b*x))*(d + e*x)^(5/2))/(2*b^2*c^2*Log[F]^2) + (F^(c*(a + b*x))*(d + e*x)^(7/2))/(b*c*Log[F])
```

**Rubi [A]** time = 0.266096, antiderivative size = 208, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$ , Rules used = {2176, 2180, 2204}

$$\frac{105\sqrt{\pi}e^{7/2}F^{c\left(\frac{a-bd}{e}\right)}\operatorname{Erfi}\left(\frac{\sqrt{b}\sqrt{c}\sqrt{\log(F)}\sqrt{d+ex}}{\sqrt{e}}\right)}{16b^{9/2}c^{9/2}\log^2(F)} + \frac{35e^2(d+ex)^{3/2}F^{c(a+bx)}}{4b^3c^3\log^3(F)} - \frac{105e^3\sqrt{d+ex}F^{c(a+bx)}}{8b^4c^4\log^4(F)} - \frac{7e(d+ex)^{5/2}F^{c(a+bx)}}{2b^2c^2\log^2(F)} + \frac{(d+ex)^{7/2}F^{c(a+bx)}}{b^2c^2\log^2(F)}$$

Antiderivative was successfully verified.

```
[In] Int[F^(c*(a + b*x))*(d + e*x)^(7/2), x]
```

```
[Out] (105*e^(7/2)*F^(c*(a - (b*d)/e))*Sqrt[Pi]*Erfi[(Sqrt[b]*Sqrt[c]*Sqrt[d + e*x]*Sqrt[Log[F]])/Sqrt[e]]/(16*b^(9/2)*c^(9/2)*Log[F]^(9/2)) - (105*e^3*F^(c*(a + b*x))*Sqrt[d + e*x])/(8*b^4*c^4*Log[F]^4) + (35*e^2*F^(c*(a + b*x))*(d + e*x)^(3/2))/(4*b^3*c^3*Log[F]^3) - (7*e*F^(c*(a + b*x))*(d + e*x)^(5/2))/(2*b^2*c^2*Log[F]^2) + (F^(c*(a + b*x))*(d + e*x)^(7/2))/(b*c*Log[F])
```

#### Rule 2176

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[((c + d*x)^m*(b*F^(g*(e + f*x)))^n)/(f*g*n*Log[F]), x] - Dist[(d*m)/(f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m] && !$UseGamma === True
```

#### Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !$UseGamma === True
```

### Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqrt[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]
```

### Rubi steps

$$\begin{aligned}
\int F^{c(ax)}(d+ex)^{7/2} dx &= \frac{F^{c(ax)}(d+ex)^{7/2}}{bc \log(F)} - \frac{(7e) \int F^{c(ax)}(d+ex)^{5/2} dx}{2bc \log(F)} \\
&= -\frac{7eF^{c(ax)}(d+ex)^{5/2}}{2b^2c^2 \log^2(F)} + \frac{F^{c(ax)}(d+ex)^{7/2}}{bc \log(F)} + \frac{(35e^2) \int F^{c(ax)}(d+ex)^{3/2} dx}{4b^2c^2 \log^2(F)} \\
&= \frac{35e^2F^{c(ax)}(d+ex)^{3/2}}{4b^3c^3 \log^3(F)} - \frac{7eF^{c(ax)}(d+ex)^{5/2}}{2b^2c^2 \log^2(F)} + \frac{F^{c(ax)}(d+ex)^{7/2}}{bc \log(F)} - \frac{(105e^3) \int F^{c(ax)}\sqrt{d+ex} dx}{8b^3c^3 \log^3(F)} \\
&= -\frac{105e^3F^{c(ax)}\sqrt{d+ex}}{8b^4c^4 \log^4(F)} + \frac{35e^2F^{c(ax)}(d+ex)^{3/2}}{4b^3c^3 \log^3(F)} - \frac{7eF^{c(ax)}(d+ex)^{5/2}}{2b^2c^2 \log^2(F)} + \frac{F^{c(ax)}(d+ex)^{7/2}}{bc \log(F)} + \dots \\
&= -\frac{105e^3F^{c(ax)}\sqrt{d+ex}}{8b^4c^4 \log^4(F)} + \frac{35e^2F^{c(ax)}(d+ex)^{3/2}}{4b^3c^3 \log^3(F)} - \frac{7eF^{c(ax)}(d+ex)^{5/2}}{2b^2c^2 \log^2(F)} + \frac{F^{c(ax)}(d+ex)^{7/2}}{bc \log(F)} + \dots \\
&= \frac{105e^{7/2}F^{c\left(a-\frac{bd}{e}\right)}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{b}\sqrt{c}\sqrt{d+ex}\sqrt{\log(F)}}{\sqrt{e}}\right)}{16b^{9/2}c^{9/2}\log^2(F)} - \frac{105e^3F^{c(ax)}\sqrt{d+ex}}{8b^4c^4 \log^4(F)} + \frac{35e^2F^{c(ax)}(d+ex)^{3/2}}{4b^3c^3 \log^3(F)} - \dots
\end{aligned}$$

**Mathematica [A]** time = 0.0296248, size = 72, normalized size = 0.35

$$\frac{e^4 F^{c\left(a-\frac{bd}{e}\right)} \sqrt{-\frac{bc \log(F)(d+ex)}{e}} \operatorname{Gamma}\left(\frac{9}{2}, -\frac{bc \log(F)(d+ex)}{e}\right)}{b^5 c^5 \log^5(F) \sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Integrate[F^(c*(a + b*x))*(d + e*x)^(7/2), x]
```

[Out]  $(e^4 F^{c(a - (b*d)/e)}) \Gamma[9/2, -((b*c*(d + e*x)*\text{Log}[F])/e)] \sqrt{-((b*c*(d + e*x)*\text{Log}[F])/e)} / (b^5 c^5 \sqrt{d + e*x} \text{Log}[F]^5)$

**Maple [F]** time = 0.02, size = 0, normalized size = 0.

$$\int F^{c(bx+a)} (ex + d)^{\frac{7}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(F^(c*(b*x+a))*(e*x+d)^(7/2),x)`

[Out] `int(F^(c*(b*x+a))*(e*x+d)^(7/2),x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (ex + d)^{\frac{7}{2}} F^{(bx+a)c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(c*(b*x+a))*(e*x+d)^(7/2),x, algorithm="maxima")`

[Out] `integrate((e*x + d)^(7/2)*F^((b*x + a)*c), x)`

**Fricas [A]** time = 1.55591, size = 516, normalized size = 2.48

$$\frac{105 \sqrt{\pi} \sqrt{-\frac{bc \log(F)}{e}} e^4 \operatorname{erf}\left(\sqrt{ex+d} \sqrt{-\frac{bc \log(F)}{e}}\right)}{\frac{bcd-ace}{F^e}} + 2 \left(105 bce^3 \log(F) - 8 \left(b^4 c^4 e^3 x^3 + 3 b^4 c^4 d e^2 x^2 + 3 b^4 c^4 d^2 e x + b^4 c^4 d^3\right) \log(F)\right)^4 + \frac{16 b^5 c^5 \log(F)^5}{16 b^5 c^5 \log(F)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(c*(b*x+a))*(e*x+d)^(7/2),x, algorithm="fricas")`

[Out]  $-1/16 * (105 * \sqrt{\pi}) * \sqrt{-b*c*\log(F)/e} * e^4 * \operatorname{erf}(\sqrt{e*x + d} * \sqrt{-b*c*\log(F)/e}) / F^{(b*c*d - a*c*e)/e} + 2 * (105 * b*c*e^3 * \log(F) - 8 * (b^4 * c^4 * e^3 * x^3 + 3 * b^4 * c^4 * d * e^2 * x^2 + 3 * b^4 * c^4 * d^2 * e * x + b^4 * c^4 * d^3) * \log(F) - 8 * (b^4 * c^4 * e^3 * x^3 + 3 * b^4 * c^4 * d * e^2 * x^2 + 3 * b^4 * c^4 * d^2 * e * x + b^4 * c^4 * d^3) * \log(F)) / (16 * b^5 * c^5 * \log(F)^5)$

$$+ 3*b^4*c^4*d*e^2*x^2 + 3*b^4*c^4*d^2*e*x + b^4*c^4*d^3)*\log(F)^4 + 28*(b^3*c^3*e^3*x^2 + 2*b^3*c^3*d*e^2*x + b^3*c^3*d^2*e)*\log(F)^3 - 70*(b^2*c^2*e^3*x + b^2*c^2*d*e^2)*\log(F)^2*\sqrt{e*x + d}*F^{(b*c*x + a*c)}/(b^5*c^5*\log(F)^5)$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(c\*(b\*x+a))\*(e\*x+d)\*\*(7/2),x)

[Out] Timed out

**Giac [B]** time = 1.37808, size = 1301, normalized size = 6.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(c\*(b\*x+a))\*(e\*x+d)^(7/2),x, algorithm="giac")

[Out] 
$$\frac{1}{16}*(8*d^3*(\sqrt{\pi})*\operatorname{erf}(-\sqrt{-b*c*e*\log(F)})*\sqrt{x*e + d}*e^{-1})*e^{-(b*c*d*\log(F) - a*c*e*\log(F))*e^{-1} + 2}/(\sqrt{-b*c*e*\log(F)}*b*c*\log(F)) + 2*\sqrt{x*e + d}*e^{(((x*e + d)*b*c*\log(F) - b*c*d*\log(F) + a*c*e*\log(F))*e^{-1} + 1)/(b*c*\log(F))} - 12*d^2*(\sqrt{\pi})*(2*b*c*d*e*\log(F) + 3*e^2)*\operatorname{erf}(-\sqrt{-b*c*e*\log(F)})*\sqrt{x*e + d}*e^{-1})*e^{-(b*c*d*\log(F) - a*c*e*\log(F))*e^{-1} + 1}/(\sqrt{-b*c*e*\log(F)}*b^2*c^2*\log(F)^2) - 2*(2*(x*e + d)^{(3/2)}*b*c*e*\log(F) - 2*\sqrt{x*e + d}*b*c*d*e*\log(F) - 3*\sqrt{x*e + d}*e^2)*e^{(((x*e + d)*b*c*\log(F) - b*c*d*\log(F) + a*c*e*\log(F))*e^{-1})}/(b^2*c^2*\log(F)^2) + 6*d*(\sqrt{\pi})*(4*b^2*c^2*d^2*e*\log(F)^2 + 12*b*c*d*e^2*\log(F) + 15*e^3)*\operatorname{erf}(-\sqrt{-b*c*e*\log(F)})*\sqrt{x*e + d}*e^{-1})*e^{-(b*c*d*\log(F) - a*c*e*\log(F) + 2*e)*e^{-1} + 1}/(\sqrt{-b*c*e*\log(F)}*b^3*c^3*\log(F)^3) + 2*(4*(x*e + d)^{(5/2)}*b^2*c^2*e*\log(F)^2 - 8*(x*e + d)^{(3/2)}*b^2*c^2*d*e*\log(F)^2 + 4*\sqrt{x*e + d}*b^2*c^2*d^2*e*\log(F)^2 - 10*(x*e + d)^{(3/2)}*b*c*e^2*\log(F) + 12*\sqrt{x*e + d}*b*c*d*e^2*\log(F) + 15*\sqrt{x*e + d}*e^3)*e^{(((x*e + d)*b*c*\log(F) - b*c*d*\log(F) + a*c*e*\log(F) - 2*e)*e^{-1})}/(b^3*c^3*\log(F)^3)*e^2 - (\sqrt{\pi})*(8*b^3*c^3*d^3*e*\log(F)^3 + 36*b^2*c^2*d^2*e^2*\log(F)^2 + 90$$

$$\begin{aligned}
& *b*c*d*e^3*\log(F) + 105*e^4*\operatorname{erf}(-\sqrt{-b*c*e*\log(F)})*\sqrt{x*e + d}*e^{-1}) \\
& *e^{-(b*c*d*\log(F) - a*c*e*\log(F) + 3*e)*e^{-1} + 1}/(\sqrt{-b*c*e*\log(F)})*b \\
& ^4*c^4*\log(F)^4 - 2*(8*(x*e + d)^{(7/2)}*b^3*c^3*e*\log(F)^3 - 24*(x*e + d)^{(5/2)}*b^3*c^3*d*e*\log(F)^3 + 24*(x*e + d)^{(3/2)}*b^3*c^3*d^2*e*\log(F)^3 - 8*\operatorname{sqrt}(x*e + d)*b^3*c^3*d^3*e*\log(F)^3 - 28*(x*e + d)^{(5/2)}*b^2*c^2*e^2*\log(F)^2 + 60*(x*e + d)^{(3/2)}*b^2*c^2*d*e^2*\log(F)^2 - 36*\operatorname{sqrt}(x*e + d)*b^2*c^2*d^2*e^2*\log(F)^2 + 70*(x*e + d)^{(3/2)}*b*c*e^3*\log(F) - 90*\operatorname{sqrt}(x*e + d)*b*c*d*e^3*\log(F) - 105*\operatorname{sqrt}(x*e + d)*e^4)*e^{((x*e + d)*b*c*\log(F) - b*c*d*\log(F) + a*c*e*\log(F) - 3*e)*e^{-1}}/(b^4*c^4*\log(F)^4)*e^3)*e^{-1}
\end{aligned}$$

### 3.40 $\int F^{c(a+bx)}(d+ex)^{5/2} dx$

**Optimal.** Leaf size=173

$$\frac{15\sqrt{\pi}e^{5/2}F^{c\left(a-\frac{bd}{e}\right)}\operatorname{Erfi}\left(\frac{\sqrt{b}\sqrt{c}\sqrt{\log(F)}\sqrt{d+ex}}{\sqrt{e}}\right)}{8b^{7/2}c^{7/2}\log^2(F)} + \frac{15e^2\sqrt{d+ex}F^{c(a+bx)}}{4b^3c^3\log^3(F)} - \frac{5e(d+ex)^{3/2}F^{c(a+bx)}}{2b^2c^2\log^2(F)} + \frac{(d+ex)^{5/2}F^{c(a+bx)}}{bc\log(F)}$$

[Out]  $(-15e^{(5/2)}F^{(c*(a - (b*d)/e)})\sqrt{\pi}\operatorname{Erfi}[(\sqrt{b}\sqrt{c}\sqrt{\log(F)}\sqrt{d + e*x})\sqrt{\log(F)}]/\sqrt{e}]/(8*b^{(7/2)}*c^{(7/2)}*\log(F)^{(7/2)}) + (15*e^2*F^{(c*(a + b*x))}\sqrt{d + e*x})/(4*b^3*c^3*\log(F)^3) - (5*e*F^{(c*(a + b*x))}*(d + e*x)^{(3/2)})/(2*b^2*c^2*\log(F)^2) + (F^{(c*(a + b*x))}*(d + e*x)^{(5/2)})/(b*c*\log(F))$

**Rubi [A]** time = 0.165517, antiderivative size = 173, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$ , Rules used = {2176, 2180, 2204}

$$\frac{15\sqrt{\pi}e^{5/2}F^{c\left(a-\frac{bd}{e}\right)}\operatorname{Erfi}\left(\frac{\sqrt{b}\sqrt{c}\sqrt{\log(F)}\sqrt{d+ex}}{\sqrt{e}}\right)}{8b^{7/2}c^{7/2}\log^2(F)} + \frac{15e^2\sqrt{d+ex}F^{c(a+bx)}}{4b^3c^3\log^3(F)} - \frac{5e(d+ex)^{3/2}F^{c(a+bx)}}{2b^2c^2\log^2(F)} + \frac{(d+ex)^{5/2}F^{c(a+bx)}}{bc\log(F)}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[F^{(c*(a + b*x))}*(d + e*x)^{(5/2)}, x]$

[Out]  $(-15e^{(5/2)}F^{(c*(a - (b*d)/e)})\sqrt{\pi}\operatorname{Erfi}[(\sqrt{b}\sqrt{c}\sqrt{\log(F)}\sqrt{d + e*x})\sqrt{\log(F)}]/\sqrt{e}]/(8*b^{(7/2)}*c^{(7/2)}*\log(F)^{(7/2)}) + (15*e^2*F^{(c*(a + b*x))}\sqrt{d + e*x})/(4*b^3*c^3*\log(F)^3) - (5*e*F^{(c*(a + b*x))}*(d + e*x)^{(3/2)})/(2*b^2*c^2*\log(F)^2) + (F^{(c*(a + b*x))}*(d + e*x)^{(5/2)})/(b*c*\log(F))$

#### Rule 2176

$\operatorname{Int}[(b_.)(F_.)^{(g_.)((e_.) + (f_.)(x_))})^{(n_.)((c_.) + (d_.)(x_))^{(m_.)}, x\_Symbol] \rightarrow \operatorname{Simp}[(c + d*x)^m*(b*F^{(g*(e + f*x))})^n/(f*g*n*\log(F)), x] - \operatorname{Dist}[(d*m)/(f*g*n*\log(F)), \operatorname{Int}[(c + d*x)^{(m-1)}*(b*F^{(g*(e + f*x))})^n, x], x] /; \operatorname{FreeQ}\{F, b, c, d, e, f, g, n\}, x] \&\& \operatorname{GtQ}[m, 0] \&\& \operatorname{IntegerQ}[2*m] \&\& !\$UseGamma === \operatorname{True}$

#### Rule 2180

```
Int[(F_)^((g_.)*(e_.) + (f_.)*(x_))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !$UseGamma === True
```

### Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_)^2), x_Symbol] :> Simp[(F^a*Sqrt[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]
```

### Rubi steps

$$\begin{aligned}
\int F^{c(ax+b)}(d+ex)^{5/2} dx &= \frac{F^{c(ax+b)}(d+ex)^{5/2}}{bc \log(F)} - \frac{(5e) \int F^{c(ax+b)}(d+ex)^{3/2} dx}{2bc \log(F)} \\
&= -\frac{5eF^{c(ax+b)}(d+ex)^{3/2}}{2b^2c^2 \log^2(F)} + \frac{F^{c(ax+b)}(d+ex)^{5/2}}{bc \log(F)} + \frac{(15e^2) \int F^{c(ax+b)}\sqrt{d+ex} dx}{4b^2c^2 \log^2(F)} \\
&= \frac{15e^2F^{c(ax+b)}\sqrt{d+ex}}{4b^3c^3 \log^3(F)} - \frac{5eF^{c(ax+b)}(d+ex)^{3/2}}{2b^2c^2 \log^2(F)} + \frac{F^{c(ax+b)}(d+ex)^{5/2}}{bc \log(F)} - \frac{(15e^3) \int \frac{F^{c(ax+b)}}{\sqrt{d+ex}} dx}{8b^3c^3 \log^3(F)} \\
&= \frac{15e^2F^{c(ax+b)}\sqrt{d+ex}}{4b^3c^3 \log^3(F)} - \frac{5eF^{c(ax+b)}(d+ex)^{3/2}}{2b^2c^2 \log^2(F)} + \frac{F^{c(ax+b)}(d+ex)^{5/2}}{bc \log(F)} - \frac{(15e^2) \text{Subst}\left(\int F^{c\left(a-\frac{bd}{e}\right)} dx\right)}{4b^3c^3 \log^3(F)} \\
&= -\frac{15e^{5/2}F^{c\left(a-\frac{bd}{e}\right)}\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{b}\sqrt{c}\sqrt{d+ex}\sqrt{\log(F)}}{\sqrt{e}}\right)}{8b^{7/2}c^{7/2} \log^2(F)} + \frac{15e^2F^{c(ax+b)}\sqrt{d+ex}}{4b^3c^3 \log^3(F)} - \frac{5eF^{c(ax+b)}(d+ex)^{3/2}}{2b^2c^2 \log^2(F)} + \frac{F^{c(ax+b)}(d+ex)^{5/2}}{bc \log(F)}
\end{aligned}$$

**Mathematica [A]** time = 0.0266816, size = 72, normalized size = 0.42

$$\frac{e^2\sqrt{d+ex}F^{c\left(a-\frac{bd}{e}\right)}\Gamma\left(\frac{7}{2}, -\frac{bc \log(F)(d+ex)}{e}\right)}{b^3c^3 \log^3(F)\sqrt{-\frac{bc \log(F)(d+ex)}{e}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[F^(c*(a + b*x))*(d + e*x)^(5/2), x]
```

```
[Out] (e^2*F^(c*(a - (b*d)/e))*Sqrt[d + e*x]*Gamma[7/2, -((b*c*(d + e*x)*Log[F])/e)])/
(b^3*c^3*Log[F]^3*Sqrt[-((b*c*(d + e*x)*Log[F])/e)])
```

---

**Maple [F]** time = 0.019, size = 0, normalized size = 0.

$$\int F^{c(bx+a)} (ex + d)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(c\*(b\*x+a))\*(e\*x+d)^(5/2), x)

[Out] int(F^(c\*(b\*x+a))\*(e\*x+d)^(5/2), x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (ex + d)^{\frac{5}{2}} F^{(bx+a)c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(c\*(b\*x+a))\*(e\*x+d)^(5/2), x, algorithm="maxima")

[Out] integrate((e\*x + d)^(5/2)\*F^((b\*x + a)\*c), x)

---

**Fricas [A]** time = 1.54212, size = 383, normalized size = 2.21

$$\frac{15\sqrt{\pi}\sqrt{-\frac{bc\log(F)}{e}}e^3\operatorname{erf}\left(\sqrt{ex+d}\sqrt{-\frac{bc\log(F)}{e}}\right)}{\frac{bcd-ace}{F\frac{e}{e}}} + 2\left(15bce^2\log(F) + 4\left(b^3c^3e^2x^2 + 2b^3c^3dex + b^3c^3d^2\right)\log(F)^3 - 10\left(b^2c^2e^2x + b^2c^2d\right)\log(F)^2\right)\sqrt{ex+d}F^{(b*c*x+a*c)}/(b^4c^4\log(F)^4)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(c\*(b\*x+a))\*(e\*x+d)^(5/2), x, algorithm="fricas")

[Out] 1/8\*(15\*sqrt(pi)\*sqrt(-b\*c\*log(F)/e)\*e^3\*erf(sqrt(e\*x + d)\*sqrt(-b\*c\*log(F)/e))/F^((b\*c\*d - a\*c\*e)/e) + 2\*(15\*b\*c\*e^2\*log(F) + 4\*(b^3\*c^3\*e^2\*x^2 + 2\*b^3\*c^3\*d\*e\*x + b^3\*c^3\*d^2)\*log(F)^3 - 10\*(b^2\*c^2\*e^2\*x + b^2\*c^2\*d\*e)\*log(F)^2)\*sqrt(e\*x + d)\*F^(b\*c\*x + a\*c)/(b^4\*c^4\*log(F)^4)



---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(c\*(b\*x+a))\*(e\*x+d)\*\*(5/2),x)

[Out] Timed out

---

**Giac [B]** time = 1.33364, size = 779, normalized size = 4.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(c\*(b\*x+a))\*(e\*x+d)^(5/2),x, algorithm="giac")

[Out] 
$$\frac{1}{8} \cdot (4d^2 \cdot (\sqrt{\pi}) \cdot \operatorname{erf}(-\sqrt{-bce \log(F)}) \cdot \sqrt{xe+d} \cdot e^{-1}) \cdot e^{-(bc \log(F) - ace \log(F)) \cdot e^{-1} + 2} / (\sqrt{-bce \log(F)} \cdot bc \log(F)) + 2 \cdot \sqrt{xe+d} \cdot e^{((xe+d) \cdot bc \log(F) - bcd \log(F) + ace \log(F)) \cdot e^{-1} + 1} / (bc \log(F))) - 4d \cdot (\sqrt{\pi}) \cdot (2bce \log(F) + 3e^2) \cdot \operatorname{erf}(-\sqrt{-bce \log(F)}) \cdot \sqrt{xe+d} \cdot e^{-1}) \cdot e^{-(bcd \log(F) - ace \log(F)) \cdot e^{-1} + 1} / (\sqrt{-bce \log(F)} \cdot b^2 c^2 \log(F)^2) - 2 \cdot (2 \cdot (xe+d)^{3/2} \cdot bce \log(F) - 2 \cdot \sqrt{xe+d} \cdot bcd \log(F) - 3 \cdot \sqrt{xe+d} \cdot e^2) \cdot e^{((xe+d) \cdot bc \log(F) - bcd \log(F) + ace \log(F)) \cdot e^{-1}} / (b^2 c^2 \log(F)^2) + (\sqrt{\pi}) \cdot (4b^2 c^2 d^2 e \log(F)^2 + 12bce^2 \log(F) + 15e^3) \cdot \operatorname{erf}(-\sqrt{-bce \log(F)}) \cdot \sqrt{xe+d} \cdot e^{-1}) \cdot e^{-(bcd \log(F) - ace \log(F) + 2e) \cdot e^{-1} + 1} / (\sqrt{-bce \log(F)} \cdot b^3 c^3 \log(F)^3) + 2 \cdot (4 \cdot (xe+d)^{5/2} \cdot b^2 c^2 e \log(F)^2 - 8 \cdot (xe+d)^{3/2} \cdot b^2 c^2 d e \log(F)^2 + 4 \cdot \sqrt{xe+d} \cdot b^2 c^2 d^2 e \log(F)^2 - 10 \cdot (xe+d)^{3/2} \cdot bce^2 \log(F) + 12 \cdot \sqrt{xe+d} \cdot bcd e^2 \log(F) + 15 \cdot \sqrt{xe+d} \cdot e^3) \cdot e^{((xe+d) \cdot bc \log(F) - bcd \log(F) + ace \log(F) - 2e) \cdot e^{-1}} / (b^3 c^3 \log(F)^3)) \cdot e^2 \cdot e^{-1}$$

### 3.41 $\int F^{c(a+bx)}(d+ex)^{3/2} dx$

**Optimal.** Leaf size=138

$$\frac{3\sqrt{\pi}e^{3/2}F^{c\left(a-\frac{bd}{e}\right)}\operatorname{Erfi}\left(\frac{\sqrt{b}\sqrt{c}\sqrt{\log(F)}\sqrt{d+ex}}{\sqrt{e}}\right)}{4b^{5/2}c^{5/2}\log^2(F)} - \frac{3e\sqrt{d+ex}F^{c(a+bx)}}{2b^2c^2\log^2(F)} + \frac{(d+ex)^{3/2}F^{c(a+bx)}}{bc\log(F)}$$

[Out] (3\*e^(3/2)\*F^(c\*(a - (b\*d)/e))\*Sqrt[Pi]\*Erfi[(Sqrt[b]\*Sqrt[c]\*Sqrt[d + e\*x]\*Sqrt[Log[F]])/Sqrt[e]]/(4\*b^(5/2)\*c^(5/2)\*Log[F]^(5/2)) - (3\*e\*F^(c\*(a + b\*x))\*Sqrt[d + e\*x])/(2\*b^2\*c^2\*Log[F]^2) + (F^(c\*(a + b\*x))\*(d + e\*x)^(3/2))/(b\*c\*Log[F])

**Rubi [A]** time = 0.11937, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$ , Rules used = {2176, 2180, 2204}

$$\frac{3\sqrt{\pi}e^{3/2}F^{c\left(a-\frac{bd}{e}\right)}\operatorname{Erfi}\left(\frac{\sqrt{b}\sqrt{c}\sqrt{\log(F)}\sqrt{d+ex}}{\sqrt{e}}\right)}{4b^{5/2}c^{5/2}\log^2(F)} - \frac{3e\sqrt{d+ex}F^{c(a+bx)}}{2b^2c^2\log^2(F)} + \frac{(d+ex)^{3/2}F^{c(a+bx)}}{bc\log(F)}$$

Antiderivative was successfully verified.

[In] Int[F^(c\*(a + b\*x))\*(d + e\*x)^(3/2), x]

[Out] (3\*e^(3/2)\*F^(c\*(a - (b\*d)/e))\*Sqrt[Pi]\*Erfi[(Sqrt[b]\*Sqrt[c]\*Sqrt[d + e\*x]\*Sqrt[Log[F]])/Sqrt[e]]/(4\*b^(5/2)\*c^(5/2)\*Log[F]^(5/2)) - (3\*e\*F^(c\*(a + b\*x))\*Sqrt[d + e\*x])/(2\*b^2\*c^2\*Log[F]^2) + (F^(c\*(a + b\*x))\*(d + e\*x)^(3/2))/(b\*c\*Log[F])

#### Rule 2176

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[((c + d*x)^m*(b*F^(g*(e + f*x)))^n)/(f*g*n*Log[F]), x] - Dist[(d*m)/(f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m] && !$UseGamma === True
```

#### Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
```

x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !\$UseGamma === True

### Rule 2204

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))<sup>2</sup>), x\_Symbol] := Simp[(F^a\*Sqrt[Pi]\*Erfi[(c + d\*x)\*Rt[b\*Log[F], 2]])/(2\*d\*Rt[b\*Log[F], 2]), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]

### Rubi steps

$$\begin{aligned} \int F^{c(ax)}(d+ex)^{3/2} dx &= \frac{F^{c(ax)}(d+ex)^{3/2}}{bc \log(F)} - \frac{(3e) \int F^{c(ax)} \sqrt{d+ex} dx}{2bc \log(F)} \\ &= -\frac{3eF^{c(ax)} \sqrt{d+ex}}{2b^2c^2 \log^2(F)} + \frac{F^{c(ax)}(d+ex)^{3/2}}{bc \log(F)} + \frac{(3e^2) \int \frac{F^{c(ax)}}{\sqrt{d+ex}} dx}{4b^2c^2 \log^2(F)} \\ &= -\frac{3eF^{c(ax)} \sqrt{d+ex}}{2b^2c^2 \log^2(F)} + \frac{F^{c(ax)}(d+ex)^{3/2}}{bc \log(F)} + \frac{(3e) \text{Subst} \left( \int F^{c\left(a-\frac{bd}{e}\right)+\frac{bcx^2}{e}} dx, x, \sqrt{d+ex} \right)}{2b^2c^2 \log^2(F)} \\ &= \frac{3e^{3/2} F^{c\left(a-\frac{bd}{e}\right)} \sqrt{\pi} \operatorname{erfi} \left( \frac{\sqrt{b}\sqrt{c}\sqrt{d+ex}\sqrt{\log(F)}}{\sqrt{e}} \right)}{4b^{5/2}c^{5/2} \log^2(F)} - \frac{3eF^{c(ax)} \sqrt{d+ex}}{2b^2c^2 \log^2(F)} + \frac{F^{c(ax)}(d+ex)^{3/2}}{bc \log(F)} \end{aligned}$$

**Mathematica [A]** time = 0.0700684, size = 63, normalized size = 0.46

$$\frac{(d+ex)^{5/2} F^{c\left(a-\frac{bd}{e}\right)} \operatorname{Gamma} \left( \frac{5}{2}, -\frac{bc \log(F)(d+ex)}{e} \right)}{e \left( -\frac{bc \log(F)(d+ex)}{e} \right)^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[F^(c\*(a + b\*x))\*(d + e\*x)^(3/2), x]

[Out] -((F^(c\*(a - (b\*d)/e))\*(d + e\*x)^(5/2)\*Gamma[5/2, -((b\*c\*(d + e\*x)\*Log[F])/e)])/(e\*(-((b\*c\*(d + e\*x)\*Log[F])/e))^(5/2)))

**Maple [F]** time = 0.02, size = 0, normalized size = 0.

$$\int F^{(bx+a)} (ex+d)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(F^(c*(b*x+a))*(e*x+d)^(3/2),x)`

[Out] `int(F^(c*(b*x+a))*(e*x+d)^(3/2),x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (ex + d)^{\frac{3}{2}} F^{(bx+a)c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(c*(b*x+a))*(e*x+d)^(3/2),x, algorithm="maxima")`

[Out] `integrate((e*x + d)^(3/2)*F^((b*x + a)*c), x)`

**Fricas [A]** time = 1.91564, size = 288, normalized size = 2.09

$$\frac{3 \sqrt{\pi} \sqrt{-\frac{bc \log(F)}{e}} e^2 \operatorname{erf}\left(\sqrt{ex+d} \sqrt{-\frac{bc \log(F)}{e}}\right)}{F \frac{bcd-ace}{e}} + 2 \left(3 bce \log(F) - 2 (b^2 c^2 ex + b^2 c^2 d) \log(F)^2\right) \sqrt{ex+d} F^{bcx+ac}$$


---


$$4 b^3 c^3 \log(F)^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(c*(b*x+a))*(e*x+d)^(3/2),x, algorithm="fricas")`

[Out] `-1/4*(3*sqrt(pi)*sqrt(-b*c*log(F)/e)*e^2*erf(sqrt(e*x + d)*sqrt(-b*c*log(F)/e))/F^((b*c*d - a*c*e)/e) + 2*(3*b*c*e*log(F) - 2*(b^2*c^2*e*x + b^2*c^2*d)*log(F)^2)*sqrt(e*x + d)*F^(b*c*x + a*c))/(b^3*c^3*log(F)^3)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(c\*(b\*x+a))\*(e\*x+d)\*\*(3/2),x)

[Out] Timed out

**Giac [B]** time = 1.19088, size = 408, normalized size = 2.96

$$\frac{1}{4} \left( 2d \left( \frac{\sqrt{\pi} \operatorname{erf}\left(-\sqrt{-bce \log(F)} \sqrt{xe + de^{(-1)}}\right) e^{-(bcd \log(F) - ace \log(F))e^{(-1)} + 2}}{\sqrt{-bce \log(F)} bc \log(F)} \right) + \frac{2 \sqrt{xe + de^{(-1)}} e^{((xe+d)bc \log(F) - bcd \log(F) + ace \log(F))e^{(-1)}}}{bc \log(F)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(c\*(b\*x+a))\*(e\*x+d)^(3/2),x, algorithm="giac")

[Out] 1/4\*(2\*d\*(sqrt(pi)\*erf(-sqrt(-b\*c\*e\*log(F))\*sqrt(x\*e + d)\*e^(-1))\*e^(-(b\*c\*d\*log(F) - a\*c\*e\*log(F))\*e^(-1) + 2)/(sqrt(-b\*c\*e\*log(F))\*b\*c\*log(F)) + 2\*sqrt(x\*e + d)\*e^(((x\*e + d)\*b\*c\*log(F) - b\*c\*d\*log(F) + a\*c\*e\*log(F))\*e^(-1) + 1)/(b\*c\*log(F))) - sqrt(pi)\*(2\*b\*c\*d\*e\*log(F) + 3\*e^2)\*erf(-sqrt(-b\*c\*e\*log(F))\*sqrt(x\*e + d)\*e^(-1))\*e^(-(b\*c\*d\*log(F) - a\*c\*e\*log(F))\*e^(-1) + 1)/(sqrt(-b\*c\*e\*log(F))\*b^2\*c^2\*log(F)^2) + 2\*(2\*(x\*e + d)^(3/2)\*b\*c\*e\*log(F) - 2\*sqrt(x\*e + d)\*b\*c\*d\*e\*log(F) - 3\*sqrt(x\*e + d)\*e^2)\*e^(((x\*e + d)\*b\*c\*log(F) - b\*c\*d\*log(F) + a\*c\*e\*log(F))\*e^(-1))/(b^2\*c^2\*log(F)^2))\*e^(-1)

### 3.42 $\int F^{c(a+bx)} \sqrt{d+ex} dx$

**Optimal.** Leaf size=105

$$\frac{\sqrt{d+ex} F^{c(a+bx)}}{bc \log(F)} - \frac{\sqrt{\pi} \sqrt{e} F^{c\left(a-\frac{bd}{e}\right)} \operatorname{Erfi}\left(\frac{\sqrt{b}\sqrt{c}\sqrt{\log(F)}\sqrt{d+ex}}{\sqrt{e}}\right)}{2b^{3/2}c^{3/2} \log^{\frac{3}{2}}(F)}$$

[Out]  $-(\operatorname{Sqrt}[e]*F^{c*(a-(b*d)/e)}*\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erfi}[(\operatorname{Sqrt}[b]*\operatorname{Sqrt}[c]*\operatorname{Sqrt}[d+e*x])* \operatorname{Sqrt}[\operatorname{Log}[F]]]/\operatorname{Sqrt}[e]])/(2*b^{(3/2)}*c^{(3/2)}*\operatorname{Log}[F]^{(3/2)}) + (F^{c*(a+b*x)})* \operatorname{Sqrt}[d+e*x])/(b*c*\operatorname{Log}[F])$

**Rubi [A]** time = 0.0772365, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$ , Rules used = {2176, 2180, 2204}

$$\frac{\sqrt{d+ex} F^{c(a+bx)}}{bc \log(F)} - \frac{\sqrt{\pi} \sqrt{e} F^{c\left(a-\frac{bd}{e}\right)} \operatorname{Erfi}\left(\frac{\sqrt{b}\sqrt{c}\sqrt{\log(F)}\sqrt{d+ex}}{\sqrt{e}}\right)}{2b^{3/2}c^{3/2} \log^{\frac{3}{2}}(F)}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[F^{c*(a+b*x)}*\operatorname{Sqrt}[d+e*x], x]$

[Out]  $-(\operatorname{Sqrt}[e]*F^{c*(a-(b*d)/e)}*\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erfi}[(\operatorname{Sqrt}[b]*\operatorname{Sqrt}[c]*\operatorname{Sqrt}[d+e*x])* \operatorname{Sqrt}[\operatorname{Log}[F]]]/\operatorname{Sqrt}[e]])/(2*b^{(3/2)}*c^{(3/2)}*\operatorname{Log}[F]^{(3/2)}) + (F^{c*(a+b*x)})* \operatorname{Sqrt}[d+e*x])/(b*c*\operatorname{Log}[F])$

#### Rule 2176

```
Int[((b_.)*(F_)^((g_.)*((e_.)+(f_.)*(x_))))^(n_.)*((c_.)+(d_.)*(x_))^(m
_.), x_Symbol] :> Simp[((c+d*x)^m*(b*F^(g*(e+f*x)))^n)/(f*g*n*Log[F]),
x] - Dist[(d*m)/(f*g*n*Log[F]), Int[(c+d*x)^(m-1)*(b*F^(g*(e+f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m]
] && !$UseGamma === True
```

#### Rule 2180

```
Int[(F_)^((g_.)*((e_.)+(f_.)*(x_)))/Sqrt[(c_.)+(d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e-(c*f)/d)+(f*g*x^2)/d), x], x, Sqrt[c+d*x
```

x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !\$UseGamma === True

### Rule 2204

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))<sup>2</sup>), x\_Symbol] :> Simp[(F^a\*Sqrt[Pi]\*Erfi[(c + d\*x)\*Rt[b\*Log[F], 2]])/(2\*d\*Rt[b\*Log[F], 2]), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]

### Rubi steps

$$\begin{aligned} \int F^{c(a+bx)} \sqrt{d+ex} dx &= \frac{F^{c(a+bx)} \sqrt{d+ex}}{bc \log(F)} - \frac{e \int \frac{F^{c(a+bx)}}{\sqrt{d+ex}} dx}{2bc \log(F)} \\ &= \frac{F^{c(a+bx)} \sqrt{d+ex}}{bc \log(F)} - \frac{\text{Subst} \left( \int F^{c \left( a - \frac{bd}{e} \right) + \frac{bcx^2}{e}} dx, x, \sqrt{d+ex} \right)}{bc \log(F)} \\ &= -\frac{\sqrt{e} F^{c \left( a - \frac{bd}{e} \right)} \sqrt{\pi} \operatorname{erfi} \left( \frac{\sqrt{b} \sqrt{c} \sqrt{d+ex} \sqrt{\log(F)}}{\sqrt{e}} \right)}{2b^{3/2} c^{3/2} \log^2(F)} + \frac{F^{c(a+bx)} \sqrt{d+ex}}{bc \log(F)} \end{aligned}$$

**Mathematica [A]** time = 0.0380728, size = 63, normalized size = 0.6

$$-\frac{(d+ex)^{3/2} F^{c \left( a - \frac{bd}{e} \right)} \Gamma \left( \frac{3}{2}, -\frac{bc \log(F)(d+ex)}{e} \right)}{e \left( -\frac{bc \log(F)(d+ex)}{e} \right)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[F^(c\*(a + b\*x))\*Sqrt[d + e\*x], x]

[Out] -((F^(c\*(a - (b\*d)/e))\*(d + e\*x)^(3/2)\*Gamma[3/2, -((b\*c\*(d + e\*x)\*Log[F])/e)])/(e\*(-((b\*c\*(d + e\*x)\*Log[F])/e))^(3/2)))

**Maple [F]** time = 0.019, size = 0, normalized size = 0.

$$\int F^{c(bx+a)} \sqrt{ex+d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(F^(c*(b*x+a))*(e*x+d)^(1/2),x)`

[Out] `int(F^(c*(b*x+a))*(e*x+d)^(1/2),x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{ex + d} F^{(bx+a)c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(c*(b*x+a))*(e*x+d)^(1/2),x, algorithm="maxima")`

[Out] `integrate(sqrt(e*x + d)*F^((b*x + a)*c), x)`

**Fricas [A]** time = 2.08921, size = 220, normalized size = 2.1

$$\frac{2\sqrt{ex+d}F^{bcx+ac}bc\log(F) + \frac{\sqrt{\pi}\sqrt{-\frac{bc\log(F)}{e}}e\operatorname{erf}\left(\sqrt{ex+d}\sqrt{-\frac{bc\log(F)}{e}}\right)}{F^{\frac{bcd-ace}{e}}}}{2b^2c^2\log(F)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(c*(b*x+a))*(e*x+d)^(1/2),x, algorithm="fricas")`

[Out] `1/2*(2*sqrt(e*x + d)*F^(b*c*x + a*c)*b*c*log(F) + sqrt(pi)*sqrt(-b*c*log(F)/e)*e*erf(sqrt(e*x + d)*sqrt(-b*c*log(F)/e))/F^((b*c*d - a*c*e)/e))/(b^2*c^2*log(F)^2)`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int F^{c(a+bx)}\sqrt{d+ex} dx$$

Verification of antiderivative is not currently implemented for this CAS.



[In] integrate(F\*\*(c\*(b\*x+a))\*(e\*x+d)\*\*(1/2),x)

[Out] Integral(F\*\*(c\*(a + b\*x))\*sqrt(d + e\*x), x)

**Giac [A]** time = 1.30169, size = 170, normalized size = 1.62

$$\frac{1}{2} \left( \frac{\sqrt{\pi} \operatorname{erf}\left(-\sqrt{-bce \log(F)} \sqrt{xe + de^{(-1)}}\right) e^{(-(bcd \log(F) - ace \log(F))e^{(-1)} + 2)}}{\sqrt{-bce \log(F)} bc \log(F)} + \frac{2 \sqrt{xe + de^{(-1)}} e^{((xe+d)bc \log(F) - bcd \log(F) + ace \log(F))e^{(-1)} + 1}}{bc \log(F)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(c\*(b\*x+a))\*(e\*x+d)^(1/2),x, algorithm="giac")

[Out] 1/2\*(sqrt(pi)\*erf(-sqrt(-b\*c\*e\*log(F))\*sqrt(x\*e + d)\*e^(-1))\*e^(-(b\*c\*d\*log(F) - a\*c\*e\*log(F))\*e^(-1) + 2)/(sqrt(-b\*c\*e\*log(F))\*b\*c\*log(F)) + 2\*sqrt(x\*e + d)\*e^(((x\*e + d)\*b\*c\*log(F) - b\*c\*d\*log(F) + a\*c\*e\*log(F))\*e^(-1) + 1)/(b\*c\*log(F))\*e^(-1)

### 3.43 $\int \frac{F^{c(a+bx)}}{\sqrt{d+ex}} dx$

**Optimal.** Leaf size=72

$$\frac{\sqrt{\pi} F^{c\left(a-\frac{bd}{e}\right)} \operatorname{Erfi}\left(\frac{\sqrt{b}\sqrt{c}\sqrt{\log(F)}\sqrt{d+ex}}{\sqrt{e}}\right)}{\sqrt{b}\sqrt{c}\sqrt{e}\sqrt{\log(F)}}$$

[Out] (F^(c\*(a - (b\*d)/e))\*Sqrt[Pi]\*Erfi[(Sqrt[b]\*Sqrt[c]\*Sqrt[d + e\*x]\*Sqrt[Log[F]])/Sqrt[e]])/(Sqrt[b]\*Sqrt[c]\*Sqrt[e]\*Sqrt[Log[F]])

**Rubi [A]** time = 0.0454783, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$ , Rules used = {2180, 2204}

$$\frac{\sqrt{\pi} F^{c\left(a-\frac{bd}{e}\right)} \operatorname{Erfi}\left(\frac{\sqrt{b}\sqrt{c}\sqrt{\log(F)}\sqrt{d+ex}}{\sqrt{e}}\right)}{\sqrt{b}\sqrt{c}\sqrt{e}\sqrt{\log(F)}}$$

Antiderivative was successfully verified.

[In] Int[F^(c\*(a + b\*x))/Sqrt[d + e\*x], x]

[Out] (F^(c\*(a - (b\*d)/e))\*Sqrt[Pi]\*Erfi[(Sqrt[b]\*Sqrt[c]\*Sqrt[d + e\*x]\*Sqrt[Log[F]])/Sqrt[e]])/(Sqrt[b]\*Sqrt[c]\*Sqrt[e]\*Sqrt[Log[F]])

#### Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !$UseGamma == True
```

#### Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_)^2)), x_Symbol] := Simp[(F^a*Sqrt[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]
```

#### Rubi steps

$$\int \frac{F^{c(a+bx)}}{\sqrt{d+ex}} dx = \frac{2 \operatorname{Subst} \left( \int F^{c\left(a-\frac{bd}{e}\right) + \frac{bcx^2}{e}} dx, x, \sqrt{d+ex} \right)}{e}$$

$$= \frac{F^{c\left(a-\frac{bd}{e}\right)} \sqrt{\pi} \operatorname{erfi} \left( \frac{\sqrt{b}\sqrt{c}\sqrt{d+ex}\sqrt{\log(F)}}{\sqrt{e}} \right)}{\sqrt{b}\sqrt{c}\sqrt{e}\sqrt{\log(F)}}$$

**Mathematica [A]** time = 0.0279241, size = 63, normalized size = 0.88

$$\frac{\sqrt{d+ex} F^{c\left(a-\frac{bd}{e}\right)} \operatorname{Gamma} \left( \frac{1}{2}, -\frac{bc \log(F)(d+ex)}{e} \right)}{e \sqrt{-\frac{bc \log(F)(d+ex)}{e}}}$$

Antiderivative was successfully verified.

[In] Integrate[F^(c\*(a + b\*x))/Sqrt[d + e\*x], x]

[Out] -((F^(c\*(a - (b\*d)/e))\*Sqrt[d + e\*x]\*Gamma[1/2, -((b\*c\*(d + e\*x)\*Log[F])/e])]/(e\*Sqrt[-((b\*c\*(d + e\*x)\*Log[F])/e])))

**Maple [F]** time = 0.022, size = 0, normalized size = 0.

$$\int F^{c(bx+a)} \frac{1}{\sqrt{ex+d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(c\*(b\*x+a))/(e\*x+d)^(1/2), x)

[Out] int(F^(c\*(b\*x+a))/(e\*x+d)^(1/2), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{(bx+a)c}}{\sqrt{ex+d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(c\*(b\*x+a))/(e\*x+d)^(1/2),x, algorithm="maxima")

[Out] integrate(F^((b\*x + a)\*c)/sqrt(e\*x + d), x)

**Fricas [A]** time = 2.04574, size = 142, normalized size = 1.97

$$-\frac{\sqrt{\pi}\sqrt{-\frac{bc\log(F)}{e}}\operatorname{erf}\left(\sqrt{ex+d}\sqrt{-\frac{bc\log(F)}{e}}\right)}{F^{\frac{bcd-ace}{e}}bc\log(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(c\*(b\*x+a))/(e\*x+d)^(1/2),x, algorithm="fricas")

[Out] -sqrt(pi)\*sqrt(-b\*c\*log(F)/e)\*erf(sqrt(e\*x + d)\*sqrt(-b\*c\*log(F)/e))/(F^((b\*c\*d - a\*c\*e)/e)\*b\*c\*log(F))

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{c(a+bx)}}{\sqrt{d+ex}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(c\*(b\*x+a))/(e\*x+d)\*\*(1/2),x)

[Out] Integral(F\*\*(c\*(a + b\*x))/sqrt(d + e\*x), x)

**Giac [A]** time = 1.25441, size = 78, normalized size = 1.08

$$-\frac{\sqrt{\pi}\operatorname{erf}\left(-\sqrt{-bce\log(F)}\sqrt{xe+de^{(-1)}}\right)e^{-(bcd\log(F)-ace\log(F))e^{(-1)}}}{\sqrt{-bce\log(F)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))/(e*x+d)^(1/2),x, algorithm="giac")
```

```
[Out] -sqrt(pi)*erf(-sqrt(-b*c*e*log(F))*sqrt(x*e + d)*e^(-1))*e^(-(b*c*d*log(F)
- a*c*e*log(F))*e^(-1))/sqrt(-b*c*e*log(F))
```

$$3.44 \quad \int \frac{F^{c(a+bx)}}{(d+ex)^{3/2}} dx$$

**Optimal.** Leaf size=97

$$\frac{2\sqrt{\pi}\sqrt{b}\sqrt{c}\sqrt{\log(F)}F^{c\left(a-\frac{bd}{e}\right)}\operatorname{Erfi}\left(\frac{\sqrt{b}\sqrt{c}\sqrt{\log(F)}\sqrt{d+ex}}{\sqrt{e}}\right)}{e^{3/2}} - \frac{2F^{c(a+bx)}}{e\sqrt{d+ex}}$$

[Out]  $(-2F^{c(a+bx)})/(e\sqrt{d+ex}) + (2\sqrt{b}\sqrt{c}F^{c(a-\frac{bd}{e})})\sqrt{\pi}\operatorname{Erfi}[(\sqrt{b}\sqrt{c}\sqrt{\log(F)}\sqrt{d+ex})/\sqrt{e}]\sqrt{\log(F)}/e^{3/2}$

**Rubi [A]** time = 0.0854085, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$ , Rules used = {2177, 2180, 2204}

$$\frac{2\sqrt{\pi}\sqrt{b}\sqrt{c}\sqrt{\log(F)}F^{c\left(a-\frac{bd}{e}\right)}\operatorname{Erfi}\left(\frac{\sqrt{b}\sqrt{c}\sqrt{\log(F)}\sqrt{d+ex}}{\sqrt{e}}\right)}{e^{3/2}} - \frac{2F^{c(a+bx)}}{e\sqrt{d+ex}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[F^{c(a+bx)}/(d+ex)^{3/2}, x]$

[Out]  $(-2F^{c(a+bx)})/(e\sqrt{d+ex}) + (2\sqrt{b}\sqrt{c}F^{c(a-\frac{bd}{e})})\sqrt{\pi}\operatorname{Erfi}[(\sqrt{b}\sqrt{c}\sqrt{\log(F)}\sqrt{d+ex})/\sqrt{e}]\sqrt{\log(F)}/e^{3/2}$

#### Rule 2177

$\operatorname{Int}[(b_*)(F_*)((g_*)((e_*) + (f_*)(x_))))^{(n_*)}((c_*) + (d_*)(x_))^{(m_*)}, x\_Symbol] \rightarrow \operatorname{Simp}[(c + d*x)^{(m+1)}*(b*F^{(g*(e+f*x))})^n]/(d*(m+1)), x] - \operatorname{Dist}[(f*g*n*\operatorname{Log}[F])/(d*(m+1)), \operatorname{Int}[(c + d*x)^{(m+1)}*(b*F^{(g*(e+f*x))})^n, x], x] /;$  FreeQ[{F, b, c, d, e, f, g, n}, x] && LtQ[m, -1] && IntegerQ[2\*m] && !\$UseGamma == True

#### Rule 2180

$\operatorname{Int}[(F_*)((g_*)((e_*) + (f_*)(x_))))/\sqrt{(c_*) + (d_*)(x_*)}, x\_Symbol] \rightarrow \operatorname{Dist}[2/d, \operatorname{Subst}[\operatorname{Int}[F^{(g*(e-(c*f)/d) + (f*g*x^2)/d)}, x], x, \sqrt{c + d*x}]$

x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !\$UseGamma === True

### Rule 2204

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_)^2), x\_Symbol] := Simp[(F^a\*Sqrt[Pi]\*Erfi[(c + d\*x)\*Rt[b\*Log[F], 2]]]/(2\*d\*Rt[b\*Log[F], 2]), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]

### Rubi steps

$$\begin{aligned} \int \frac{F^{c(ax+b)}}{(d+ex)^{3/2}} dx &= -\frac{2F^{c(ax+b)}}{e\sqrt{d+ex}} + \frac{(2bc \log(F)) \int \frac{F^{c(ax+b)}}{\sqrt{d+ex}} dx}{e} \\ &= -\frac{2F^{c(ax+b)}}{e\sqrt{d+ex}} + \frac{(4bc \log(F)) \text{Subst}\left(\int F^{c\left(a-\frac{bd}{e}\right)+\frac{bcx^2}{e}} dx, x, \sqrt{d+ex}\right)}{e^2} \\ &= -\frac{2F^{c(ax+b)}}{e\sqrt{d+ex}} + \frac{2\sqrt{b}\sqrt{c}F^{c\left(a-\frac{bd}{e}\right)}\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{b}\sqrt{c}\sqrt{d+ex}\sqrt{\log(F)}}{\sqrt{e}}\right)\sqrt{\log(F)}}{e^{3/2}} \end{aligned}$$

**Mathematica [A]** time = 0.0744716, size = 75, normalized size = 0.77

$$\frac{2\left(F^{c(ax+b)} - F^{c\left(a-\frac{bd}{e}\right)}\sqrt{-\frac{bc \log(F)(d+ex)}{e}}\Gamma\left(\frac{1}{2}, -\frac{bc \log(F)(d+ex)}{e}\right)\right)}{e\sqrt{d+ex}}$$

Antiderivative was successfully verified.

[In] Integrate[F^(c\*(a + b\*x))/(d + e\*x)^(3/2), x]

[Out] (-2\*(F^(c\*(a + b\*x)) - F^(c\*(a - (b\*d)/e))\*Gamma[1/2, -((b\*c\*(d + e\*x)\*Log[F])/e)]\*Sqrt[-((b\*c\*(d + e\*x)\*Log[F])/e)])/(e\*Sqrt[d + e\*x])

**Maple [F]** time = 0.02, size = 0, normalized size = 0.

$$\int F^{c(bx+a)} (ex + d)^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(F^(c*(b*x+a))/(e*x+d)^(3/2),x)`

[Out] `int(F^(c*(b*x+a))/(e*x+d)^(3/2),x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{(bx+a)c}}{(ex+d)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(c*(b*x+a))/(e*x+d)^(3/2),x, algorithm="maxima")`

[Out] `integrate(F^((b*x + a)*c)/(e*x + d)^(3/2), x)`

**Fricas [A]** time = 1.83572, size = 205, normalized size = 2.11

$$\frac{2 \left( \frac{\sqrt{\pi}(ex+d)\sqrt{-\frac{bc \log(F)}{e}} \operatorname{erf}\left(\sqrt{ex+d}\sqrt{-\frac{bc \log(F)}{e}}\right)}{F^{\frac{bcd-ace}{e}}} + \sqrt{ex+d}F^{bcx+ac} \right)}{e^2x + de}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(c*(b*x+a))/(e*x+d)^(3/2),x, algorithm="fricas")`

[Out] `-2*(sqrt(pi)*(e*x + d)*sqrt(-b*c*log(F)/e)*erf(sqrt(e*x + d)*sqrt(-b*c*log(F)/e)))/F^((b*c*d - a*c*e)/e) + sqrt(e*x + d)*F^(b*c*x + a*c))/(e^2*x + d*e)`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{c(a+bx)}}{(d+ex)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.



```
[In] integrate(F**(c*(b*x+a))/(e*x+d)**(3/2),x)
```

```
[Out] Integral(F**(c*(a + b*x))/(d + e*x)**(3/2), x)
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{(bx+a)c}}{(ex+d)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))/(e*x+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(F^((b*x + a)*c)/(e*x + d)^(3/2), x)
```

$$3.45 \quad \int \frac{F^{c(a+bx)}}{(d+ex)^{5/2}} dx$$

**Optimal.** Leaf size=130

$$\frac{4\sqrt{\pi}b^{3/2}c^{3/2}\log^2(F)F^{c\left(a-\frac{bd}{e}\right)}\operatorname{Erfi}\left(\frac{\sqrt{b}\sqrt{c}\sqrt{\log(F)}\sqrt{d+ex}}{\sqrt{e}}\right)}{3e^{5/2}} - \frac{4bc\log(F)F^{c(a+bx)}}{3e^2\sqrt{d+ex}} - \frac{2F^{c(a+bx)}}{3e(d+ex)^{3/2}}$$

[Out]  $(-2F^{c(a+bx)})/(3e(d+ex)^{3/2}) - (4b^2c^2F^{c(a+bx)}\operatorname{Log}[F])/(3e^2\sqrt{d+ex}) + (4b^{3/2}c^{3/2}F^{c(a-(b*d)/e)}\sqrt{\pi}\operatorname{Erfi}[(\sqrt{b}\sqrt{c}\sqrt{\log[F]})/\sqrt{e}])\operatorname{Log}[F]^{3/2})/(3e^{5/2})$

**Rubi [A]** time = 0.119372, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$ , Rules used = {2177, 2180, 2204}

$$\frac{4\sqrt{\pi}b^{3/2}c^{3/2}\log^2(F)F^{c\left(a-\frac{bd}{e}\right)}\operatorname{Erfi}\left(\frac{\sqrt{b}\sqrt{c}\sqrt{\log(F)}\sqrt{d+ex}}{\sqrt{e}}\right)}{3e^{5/2}} - \frac{4bc\log(F)F^{c(a+bx)}}{3e^2\sqrt{d+ex}} - \frac{2F^{c(a+bx)}}{3e(d+ex)^{3/2}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[F^{c(a+bx)}/(d+ex)^{5/2}, x]$

[Out]  $(-2F^{c(a+bx)})/(3e(d+ex)^{3/2}) - (4b^2c^2F^{c(a+bx)}\operatorname{Log}[F])/(3e^2\sqrt{d+ex}) + (4b^{3/2}c^{3/2}F^{c(a-(b*d)/e)}\sqrt{\pi}\operatorname{Erfi}[(\sqrt{b}\sqrt{c}\sqrt{\log[F]})/\sqrt{e}])\operatorname{Log}[F]^{3/2})/(3e^{5/2})$

### Rule 2177

$\operatorname{Int}[(b_*)*(F_*)^{((g_*)*((e_*) + (f_*)*(x_*)))^{(n_*)*((c_*) + (d_*)*(x_*))^{(m_*)}, x\_Symbol] :> \operatorname{Simp}[(c + d*x)^{(m+1)}*(b*F^{(g*(e+f*x)))^n}/(d*(m+1)), x] - \operatorname{Dist}[(f*g*n*\operatorname{Log}[F])/(d*(m+1)), \operatorname{Int}[(c + d*x)^{(m+1)}*(b*F^{(g*(e+f*x)))^n}, x], x] /; \operatorname{FreeQ}\{F, b, c, d, e, f, g, n\}, x] \&\& \operatorname{LtQ}[m, -1] \&\& \operatorname{IntegerQ}[2*m] \&\& !\$UseGamma == True$

### Rule 2180

$\operatorname{Int}[(F_*)^{((g_*)*((e_*) + (f_*)*(x_*)))}/\operatorname{Sqrt}[(c_*) + (d_*)*(x_*)], x\_Symbol] :> \operatorname{Dist}[2/d, \operatorname{Subst}[\operatorname{Int}[F^{(g*(e-(c*f)/d) + (f*g*x^2)/d)}, x], x], \operatorname{Sqrt}[c + d*$

x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !\$UseGamma === True

### Rule 2204

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))<sup>2</sup>), x\_Symbol] := Simp[(F^a\*Sqrt[Pi]\*Erfi[(c + d\*x)\*Rt[b\*Log[F], 2]])/(2\*d\*Rt[b\*Log[F], 2]), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]

### Rubi steps

$$\begin{aligned} \int \frac{F^{c(a+bx)}}{(d+ex)^{5/2}} dx &= -\frac{2F^{c(a+bx)}}{3e(d+ex)^{3/2}} + \frac{(2bc \log(F)) \int \frac{F^{c(a+bx)}}{(d+ex)^{3/2}} dx}{3e} \\ &= -\frac{2F^{c(a+bx)}}{3e(d+ex)^{3/2}} - \frac{4bcF^{c(a+bx)} \log(F)}{3e^2 \sqrt{d+ex}} + \frac{(4b^2c^2 \log^2(F)) \int \frac{F^{c(a+bx)}}{\sqrt{d+ex}} dx}{3e^2} \\ &= -\frac{2F^{c(a+bx)}}{3e(d+ex)^{3/2}} - \frac{4bcF^{c(a+bx)} \log(F)}{3e^2 \sqrt{d+ex}} + \frac{(8b^2c^2 \log^2(F)) \text{Subst}\left(\int F^{c\left(a-\frac{bd}{e}\right)+\frac{bcx^2}{e}} dx, x, \sqrt{d+ex}\right)}{3e^3} \\ &= -\frac{2F^{c(a+bx)}}{3e(d+ex)^{3/2}} - \frac{4bcF^{c(a+bx)} \log(F)}{3e^2 \sqrt{d+ex}} + \frac{4b^{3/2}c^{3/2}F^{c\left(a-\frac{bd}{e}\right)} \sqrt{\pi} \operatorname{erfi}\left(\frac{\sqrt{b}\sqrt{c}\sqrt{d+ex}\sqrt{\log(F)}}{\sqrt{e}}\right) \log^2(F)}{3e^{5/2}} \end{aligned}$$

**Mathematica [A]** time = 0.234234, size = 92, normalized size = 0.71

$$\frac{2 \left( 2eF^{c\left(a-\frac{bd}{e}\right)} \left( -\frac{bc \log(F)(d+ex)}{e} \right)^{3/2} \operatorname{Gamma}\left(\frac{1}{2}, -\frac{bc \log(F)(d+ex)}{e}\right) + F^{c(a+bx)}(2bc \log(F)(d+ex) + e) \right)}{3e^2(d+ex)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[F^(c\*(a + b\*x))/(d + e\*x)^(5/2), x]

[Out] (-2\*(2\*e\*F^(c\*(a - (b\*d)/e))\*Gamma[1/2, -((b\*c\*(d + e\*x)\*Log[F])/e)]\*(-((b\*c\*(d + e\*x)\*Log[F])/e))^(3/2) + F^(c\*(a + b\*x))\*(e + 2\*b\*c\*(d + e\*x)\*Log[F]))/(3\*e^2\*(d + e\*x)^(3/2))

**Maple [F]** time = 0.018, size = 0, normalized size = 0.

$$\int F^{c(bx+a)} (ex + d)^{-\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(c\*(b\*x+a))/(e\*x+d)^(5/2),x)

[Out] int(F^(c\*(b\*x+a))/(e\*x+d)^(5/2),x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{(bx+a)c}}{(ex + d)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(c\*(b\*x+a))/(e\*x+d)^(5/2),x, algorithm="maxima")

[Out] integrate(F^((b\*x + a)\*c)/(e\*x + d)^(5/2), x)

**Fricas [A]** time = 1.57267, size = 327, normalized size = 2.52

$$2 \left( \frac{2\sqrt{\pi}(bce^2x^2 + 2bcdex + bcd^2)\sqrt{-\frac{bc\log(F)}{e}} \operatorname{erf}\left(\sqrt{ex+d}\sqrt{-\frac{bc\log(F)}{e}}\right)\log(F)}{\frac{bcd-ace}{F^e}} + \sqrt{ex+d}(2(bcex + bcd)\log(F) + e)F^{bcx+ac} \right) \\ \hline 3(e^4x^2 + 2de^3x + d^2e^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(c\*(b\*x+a))/(e\*x+d)^(5/2),x, algorithm="fricas")

[Out] -2/3\*(2\*sqrt(pi)\*(b\*c\*e^2\*x^2 + 2\*b\*c\*d\*e\*x + b\*c\*d^2)\*sqrt(-b\*c\*log(F)/e)\*  
erf(sqrt(e\*x + d)\*sqrt(-b\*c\*log(F)/e))\*log(F)/F^((b\*c\*d - a\*c\*e)/e) + sqrt(  
e\*x + d)\*(2\*(b\*c\*e\*x + b\*c\*d)\*log(F) + e)\*F^(b\*c\*x + a\*c))/(e^4\*x^2 + 2\*d\*e  
^3\*x + d^2\*e^2)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(c\*(b\*x+a))/(e\*x+d)\*\*(5/2), x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{(bx+a)c}}{(ex+d)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(c\*(b\*x+a))/(e\*x+d)^(5/2), x, algorithm="giac")

[Out] integrate(F^((b\*x + a)\*c)/(e\*x + d)^(5/2), x)

$$3.46 \quad \int \frac{F^{c(a+bx)}}{(d+ex)^{7/2}} dx$$

**Optimal.** Leaf size=165

$$\frac{8\sqrt{\pi}b^{5/2}c^{5/2}\log^2(F)F^{c\left(a-\frac{bd}{e}\right)}\operatorname{Erfi}\left(\frac{\sqrt{b}\sqrt{c}\sqrt{\log(F)}\sqrt{d+ex}}{\sqrt{e}}\right)}{15e^{7/2}} - \frac{8b^2c^2\log^2(F)F^{c(a+bx)}}{15e^3\sqrt{d+ex}} - \frac{4bc\log(F)F^{c(a+bx)}}{15e^2(d+ex)^{3/2}} - \frac{2F^{c(a+bx)}}{5e(d+ex)^{5/2}}$$

[Out]  $(-2F^{c(a+bx)})/(5e(d+ex)^{5/2}) - (4bcF^{c(a+bx)}\operatorname{Log}[F])/(15e^2(d+ex)^{3/2}) - (8b^2c^2F^{c(a+bx)}\operatorname{Log}[F]^2)/(15e^3\operatorname{Sqrt}[d+ex]) + (8b^{5/2}c^{5/2}F^{c(a-(bd)/e)}\operatorname{Sqrt}[\pi]\operatorname{Erfi}[(\operatorname{Sqrt}[b]\operatorname{Sqrt}[c]\operatorname{Sqrt}[d+ex]\operatorname{Sqrt}[\operatorname{Log}[F]])/\operatorname{Sqrt}[e]]\operatorname{Log}[F]^{5/2})/(15e^{7/2})$

**Rubi [A]** time = 0.164663, antiderivative size = 165, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$ , Rules used = {2177, 2180, 2204}

$$\frac{8\sqrt{\pi}b^{5/2}c^{5/2}\log^2(F)F^{c\left(a-\frac{bd}{e}\right)}\operatorname{Erfi}\left(\frac{\sqrt{b}\sqrt{c}\sqrt{\log(F)}\sqrt{d+ex}}{\sqrt{e}}\right)}{15e^{7/2}} - \frac{8b^2c^2\log^2(F)F^{c(a+bx)}}{15e^3\sqrt{d+ex}} - \frac{4bc\log(F)F^{c(a+bx)}}{15e^2(d+ex)^{3/2}} - \frac{2F^{c(a+bx)}}{5e(d+ex)^{5/2}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[F^{c(a+bx)}/(d+ex)^{7/2}, x]$

[Out]  $(-2F^{c(a+bx)})/(5e(d+ex)^{5/2}) - (4bcF^{c(a+bx)}\operatorname{Log}[F])/(15e^2(d+ex)^{3/2}) - (8b^2c^2F^{c(a+bx)}\operatorname{Log}[F]^2)/(15e^3\operatorname{Sqrt}[d+ex]) + (8b^{5/2}c^{5/2}F^{c(a-(bd)/e)}\operatorname{Sqrt}[\pi]\operatorname{Erfi}[(\operatorname{Sqrt}[b]\operatorname{Sqrt}[c]\operatorname{Sqrt}[d+ex]\operatorname{Sqrt}[\operatorname{Log}[F]])/\operatorname{Sqrt}[e]]\operatorname{Log}[F]^{5/2})/(15e^{7/2})$

### Rule 2177

$\operatorname{Int}[(b_*)(F_*)^{((g_*)((e_*) + (f_*)(x_)))})^{(n_*)((c_*) + (d_*)(x_))}^{(m_*)}, x\_Symbol] \rightarrow \operatorname{Simp}[(c + dx)^{(m+1)}(bF^{g(e+fx)})^n/(d(m+1)), x] - \operatorname{Dist}[(f*g*n*\operatorname{Log}[F])/(d(m+1)), \operatorname{Int}[(c + dx)^{(m+1)}(bF^{g(e+fx)})^n, x], x] /;$  FreeQ[{F, b, c, d, e, f, g, n}, x] && LtQ[m, -1] && IntegerQ[2\*m] && !\$UseGamma == True

### Rule 2180

$\operatorname{Int}[(F_*)^{((g_*)((e_*) + (f_*)(x_)))}/\operatorname{Sqrt}[(c_*) + (d_*)(x_)], x\_Symbol] \rightarrow \operatorname{Dist}[2/d, \operatorname{Subst}[\operatorname{Int}[F^{g(e-(cf)/d)} + (f*g*x^2)/d], x], x, \operatorname{Sqrt}[c + dx]$

x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !\$UseGamma === True

### Rule 2204

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(2)), x\_Symbol] := Simp[(F^a\*Sqr  
t[Pi]\*Erfi[(c + d\*x)\*Rt[b\*Log[F], 2]])/(2\*d\*Rt[b\*Log[F], 2]), x] /; FreeQ[{  
F, a, b, c, d}, x] && PosQ[b]

### Rubi steps

$$\begin{aligned} \int \frac{F^{c(ax)} dx}{(d+ex)^{7/2}} &= -\frac{2F^{c(ax)}}{5e(d+ex)^{5/2}} + \frac{(2bc \log(F)) \int \frac{F^{c(ax)}}{(d+ex)^{5/2}} dx}{5e} \\ &= -\frac{2F^{c(ax)}}{5e(d+ex)^{5/2}} - \frac{4bcF^{c(ax)} \log(F)}{15e^2(d+ex)^{3/2}} + \frac{(4b^2c^2 \log^2(F)) \int \frac{F^{c(ax)}}{(d+ex)^{3/2}} dx}{15e^2} \\ &= -\frac{2F^{c(ax)}}{5e(d+ex)^{5/2}} - \frac{4bcF^{c(ax)} \log(F)}{15e^2(d+ex)^{3/2}} - \frac{8b^2c^2F^{c(ax)} \log^2(F)}{15e^3\sqrt{d+ex}} + \frac{(8b^3c^3 \log^3(F)) \int \frac{F^{c(ax)}}{\sqrt{d+ex}} dx}{15e^3} \\ &= -\frac{2F^{c(ax)}}{5e(d+ex)^{5/2}} - \frac{4bcF^{c(ax)} \log(F)}{15e^2(d+ex)^{3/2}} - \frac{8b^2c^2F^{c(ax)} \log^2(F)}{15e^3\sqrt{d+ex}} + \frac{(16b^3c^3 \log^3(F)) \text{Subst}\left(\int F^{c\left(a-\frac{bd}{e}\right)+\frac{bcx}{e}} dx\right)}{15e^4} \\ &= -\frac{2F^{c(ax)}}{5e(d+ex)^{5/2}} - \frac{4bcF^{c(ax)} \log(F)}{15e^2(d+ex)^{3/2}} - \frac{8b^2c^2F^{c(ax)} \log^2(F)}{15e^3\sqrt{d+ex}} + \frac{8b^{5/2}c^{5/2}F^{c\left(a-\frac{bd}{e}\right)}\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{b}\sqrt{c}\sqrt{d+ex}\sqrt{10}}{\sqrt{e}}\right)}{15e^{7/2}} \end{aligned}$$

**Mathematica [A]** time = 0.137881, size = 118, normalized size = 0.72

$$\frac{2\left(-2bc \log(F)(d+ex)\left(2eF^{c\left(a-\frac{bd}{e}\right)}\left(-\frac{bc \log(F)(d+ex)}{e}\right)^{3/2}\Gamma\left(\frac{1}{2}, -\frac{bc \log(F)(d+ex)}{e}\right) + F^{c(ax)}(2bc \log(F)(d+ex) + e)\right) - 3\right)}{15e^3(d+ex)^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[F^(c\*(a + b\*x))/(d + e\*x)^(7/2), x]

[Out] (2\*(-3\*e^2\*F^(c\*(a + b\*x)) - 2\*b\*c\*(d + e\*x)\*Log[F]\*(2\*e\*F^(c\*(a - (b\*d)/e)) \*Gamma[1/2, -((b\*c\*(d + e\*x)\*Log[F])/e)]\*(-((b\*c\*(d + e\*x)\*Log[F])/e))^(3/2) + F^(c\*(a + b\*x))\*(e + 2\*b\*c\*(d + e\*x)\*Log[F])))/(15\*e^3\*(d + e\*x)^(5/2))

---

**Maple [F]** time = 0.019, size = 0, normalized size = 0.

$$\int F^{c(bx+a)} (ex + d)^{-\frac{7}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(c\*(b\*x+a))/(e\*x+d)^(7/2), x)

[Out] int(F^(c\*(b\*x+a))/(e\*x+d)^(7/2), x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{(bx+a)c}}{(ex + d)^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(c\*(b\*x+a))/(e\*x+d)^(7/2), x, algorithm="maxima")

[Out] integrate(F^((b\*x + a)\*c)/(e\*x + d)^(7/2), x)

---

**Fricas [A]** time = 1.56885, size = 497, normalized size = 3.01

$$2 \left( \frac{4 \sqrt{\pi} (b^2 c^2 e^3 x^3 + 3 b^2 c^2 d e^2 x^2 + 3 b^2 c^2 d^2 e x + b^2 c^2 d^3) \sqrt{-\frac{bc \log(F)}{e}} \operatorname{erf}\left(\sqrt{ex+d} \sqrt{-\frac{bc \log(F)}{e}}\right) \log(F)^2}{F^{\frac{bcd-ace}{e}}} + (4 (b^2 c^2 e^2 x^2 + 2 b^2 c^2 d e x + b^2 c^2 d^2) \log(F)^2 \right)$$


---


$$15 (e^6 x^3 + 3 d e^5 x^2 + 3 d^2 e^4 x + d^3 e^3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(c\*(b\*x+a))/(e\*x+d)^(7/2), x, algorithm="fricas")

[Out] -2/15\*(4\*sqrt(pi)\*(b^2\*c^2\*e^3\*x^3 + 3\*b^2\*c^2\*d\*e^2\*x^2 + 3\*b^2\*c^2\*d^2\*e\*x + b^2\*c^2\*d^3)\*sqrt(-b\*c\*log(F)/e)\*erf(sqrt(e\*x + d)\*sqrt(-b\*c\*log(F)/e))\*log(F)^2/F^((b\*c\*d - a\*c\*e)/e) + (4\*(b^2\*c^2\*e^2\*x^2 + 2\*b^2\*c^2\*d\*e\*x + b^2\*c^2\*d^2)\*log(F)^2 + 3\*e^2 + 2\*(b\*c\*e^2\*x + b\*c\*d\*e)\*log(F))\*sqrt(e\*x + d)



) $F^{(b*c*x + a*c)}$ /( $e^{6*x^3} + 3*d*e^{5*x^2} + 3*d^2*e^{4*x} + d^3*e^3$ )

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate( $F^{(c*(b*x+a))}$ /( $e*x+d$ ) $^{(7/2)}$ , x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{(bx+a)c}}{(ex+d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate( $F^{(c*(b*x+a))}$ /( $e*x+d$ ) $^{(7/2)}$ , x, algorithm="giac")

[Out] integrate( $F^{((b*x + a)*c)}$ /( $e*x + d$ ) $^{(7/2)}$ , x)

$$3.47 \quad \int \frac{F^{c(a+bx)}}{(d+ex)^{9/2}} dx$$

**Optimal.** Leaf size=200

$$\frac{16\sqrt{\pi}b^{7/2}c^{7/2}\log^2(F)F^{c\left(a-\frac{bd}{e}\right)}\operatorname{Erfi}\left(\frac{\sqrt{b}\sqrt{c}\sqrt{\log(F)}\sqrt{d+ex}}{\sqrt{e}}\right)}{105e^{9/2}} - \frac{16b^3c^3\log^3(F)F^{c(a+bx)}}{105e^4\sqrt{d+ex}} - \frac{8b^2c^2\log^2(F)F^{c(a+bx)}}{105e^3(d+ex)^{3/2}} - \frac{4bc\log(F)F^{c(a+bx)}}{35e^2(d+ex)^{5/2}}$$

[Out]  $(-2F^{c(a+bx)})/(7e(d+ex)^{7/2}) - (4b^3c^3F^{c(a+bx)}\operatorname{Log}[F])/(35e^2(d+ex)^{5/2}) - (8b^2c^2F^{c(a+bx)}\operatorname{Log}[F]^2)/(105e^3(d+ex)^{3/2}) - (16b^3c^3F^{c(a+bx)}\operatorname{Log}[F]^3)/(105e^4\sqrt{d+ex}) + (16b^{7/2}c^{7/2}F^{c(a-(bd)/e)}\sqrt{\pi}\operatorname{Erfi}[(\sqrt{b}\sqrt{c}\sqrt{\log[F]})/\sqrt{e}])/(105e^{9/2})$

**Rubi [A]** time = 0.203384, antiderivative size = 200, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$ , Rules used = {2177, 2180, 2204}

$$\frac{16\sqrt{\pi}b^{7/2}c^{7/2}\log^2(F)F^{c\left(a-\frac{bd}{e}\right)}\operatorname{Erfi}\left(\frac{\sqrt{b}\sqrt{c}\sqrt{\log(F)}\sqrt{d+ex}}{\sqrt{e}}\right)}{105e^{9/2}} - \frac{16b^3c^3\log^3(F)F^{c(a+bx)}}{105e^4\sqrt{d+ex}} - \frac{8b^2c^2\log^2(F)F^{c(a+bx)}}{105e^3(d+ex)^{3/2}} - \frac{4bc\log(F)F^{c(a+bx)}}{35e^2(d+ex)^{5/2}}$$

Antiderivative was successfully verified.

[In]  $\int F^{c(a+bx)}/(d+ex)^{9/2}, x]$

[Out]  $(-2F^{c(a+bx)})/(7e(d+ex)^{7/2}) - (4b^3c^3F^{c(a+bx)}\operatorname{Log}[F])/(35e^2(d+ex)^{5/2}) - (8b^2c^2F^{c(a+bx)}\operatorname{Log}[F]^2)/(105e^3(d+ex)^{3/2}) - (16b^3c^3F^{c(a+bx)}\operatorname{Log}[F]^3)/(105e^4\sqrt{d+ex}) + (16b^{7/2}c^{7/2}F^{c(a-(bd)/e)}\sqrt{\pi}\operatorname{Erfi}[(\sqrt{b}\sqrt{c}\sqrt{\log[F]})/\sqrt{e}])/(105e^{9/2})$

### Rule 2177

$\operatorname{Int}[(b_.)(F_)^{((g_.)((e_.) + (f_.)(x_)))^{(n_.)((c_.) + (d_.)(x_))^{(m_.)}, x\_Symbol] := \operatorname{Simp}[(c + dx)^{(m+1)}(bF^{g(e+fx)})^n]/(d(m+1)), x] - \operatorname{Dist}[(f^n \operatorname{Log}[F])/(d(m+1)), \operatorname{Int}[(c + dx)^{(m+1)}(bF^{g(e+fx)})^n, x], x] /;$  FreeQ[{F, b, c, d, e, f, g, n}, x] && LtQ[m, -1] && IntegerQ[2\*m] && !\$UseGamma == True

### Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !$UseGamma === True
```

### Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_)^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

### Rubi steps

$$\begin{aligned} \int \frac{F^{c(ax+bx)}}{(d+ex)^{9/2}} dx &= -\frac{2F^{c(ax+bx)}}{7e(d+ex)^{7/2}} + \frac{(2bc \log(F)) \int \frac{F^{c(ax+bx)}}{(d+ex)^{7/2}} dx}{7e} \\ &= -\frac{2F^{c(ax+bx)}}{7e(d+ex)^{7/2}} - \frac{4bcF^{c(ax+bx)} \log(F)}{35e^2(d+ex)^{5/2}} + \frac{(4b^2c^2 \log^2(F)) \int \frac{F^{c(ax+bx)}}{(d+ex)^{5/2}} dx}{35e^2} \\ &= -\frac{2F^{c(ax+bx)}}{7e(d+ex)^{7/2}} - \frac{4bcF^{c(ax+bx)} \log(F)}{35e^2(d+ex)^{5/2}} - \frac{8b^2c^2F^{c(ax+bx)} \log^2(F)}{105e^3(d+ex)^{3/2}} + \frac{(8b^3c^3 \log^3(F)) \int \frac{F^{c(ax+bx)}}{(d+ex)^{3/2}} dx}{105e^3} \\ &= -\frac{2F^{c(ax+bx)}}{7e(d+ex)^{7/2}} - \frac{4bcF^{c(ax+bx)} \log(F)}{35e^2(d+ex)^{5/2}} - \frac{8b^2c^2F^{c(ax+bx)} \log^2(F)}{105e^3(d+ex)^{3/2}} - \frac{16b^3c^3F^{c(ax+bx)} \log^3(F)}{105e^4\sqrt{d+ex}} + \frac{(16b^4c^4 \log^4(F)) \int \frac{F^{c(ax+bx)}}{\sqrt{d+ex}} dx}{105e^4} \\ &= -\frac{2F^{c(ax+bx)}}{7e(d+ex)^{7/2}} - \frac{4bcF^{c(ax+bx)} \log(F)}{35e^2(d+ex)^{5/2}} - \frac{8b^2c^2F^{c(ax+bx)} \log^2(F)}{105e^3(d+ex)^{3/2}} - \frac{16b^3c^3F^{c(ax+bx)} \log^3(F)}{105e^4\sqrt{d+ex}} + \frac{(32b^4c^4 \log^4(F)) \int \frac{F^{c(ax+bx)}}{\sqrt{d+ex}} dx}{105e^4} \\ &= -\frac{2F^{c(ax+bx)}}{7e(d+ex)^{7/2}} - \frac{4bcF^{c(ax+bx)} \log(F)}{35e^2(d+ex)^{5/2}} - \frac{8b^2c^2F^{c(ax+bx)} \log^2(F)}{105e^3(d+ex)^{3/2}} - \frac{16b^3c^3F^{c(ax+bx)} \log^3(F)}{105e^4\sqrt{d+ex}} + \frac{16b^{7/2}c^{7/2}F^{c(ax+bx)}}{105e^4} \end{aligned}$$

**Mathematica [A]** time = 0.165273, size = 144, normalized size = 0.72

$$\frac{2 \left( 2bc \log(F)(d+ex) \left( -2bc \log(F)(d+ex) \left( 2eF^{c \left( a - \frac{bd}{e} \right)} \left( -\frac{bc \log(F)(d+ex)}{e} \right)^{3/2} \Gamma\left(\frac{1}{2}, -\frac{bc \log(F)(d+ex)}{e}\right) + F^{c(ax+bx)}(2bc \log(F)) \right) \right)}{105e^4(d+ex)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[F^(c*(a + b*x))/(d + e*x)^(9/2), x]
```

[Out]  $(2*(-15*e^3*F^{c*(a + b*x)} + 2*b*c*(d + e*x)*\text{Log}[F]*(-3*e^2*F^{c*(a + b*x)}) - 2*b*c*(d + e*x)*\text{Log}[F]*(2*e*F^{c*(a - (b*d)/e)})*\text{Gamma}[1/2, -((b*c*(d + e*x)*\text{Log}[F])/e)]*(-((b*c*(d + e*x)*\text{Log}[F])/e))^{3/2} + F^{c*(a + b*x)}*(e + 2*b*c*(d + e*x)*\text{Log}[F])))/(105*e^4*(d + e*x)^{7/2})$

**Maple [F]** time = 0.019, size = 0, normalized size = 0.

$$\int F^{c(bx+a)} (ex + d)^{-\frac{9}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(F^{c*(b*x+a)}/(e*x+d)^{9/2},x)`

[Out] `int(F^{c*(b*x+a)}/(e*x+d)^{9/2},x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{(bx+a)c}}{(ex + d)^{\frac{9}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^{c*(b*x+a)}/(e*x+d)^{9/2},x, algorithm="maxima")`

[Out] `integrate(F^{((b*x + a)*c)}/(e*x + d)^{9/2}, x)`

**Fricas [A]** time = 1.59008, size = 679, normalized size = 3.4

$$2 \left( \frac{8 \sqrt{\pi} (b^3 c^3 e^4 x^4 + 4 b^3 c^3 d e^3 x^3 + 6 b^3 c^3 d^2 e^2 x^2 + 4 b^3 c^3 d^3 e x + b^3 c^3 d^4) \sqrt{-\frac{bc \log(F)}{e}} \operatorname{erf}\left(\sqrt{ex+d} \sqrt{-\frac{bc \log(F)}{e}}\right) \log(F)^3}{F^{\frac{bcd-ace}{e}}} + (8 (b^3 c^3 e^3 x^3 + 3 b^3 c^3 d e^2 x^2 + 3 b^3 c^3 d^2 e x + b^3 c^3 d^3) \log(F)^3 + 105 (e^8 x^4 + 4 d e^7 x^3 + \dots) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^{c*(b*x+a)}/(e*x+d)^{9/2},x, algorithm="fricas")`

```
[Out] -2/105*(8*sqrt(pi)*(b^3*c^3*e^4*x^4 + 4*b^3*c^3*d*e^3*x^3 + 6*b^3*c^3*d^2*e^2*x^2 + 4*b^3*c^3*d^3*e*x + b^3*c^3*d^4)*sqrt(-b*c*log(F)/e)*erf(sqrt(e*x + d)*sqrt(-b*c*log(F)/e))*log(F)^3/F^((b*c*d - a*c*e)/e) + (8*(b^3*c^3*e^3*x^3 + 3*b^3*c^3*d*e^2*x^2 + 3*b^3*c^3*d^2*e*x + b^3*c^3*d^3)*log(F)^3 + 15*e^3 + 4*(b^2*c^2*e^3*x^2 + 2*b^2*c^2*d*e^2*x + b^2*c^2*d^2*e)*log(F)^2 + 6*(b*c*e^3*x + b*c*d*e^2)*log(F))*sqrt(e*x + d)*F^(b*c*x + a*c))/(e^8*x^4 + 4*d*e^7*x^3 + 6*d^2*e^6*x^2 + 4*d^3*e^5*x + d^4*e^4)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(c*(b*x+a))/(e*x+d)**(9/2),x)
```

```
[Out] Timed out
```

**Giac [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))/(e*x+d)^(9/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError
```

### 3.48 $\int e^{-bx} x^{13/2} dx$

**Optimal.** Leaf size=151

$$\frac{135135\sqrt{\pi}\operatorname{Erf}(\sqrt{b}\sqrt{x})}{128b^{15/2}} - \frac{13x^{11/2}e^{-bx}}{2b^2} - \frac{143x^{9/2}e^{-bx}}{4b^3} - \frac{1287x^{7/2}e^{-bx}}{8b^4} - \frac{9009x^{5/2}e^{-bx}}{16b^5} - \frac{45045x^{3/2}e^{-bx}}{32b^6} - \frac{135135\sqrt{xe^{-bx}}}{64b^7}$$

[Out]  $(-135135\sqrt{x})/(64*b^7*E^{(b*x)}) - (45045*x^{(3/2)})/(32*b^6*E^{(b*x)}) - (9009*x^{(5/2)})/(16*b^5*E^{(b*x)}) - (1287*x^{(7/2)})/(8*b^4*E^{(b*x)}) - (143*x^{(9/2)})/(4*b^3*E^{(b*x)}) - (13*x^{(11/2)})/(2*b^2*E^{(b*x)}) - x^{(13/2)}/(b*E^{(b*x)}) + (135135*\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erf}[\operatorname{Sqrt}[b]*\operatorname{Sqrt}[x]])/(128*b^{(15/2)})$

**Rubi [A]** time = 0.154831, antiderivative size = 151, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 3, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$ , Rules used = {2176, 2180, 2205}

$$\frac{135135\sqrt{\pi}\operatorname{Erf}(\sqrt{b}\sqrt{x})}{128b^{15/2}} - \frac{13x^{11/2}e^{-bx}}{2b^2} - \frac{143x^{9/2}e^{-bx}}{4b^3} - \frac{1287x^{7/2}e^{-bx}}{8b^4} - \frac{9009x^{5/2}e^{-bx}}{16b^5} - \frac{45045x^{3/2}e^{-bx}}{32b^6} - \frac{135135\sqrt{xe^{-bx}}}{64b^7}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[x^{(13/2)}/E^{(b*x)}, x]$

[Out]  $(-135135\sqrt{x})/(64*b^7*E^{(b*x)}) - (45045*x^{(3/2)})/(32*b^6*E^{(b*x)}) - (9009*x^{(5/2)})/(16*b^5*E^{(b*x)}) - (1287*x^{(7/2)})/(8*b^4*E^{(b*x)}) - (143*x^{(9/2)})/(4*b^3*E^{(b*x)}) - (13*x^{(11/2)})/(2*b^2*E^{(b*x)}) - x^{(13/2)}/(b*E^{(b*x)}) + (135135*\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erf}[\operatorname{Sqrt}[b]*\operatorname{Sqrt}[x]])/(128*b^{(15/2)})$

#### Rule 2176

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[((c + d*x)^m*(b*F^(g*(e + f*x)))^n)/(f*g*n*Log[F]), x] - Dist[(d*m)/(f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m] && !$UseGamma === True
```

#### Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !$UseGamma === True
```

Rule 2205

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))<sup>2</sup>), x\_Symbol] := Simp[(F^a\*Sqrt[Pi]\*Erf[(c + d\*x)\*Rt[-(b\*Log[F]), 2]])/(2\*d\*Rt[-(b\*Log[F]), 2]), x] /; FreeQ[{F, a, b, c, d}, x] && NegQ[b]

Rubi steps

$$\begin{aligned}
 \int e^{-bx}x^{13/2} dx &= -\frac{e^{-bx}x^{13/2}}{b} + \frac{13 \int e^{-bx}x^{11/2} dx}{2b} \\
 &= -\frac{13e^{-bx}x^{11/2}}{2b^2} - \frac{e^{-bx}x^{13/2}}{b} + \frac{143 \int e^{-bx}x^{9/2} dx}{4b^2} \\
 &= -\frac{143e^{-bx}x^{9/2}}{4b^3} - \frac{13e^{-bx}x^{11/2}}{2b^2} - \frac{e^{-bx}x^{13/2}}{b} + \frac{1287 \int e^{-bx}x^{7/2} dx}{8b^3} \\
 &= -\frac{1287e^{-bx}x^{7/2}}{8b^4} - \frac{143e^{-bx}x^{9/2}}{4b^3} - \frac{13e^{-bx}x^{11/2}}{2b^2} - \frac{e^{-bx}x^{13/2}}{b} + \frac{9009 \int e^{-bx}x^{5/2} dx}{16b^4} \\
 &= -\frac{9009e^{-bx}x^{5/2}}{16b^5} - \frac{1287e^{-bx}x^{7/2}}{8b^4} - \frac{143e^{-bx}x^{9/2}}{4b^3} - \frac{13e^{-bx}x^{11/2}}{2b^2} - \frac{e^{-bx}x^{13/2}}{b} + \frac{45045 \int e^{-bx}x^{3/2} dx}{32b^5} \\
 &= -\frac{45045e^{-bx}x^{3/2}}{32b^6} - \frac{9009e^{-bx}x^{5/2}}{16b^5} - \frac{1287e^{-bx}x^{7/2}}{8b^4} - \frac{143e^{-bx}x^{9/2}}{4b^3} - \frac{13e^{-bx}x^{11/2}}{2b^2} - \frac{e^{-bx}x^{13/2}}{b} + \frac{135135 \int e^{-bx} dx}{64b^6} \\
 &= -\frac{135135e^{-bx}\sqrt{x}}{64b^7} - \frac{45045e^{-bx}x^{3/2}}{32b^6} - \frac{9009e^{-bx}x^{5/2}}{16b^5} - \frac{1287e^{-bx}x^{7/2}}{8b^4} - \frac{143e^{-bx}x^{9/2}}{4b^3} - \frac{13e^{-bx}x^{11/2}}{2b^2} - \frac{e^{-bx}x^{13/2}}{b} \\
 &= -\frac{135135e^{-bx}\sqrt{x}}{64b^7} - \frac{45045e^{-bx}x^{3/2}}{32b^6} - \frac{9009e^{-bx}x^{5/2}}{16b^5} - \frac{1287e^{-bx}x^{7/2}}{8b^4} - \frac{143e^{-bx}x^{9/2}}{4b^3} - \frac{13e^{-bx}x^{11/2}}{2b^2} - \frac{e^{-bx}x^{13/2}}{b} \\
 &= -\frac{135135e^{-bx}\sqrt{x}}{64b^7} - \frac{45045e^{-bx}x^{3/2}}{32b^6} - \frac{9009e^{-bx}x^{5/2}}{16b^5} - \frac{1287e^{-bx}x^{7/2}}{8b^4} - \frac{143e^{-bx}x^{9/2}}{4b^3} - \frac{13e^{-bx}x^{11/2}}{2b^2} - \frac{e^{-bx}x^{13/2}}{b}
 \end{aligned}$$

**Mathematica [A]** time = 0.0041943, size = 24, normalized size = 0.16

$$-\frac{\sqrt{bx}\Gamma\left(\frac{15}{2}, bx\right)}{b^8\sqrt{x}}$$

Antiderivative was successfully verified.

[In] Integrate[x^(13/2)/E^(b\*x), x]

[Out] -((Sqrt[b\*x]\*Gamma[15/2, b\*x])/(b^8\*Sqrt[x]))

---

**Maple [A]** time = 0.135, size = 145, normalized size = 1.

$$-\frac{e^{-bx}}{b}x^{\frac{13}{2}} + 13\frac{1}{b}\left(-\frac{1}{2}\frac{x^{11/2}e^{-bx}}{b} + 11/2\frac{1}{b}\left(-\frac{1}{2}\frac{x^{9/2}e^{-bx}}{b} + 9/2\frac{1}{b}\left(-\frac{1}{2}\frac{x^{7/2}e^{-bx}}{b} + 7/2\frac{1}{b}\left(-\frac{1}{2}\frac{x^{5/2}e^{-bx}}{b} + 5/2\frac{1}{b}\left(-\frac{1}{2}\frac{x^{3/2}e^{-bx}}{b} + 1/2\frac{1}{b}\left(-\frac{1}{2}\frac{x^{1/2}e^{-bx}}{b} + \frac{1}{4}b^{3/2}\operatorname{erf}\left(\frac{x^{1/2}}{b^{1/2}}\right)\right)\right)\right)\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^(13/2)/exp(b*x),x)`

[Out] `-1/b*x^(13/2)*exp(-b*x)+13/b*(-1/2/b*x^(11/2)*exp(-b*x)+11/2/b*(-1/2/b*x^(9/2)*exp(-b*x)+9/2/b*(-1/2/b*x^(7/2)*exp(-b*x)+7/2/b*(-1/2/b*x^(5/2)*exp(-b*x)+5/2/b*(-1/2/b*x^(3/2)*exp(-b*x)+3/2/b*(-1/2/b*x^(1/2)*exp(-b*x)+1/4/b^(3/2)*Pi^(1/2)*erf(b^(1/2)*x^(1/2))))))`

---

**Maxima [A]** time = 1.07258, size = 107, normalized size = 0.71

$$\frac{\left(64b^6x^{\frac{13}{2}} + 416b^5x^{\frac{11}{2}} + 2288b^4x^{\frac{9}{2}} + 10296b^3x^{\frac{7}{2}} + 36036b^2x^{\frac{5}{2}} + 90090bx^{\frac{3}{2}} + 135135\sqrt{x}\right)e^{-bx}}{64b^7} + \frac{135135\sqrt{\pi}\operatorname{erf}\left(\sqrt{b}x^{1/2}\right)}{128b^{\frac{15}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^(13/2)/exp(b*x),x, algorithm="maxima")`

[Out] `-1/64*(64*b^6*x^(13/2) + 416*b^5*x^(11/2) + 2288*b^4*x^(9/2) + 10296*b^3*x^(7/2) + 36036*b^2*x^(5/2) + 90090*b*x^(3/2) + 135135*sqrt(x))*e^(-b*x)/b^7 + 135135/128*sqrt(pi)*erf(sqrt(b)*sqrt(x))/b^(15/2)`

---

**Fricas [A]** time = 1.51932, size = 242, normalized size = 1.6

$$\frac{2\left(64b^7x^6 + 416b^6x^5 + 2288b^5x^4 + 10296b^4x^3 + 36036b^3x^2 + 90090b^2x + 135135b\right)\sqrt{x}e^{-bx} - 135135\sqrt{\pi}\sqrt{b}\operatorname{erf}\left(\sqrt{b}x^{1/2}\right)}{128b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^(13/2)/exp(b*x),x, algorithm="fricas")`



[Out] 
$$-1/128*(2*(64*b^7*x^6 + 416*b^6*x^5 + 2288*b^5*x^4 + 10296*b^4*x^3 + 36036*b^3*x^2 + 90090*b^2*x + 135135*b)*\sqrt{x})e^{-bx} - 135135*\sqrt{\pi}*\sqrt{b}*\operatorname{erf}(\sqrt{b}*\sqrt{x}))/b^8$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**(13/2)/exp(b*x), x)`

[Out] Timed out

**Giac [A]** time = 1.1608, size = 108, normalized size = 0.72

$$\frac{\left(64b^6x^{\frac{13}{2}} + 416b^5x^{\frac{11}{2}} + 2288b^4x^{\frac{9}{2}} + 10296b^3x^{\frac{7}{2}} + 36036b^2x^{\frac{5}{2}} + 90090bx^{\frac{3}{2}} + 135135\sqrt{x}\right)e^{-bx}}{64b^7} - \frac{135135\sqrt{\pi}\operatorname{erf}\left(-\sqrt{b}\sqrt{x}\right)}{128b^{\frac{15}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^(13/2)/exp(b*x), x, algorithm="giac")`

[Out] 
$$-1/64*(64*b^6*x^{(13/2)} + 416*b^5*x^{(11/2)} + 2288*b^4*x^{(9/2)} + 10296*b^3*x^{(7/2)} + 36036*b^2*x^{(5/2)} + 90090*b*x^{(3/2)} + 135135*\sqrt{x})*e^{-bx}/b^7 - 135135/128*\sqrt{\pi}*\operatorname{erf}(-\sqrt{b}*\sqrt{x}))/b^{(15/2)}$$

### 3.49 $\int F^{c(a+bx)}(d+ex)^{4/3} dx$

**Optimal.** Leaf size=71

$$\frac{e^{\sqrt[3]{d+ex}} F^{c\left(a-\frac{bd}{e}\right)} \text{Gamma}\left(\frac{7}{3}, -\frac{bc \log(F)(d+ex)}{e}\right)}{b^2 c^2 \log^2(F) \sqrt[3]{-\frac{bc \log(F)(d+ex)}{e}}}$$

[Out]  $-\left(\left(e F^{c(a-(b*d)/e)}\right) * (d+e*x)^{(1/3)} * \text{Gamma}\left[7/3, -\left((b*c*(d+e*x)*\text{Log}[F])/e\right)\right]\right) / \left(b^2*c^2*\text{Log}[F]^2 * \left(-\left((b*c*(d+e*x)*\text{Log}[F])/e\right)\right)^{(1/3)}\right)$

**Rubi [A]** time = 0.0313849, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$ , Rules used = {2181}

$$\frac{e^{\sqrt[3]{d+ex}} F^{c\left(a-\frac{bd}{e}\right)} \text{Gamma}\left(\frac{7}{3}, -\frac{bc \log(F)(d+ex)}{e}\right)}{b^2 c^2 \log^2(F) \sqrt[3]{-\frac{bc \log(F)(d+ex)}{e}}}$$

Antiderivative was successfully verified.

[In] Int[F^(c\*(a + b\*x))\*(d + e\*x)^(4/3), x]

[Out]  $-\left(\left(e F^{c(a-(b*d)/e)}\right) * (d+e*x)^{(1/3)} * \text{Gamma}\left[7/3, -\left((b*c*(d+e*x)*\text{Log}[F])/e\right)\right]\right) / \left(b^2*c^2*\text{Log}[F]^2 * \left(-\left((b*c*(d+e*x)*\text{Log}[F])/e\right)\right)^{(1/3)}\right)$

#### Rule 2181

```
Int[(F_)^((g_)*((e_) + (f_)*(x_)))*((c_) + (d_)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Log[F])/d))*(c + d*x])]/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F])*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !IntegerQ[m]
```

#### Rubi steps

$$\int F^{c(a+bx)}(d+ex)^{4/3} dx = -\frac{e F^{c\left(a-\frac{bd}{e}\right)} \sqrt[3]{d+ex} \Gamma\left(\frac{7}{3}, -\frac{bc(d+ex) \log(F)}{e}\right)}{b^2 c^2 \log^2(F) \sqrt[3]{-\frac{bc(d+ex) \log(F)}{e}}}$$

**Mathematica [A]** time = 0.0802949, size = 63, normalized size = 0.89

$$\frac{(d + ex)^{7/3} F^{c\left(a - \frac{bd}{e}\right)} \text{Gamma}\left(\frac{7}{3}, -\frac{bc \log(F)(d+ex)}{e}\right)}{e\left(-\frac{bc \log(F)(d+ex)}{e}\right)^{7/3}}$$

Antiderivative was successfully verified.

[In] Integrate[F^(c\*(a + b\*x))\*(d + e\*x)^(4/3), x]

[Out] -((F^(c\*(a - (b\*d)/e))\*(d + e\*x)^(7/3)\*Gamma[7/3, -((b\*c\*(d + e\*x)\*Log[F])/e)])/(e\*(-((b\*c\*(d + e\*x)\*Log[F])/e))^(7/3)))

**Maple [F]** time = 0.018, size = 0, normalized size = 0.

$$\int F^{c(bx+a)} (ex + d)^{\frac{4}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(c\*(b\*x+a))\*(e\*x+d)^(4/3), x)

[Out] int(F^(c\*(b\*x+a))\*(e\*x+d)^(4/3), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (ex + d)^{\frac{4}{3}} F^{(bx+a)c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(c\*(b\*x+a))\*(e\*x+d)^(4/3), x, algorithm="maxima")

[Out] integrate((e\*x + d)^(4/3)\*F^((b\*x + a)\*c), x)

**Fricas [A]** time = 1.57135, size = 281, normalized size = 3.96

$$\frac{4 \left( -\frac{bc \log(F)}{e} \right)^{\frac{2}{3}} e^{2\Gamma\left(\frac{1}{3}, -\frac{(bcex+bcd) \log(F)}{e}\right)} - 3 \left( 4bce \log(F) - 3(b^2c^2ex + b^2c^2d) \log(F)^2 \right) (ex + d)^{\frac{1}{3}} F^{bcx+ac}}{F^{\frac{bcd-ace}{e}} \cdot 9b^3c^3 \log(F)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(c\*(b\*x+a))\*(e\*x+d)^(4/3),x, algorithm="fricas")

[Out] 1/9\*(4\*(-b\*c\*log(F)/e)^(2/3)\*e^2\*gamma(1/3, -(b\*c\*e\*x + b\*c\*d)\*log(F)/e)/F^((b\*c\*d - a\*c\*e)/e) - 3\*(4\*b\*c\*e\*log(F) - 3\*(b^2\*c^2\*e\*x + b^2\*c^2\*d)\*log(F)^2)\*(e\*x + d)^(1/3)\*F^(b\*c\*x + a\*c)/(b^3\*c^3\*log(F)^3)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(c\*(b\*x+a))\*(e\*x+d)\*\*(4/3),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (ex + d)^{\frac{4}{3}} F^{(bx+a)c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(c\*(b\*x+a))\*(e\*x+d)^(4/3),x, algorithm="giac")

[Out] integrate((e\*x + d)^(4/3)\*F^((b\*x + a)\*c), x)

### 3.50 $\int \left(F^{c(a+bx)}\right)^n (d+ex)^{4/3} dx$

**Optimal.** Leaf size=98

$$\frac{e\sqrt[3]{d+ex} \left(F^{c(a+bx)}\right)^n F^{cn\left(a-\frac{bd}{e}\right)-cn(a+bx)} \text{Gamma}\left(\frac{7}{3}, -\frac{bcn \log(F)(d+ex)}{e}\right)}{b^2 c^2 n^2 \log^2(F) \sqrt[3]{-\frac{bcn \log(F)(d+ex)}{e}}}$$

[Out]  $-\left(\left(eF^{c\left(a-\frac{bd}{e}\right)n-cn(a+bx)}\right)\left(F^{c(a+bx)}\right)^n(d+ex)^{\frac{1}{3}}\text{Gamma}\left[\frac{7}{3}, -\frac{(b^2cn^2(d+ex)\text{Log}[F])}{e}\right]\right)/\left(b^2c^2n^2\text{Log}[F]^2\left(-\left(b^2cn^2(d+ex)\text{Log}[F]\right)/e\right)^{\frac{1}{3}}\right)$

**Rubi [A]** time = 0.104381, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$ , Rules used = {2182, 2181}

$$\frac{e\sqrt[3]{d+ex} \left(F^{c(a+bx)}\right)^n F^{cn\left(a-\frac{bd}{e}\right)-cn(a+bx)} \text{Gamma}\left(\frac{7}{3}, -\frac{bcn \log(F)(d+ex)}{e}\right)}{b^2 c^2 n^2 \log^2(F) \sqrt[3]{-\frac{bcn \log(F)(d+ex)}{e}}}$$

Antiderivative was successfully verified.

[In] Int[(F^(c\*(a + b\*x)))^n\*(d + e\*x)^(4/3), x]

[Out]  $-\left(\left(eF^{c\left(a-\frac{bd}{e}\right)n-cn(a+bx)}\right)\left(F^{c(a+bx)}\right)^n(d+ex)^{\frac{1}{3}}\text{Gamma}\left[\frac{7}{3}, -\frac{(b^2cn^2(d+ex)\text{Log}[F])}{e}\right]\right)/\left(b^2c^2n^2\text{Log}[F]^2\left(-\left(b^2cn^2(d+ex)\text{Log}[F]\right)/e\right)^{\frac{1}{3}}\right)$

#### Rule 2182

Int[((b\_.)\*(F\_)^(g\_.)\*((e\_.) + (f\_.)\*(x\_)))^(n\_)\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] :> Dist[(b\*F^(g\*(e + f\*x)))^n/F^(g\*n\*(e + f\*x)), Int[(c + d\*x)^m\*F^(g\*n\*(e + f\*x)), x], x] /; FreeQ[{F, b, c, d, e, f, g, m, n}, x]

#### Rule 2181

Int[(F\_)^(g\_.)\*((e\_.) + (f\_.)\*(x\_))\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] :> -Simp[(F^(g\*(e - (c\*f)/d))\*(c + d\*x)^FracPart[m]\*Gamma[m + 1, (-((f\*g\*Log[F])/d))\*(c + d\*x])]/(d\*(-((f\*g\*Log[F])/d))^(IntPart[m] + 1)\*(-((f\*g\*Log[F])\*(c + d\*x)/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !I

ntegerQ [m]

### Rubi steps

$$\int (F^{c(a+bx)})^n (d+ex)^{4/3} dx = (F^{-cn(a+bx)} (F^{c(a+bx)})^n) \int F^{cn(a+bx)} (d+ex)^{4/3} dx$$

$$= -\frac{e F^{c\left(a-\frac{bd}{e}\right)n-cn(a+bx)} (F^{c(a+bx)})^n \sqrt[3]{d+ex} \Gamma\left(\frac{7}{3}, -\frac{bcn(d+ex)\log(F)}{e}\right)}{b^2 c^2 n^2 \log^2(F) \sqrt[3]{-\frac{bcn(d+ex)\log(F)}{e}}}$$

**Mathematica [A]** time = 0.134194, size = 78, normalized size = 0.8

$$\frac{(d+ex)^{7/3} (F^{c(a+bx)})^n F^{-\frac{bcn(d+ex)}{e}} \text{Gamma}\left(\frac{7}{3}, -\frac{bcn\log(F)(d+ex)}{e}\right)}{e\left(-\frac{bcn\log(F)(d+ex)}{e}\right)^{7/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(F^(c\*(a + b\*x)))^n\*(d + e\*x)^(4/3), x]

[Out] -(((F^(c\*(a + b\*x)))^n\*(d + e\*x)^(7/3)\*Gamma[7/3, -((b\*c\*n\*(d + e\*x)\*Log[F])  
(e))]/(e\*F^((b\*c\*n\*(d + e\*x))/e)\*(-(b\*c\*n\*(d + e\*x)\*Log[F])/e))^(7/3)))

**Maple [F]** time = 0.044, size = 0, normalized size = 0.

$$\int (F^{c(bx+a)})^n (ex+d)^{\frac{4}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((F^(c\*(b\*x+a)))^n\*(e\*x+d)^(4/3), x)

[Out] int((F^(c\*(b\*x+a)))^n\*(e\*x+d)^(4/3), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (ex+d)^{\frac{4}{3}} (F^{(bx+a)c})^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((F^(c\*(b\*x+a)))^n\*(e\*x+d)^(4/3), x, algorithm="maxima")

[Out] integrate((e\*x + d)^(4/3)\*(F^((b\*x + a)\*c))^n, x)

**Fricas [A]** time = 1.55899, size = 316, normalized size = 3.22

$$\frac{4 \left( -\frac{bcn \log(F)}{e} \right)^{\frac{2}{3}} e^{2\Gamma\left(\frac{1}{3}, -\frac{(bcnx+bcdn) \log(F)}{e}\right)} - 3 \left( 4bcn \log(F) - 3 \left( b^2c^2en^2x + b^2c^2dn^2 \right) \log(F)^2 \right) (ex + d)^{\frac{1}{3}} F^{bcnx+acn}}{F^{\frac{(bcd-ace)n}{e}}} 9b^3c^3n^3 \log(F)^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((F^(c\*(b\*x+a)))^n\*(e\*x+d)^(4/3), x, algorithm="fricas")

[Out]  $\frac{1}{9} * (4 * (-b*c*n*log(F)/e)^{(2/3)} * e^{2*gamma(1/3, -(b*c*e*n*x + b*c*d*n)*log(F)/e)} / F^{((b*c*d - a*c*e)*n/e)} - 3 * (4*b*c*e*n*log(F) - 3*(b^2*c^2*e*n^2*x + b^2*c^2*d*n^2)*log(F)^2) * (e*x + d)^{(1/3)} * F^{(b*c*n*x + a*c*n)}) / (b^3*c^3*n^3*log(F)^3)$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((F\*\*(c\*(b\*x+a)))\*\*n\*(e\*x+d)\*\*(4/3), x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (ex + d)^{\frac{4}{3}} (F^{(bx+a)c})^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((F^(c*(b*x+a)))^n*(e*x+d)^(4/3),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^(4/3)*(F^((b*x + a)*c))^n, x)
```



### 3.51 $\int F^{c(a+bx)}(d+ex) dx$

**Optimal.** Leaf size=48

$$\frac{(d+ex)F^{c(a+bx)}}{bc \log(F)} - \frac{eF^{c(a+bx)}}{b^2c^2 \log^2(F)}$$

[Out]  $-\left(\frac{eF^{c(a+bx)}}{b^2c^2 \log^2(F)}\right) + \frac{F^{c(a+bx)}(d+ex)}{bc \log(F)}$

**Rubi [A]** time = 0.0171887, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$ , Rules used = {2176, 2194}

$$\frac{(d+ex)F^{c(a+bx)}}{bc \log(F)} - \frac{eF^{c(a+bx)}}{b^2c^2 \log^2(F)}$$

Antiderivative was successfully verified.

[In] Int[F<sup>c(a+bx)</sup>(d+ex), x]

[Out]  $-\left(\frac{eF^{c(a+bx)}}{b^2c^2 \log^2(F)}\right) + \frac{F^{c(a+bx)}(d+ex)}{bc \log(F)}$

#### Rule 2176

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[((c + d*x)^m*(b*F^(g*(e + f*x)))^n)/(f*g*n*Log[F]),
x] - Dist[(d*m)/(f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m]
] && !$UseGamma == True
```

#### Rule 2194

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

#### Rubi steps

$$\int F^{c(a+bx)}(d+ex) dx = \frac{F^{c(a+bx)}(d+ex)}{bc \log(F)} - \frac{e \int F^{c(a+bx)} dx}{bc \log(F)}$$

$$= -\frac{e F^{c(a+bx)}}{b^2 c^2 \log^2(F)} + \frac{F^{c(a+bx)}(d+ex)}{bc \log(F)}$$

**Mathematica [A]** time = 0.0231951, size = 34, normalized size = 0.71

$$\frac{F^{c(a+bx)}(bc \log(F)(d+ex) - e)}{b^2 c^2 \log^2(F)}$$

Antiderivative was successfully verified.

[In] Integrate[F^(c\*(a + b\*x))\*(d + e\*x), x]

[Out] (F^(c\*(a + b\*x))\*(-e + b\*c\*(d + e\*x)\*Log[F]))/(b^2\*c^2\*Log[F]^2)

**Maple [A]** time = 0., size = 38, normalized size = 0.8

$$\frac{(\ln(F) bcex + \ln(F) bcd - e) F^{c(bx+a)}}{b^2 c^2 (\ln(F))^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(c\*(b\*x+a))\*(e\*x+d), x)

[Out] (ln(F)\*b\*c\*e\*x+ln(F)\*b\*c\*d-e)\*F^(c\*(b\*x+a))/b^2/c^2/ln(F)^2

**Maxima [A]** time = 1.04137, size = 81, normalized size = 1.69

$$\frac{F^{bcx+ac}d}{bc \log(F)} + \frac{(F^{ac}bcx \log(F) - F^{ac})F^{bcx}e}{b^2 c^2 \log(F)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(c\*(b\*x+a))\*(e\*x+d), x, algorithm="maxima")

[Out]  $F^{(b*c*x + a*c)*d/(b*c*\log(F))} + (F^{(a*c)*b*c*x*\log(F)} - F^{(a*c)})*F^{(b*c*x)} * e/(b^2*c^2*\log(F)^2)$

**Fricas [A]** time = 1.54877, size = 90, normalized size = 1.88

$$\frac{((bcex + bcd) \log(F) - e)F^{bcx+ac}}{b^2c^2 \log(F)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(c*(b*x+a))*(e*x+d),x, algorithm="fricas")`

[Out]  $((b*c*e*x + b*c*d)*\log(F) - e)*F^{(b*c*x + a*c)}/(b^2*c^2*\log(F)^2)$

**Sympy [A]** time = 0.128976, size = 60, normalized size = 1.25

$$\begin{cases} \frac{F^{c(a+bx)}(bcd \log(F) + bcex \log(F) - e)}{b^2c^2 \log(F)^2} & \text{for } b^2c^2 \log(F)^2 \neq 0 \\ dx + \frac{ex^2}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F**(c*(b*x+a))*(e*x+d),x)`

[Out] `Piecewise((F**(c*(a + b*x))*(b*c*d*log(F) + b*c*e*x*log(F) - e)/(b**2*c**2*log(F)**2), Ne(b**2*c**2*log(F)**2, 0)), (d*x + e*x**2/2, True))`

**Giac [C]** time = 1.29588, size = 1462, normalized size = 30.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(c*(b*x+a))*(e*x+d),x, algorithm="giac")`

[Out]  $(2*((\pi*b^2*c^2*\log(\text{abs}(F))*\text{sgn}(F) - \pi*b^2*c^2*\log(\text{abs}(F)))*(\pi*b*c*x*\text{sgn}(F) - \pi*b*c*x)/((\pi^2*b^2*c^2*\text{sgn}(F) - \pi^2*b^2*c^2 + 2*b^2*c^2*\log(\text{abs}(F)))$

$$\begin{aligned}
&^2 + 4*(\pi*b^2*c^2*\log(\text{abs}(F))*\text{sgn}(F) - \pi*b^2*c^2*\log(\text{abs}(F)))^2 + (\pi \\
&^2*b^2*c^2*\text{sgn}(F) - \pi^2*b^2*c^2 + 2*b^2*c^2*\log(\text{abs}(F))^2)*(b*c*x*\log(\text{abs}( \\
&F)) - 1)/((\pi^2*b^2*c^2*\text{sgn}(F) - \pi^2*b^2*c^2 + 2*b^2*c^2*\log(\text{abs}(F))^2)^2 \\
&+ 4*(\pi*b^2*c^2*\log(\text{abs}(F))*\text{sgn}(F) - \pi*b^2*c^2*\log(\text{abs}(F)))^2))*\cos(-1/2*\pi \\
&i*b*c*x*\text{sgn}(F) + 1/2*\pi*b*c*x - 1/2*\pi*a*c*\text{sgn}(F) + 1/2*\pi*a*c) + ((\pi^2*b^ \\
&2*c^2*\text{sgn}(F) - \pi^2*b^2*c^2 + 2*b^2*c^2*\log(\text{abs}(F))^2)*(\pi*b*c*x*\text{sgn}(F) - \pi \\
&i*b*c*x))/((\pi^2*b^2*c^2*\text{sgn}(F) - \pi^2*b^2*c^2 + 2*b^2*c^2*\log(\text{abs}(F))^2)^2 \\
&+ 4*(\pi*b^2*c^2*\log(\text{abs}(F))*\text{sgn}(F) - \pi*b^2*c^2*\log(\text{abs}(F)))^2) - 4*(\pi*b^2 \\
&*c^2*\log(\text{abs}(F))*\text{sgn}(F) - \pi*b^2*c^2*\log(\text{abs}(F)))*(b*c*x*\log(\text{abs}(F)) - 1)/( \\
&(\pi^2*b^2*c^2*\text{sgn}(F) - \pi^2*b^2*c^2 + 2*b^2*c^2*\log(\text{abs}(F))^2)^2 + 4*(\pi*b^ \\
&2*c^2*\log(\text{abs}(F))*\text{sgn}(F) - \pi*b^2*c^2*\log(\text{abs}(F)))^2))*\sin(-1/2*\pi*b*c*x*\text{sgn} \\
&n(F) + 1/2*\pi*b*c*x - 1/2*\pi*a*c*\text{sgn}(F) + 1/2*\pi*a*c))*e^(b*c*x*\log(\text{abs}(F)) \\
&+ a*c*\log(\text{abs}(F)) + 1) - 1/2*I*((2*\pi*b*c*x*\text{sgn}(F) - 2*\pi*b*c*x - 4*I*b*c* \\
&x*\log(\text{abs}(F)) + 4*I)*e^(1/2*I*\pi*b*c*x*\text{sgn}(F) - 1/2*I*\pi*b*c*x + 1/2*I*\pi*a \\
&*c*\text{sgn}(F) - 1/2*I*\pi*a*c)/(2*\pi^2*b^2*c^2*\text{sgn}(F) + 4*I*\pi*b^2*c^2*\log(\text{abs}(F) \\
&))*\text{sgn}(F) - 2*\pi^2*b^2*c^2 - 4*I*\pi*b^2*c^2*\log(\text{abs}(F)) + 4*b^2*c^2*\log(\text{abs} \\
&(F))^2) + (2*\pi*b*c*x*\text{sgn}(F) - 2*\pi*b*c*x + 4*I*b*c*x*\log(\text{abs}(F)) - 4*I)*e^ \\
&(-1/2*I*\pi*b*c*x*\text{sgn}(F) + 1/2*I*\pi*b*c*x - 1/2*I*\pi*a*c*\text{sgn}(F) + 1/2*I*\pi*a \\
&*c)/(2*\pi^2*b^2*c^2*\text{sgn}(F) - 4*I*\pi*b^2*c^2*\log(\text{abs}(F))*\text{sgn}(F) - 2*\pi^2*b^2 \\
&*c^2 + 4*I*\pi*b^2*c^2*\log(\text{abs}(F)) + 4*b^2*c^2*\log(\text{abs}(F))^2))*e^(b*c*x*\log( \\
&\text{abs}(F)) + a*c*\log(\text{abs}(F)) + 1) + 2*(2*b*c*d*\cos(-1/2*\pi*b*c*x*\text{sgn}(F) + 1/2* \\
&\pi*b*c*x - 1/2*\pi*a*c*\text{sgn}(F) + 1/2*\pi*a*c)*\log(\text{abs}(F))/(4*b^2*c^2*\log(\text{abs}(F) \\
&))^2 + (\pi*b*c*\text{sgn}(F) - \pi*b*c)^2) - (\pi*b*c*\text{sgn}(F) - \pi*b*c)*d*\sin(-1/2*\pi \\
&*b*c*x*\text{sgn}(F) + 1/2*\pi*b*c*x - 1/2*\pi*a*c*\text{sgn}(F) + 1/2*\pi*a*c)/(4*b^2*c^2* \\
&\log(\text{abs}(F))^2 + (\pi*b*c*\text{sgn}(F) - \pi*b*c)^2))*e^(b*c*x*\log(\text{abs}(F)) + a*c*\log( \\
&\text{abs}(F))) - 1/2*I*(-2*I*d*e^(1/2*I*\pi*b*c*x*\text{sgn}(F) - 1/2*I*\pi*b*c*x + 1/2*I* \\
&\pi*a*c*\text{sgn}(F) - 1/2*I*\pi*a*c)/(I*\pi*b*c*\text{sgn}(F) - I*\pi*b*c + 2*b*c*\log(\text{abs}(F) \\
&))) + 2*I*d*e^(-1/2*I*\pi*b*c*x*\text{sgn}(F) + 1/2*I*\pi*b*c*x - 1/2*I*\pi*a*c*\text{sgn}(F) \\
&+ 1/2*I*\pi*a*c)/(-I*\pi*b*c*\text{sgn}(F) + I*\pi*b*c + 2*b*c*\log(\text{abs}(F))))*e^(b*c \\
&*x*\log(\text{abs}(F)) + a*c*\log(\text{abs}(F)))
\end{aligned}$$

### 3.52 $\int F^{c(a+bx)} (d + ex + fx^2) dx$

**Optimal.** Leaf size=135

$$-\frac{eF^{c(a+bx)}}{b^2c^2 \log^2(F)} - \frac{2fxF^{c(a+bx)}}{b^2c^2 \log^2(F)} + \frac{2fF^{c(a+bx)}}{b^3c^3 \log^3(F)} + \frac{dF^{c(a+bx)}}{bc \log(F)} + \frac{exF^{c(a+bx)}}{bc \log(F)} + \frac{fx^2F^{c(a+bx)}}{bc \log(F)}$$

[Out]  $(2*f*F^{(c*(a + b*x))})/(b^3*c^3*\text{Log}[F]^3) - (e*F^{(c*(a + b*x))})/(b^2*c^2*\text{Log}[F]^2) - (2*f*F^{(c*(a + b*x))*x})/(b^2*c^2*\text{Log}[F]^2) + (d*F^{(c*(a + b*x))})/(b*c*\text{Log}[F]) + (e*F^{(c*(a + b*x))*x})/(b*c*\text{Log}[F]) + (f*F^{(c*(a + b*x))*x^2})/(b*c*\text{Log}[F])$

**Rubi [A]** time = 0.102553, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 3, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.15$ , Rules used = {2196, 2194, 2176}

$$-\frac{eF^{c(a+bx)}}{b^2c^2 \log^2(F)} - \frac{2fxF^{c(a+bx)}}{b^2c^2 \log^2(F)} + \frac{2fF^{c(a+bx)}}{b^3c^3 \log^3(F)} + \frac{dF^{c(a+bx)}}{bc \log(F)} + \frac{exF^{c(a+bx)}}{bc \log(F)} + \frac{fx^2F^{c(a+bx)}}{bc \log(F)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[F^{(c*(a + b*x))}*(d + e*x + f*x^2), x]$

[Out]  $(2*f*F^{(c*(a + b*x))})/(b^3*c^3*\text{Log}[F]^3) - (e*F^{(c*(a + b*x))})/(b^2*c^2*\text{Log}[F]^2) - (2*f*F^{(c*(a + b*x))*x})/(b^2*c^2*\text{Log}[F]^2) + (d*F^{(c*(a + b*x))})/(b*c*\text{Log}[F]) + (e*F^{(c*(a + b*x))*x})/(b*c*\text{Log}[F]) + (f*F^{(c*(a + b*x))*x^2})/(b*c*\text{Log}[F])$

#### Rule 2196

$\text{Int}[(F_)^{((c_.)*(v_))*}(u_), x\_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[F^{(c*\text{ExpandToSum}[v, x])}, u, x], x] /; \text{FreeQ}\{F, c\}, x] \ \&\& \ \text{PolynomialQ}[u, x] \ \&\& \ \text{LinearQ}[v, x] \ \&\& \ !\$UseGamma == True$

#### Rule 2194

$\text{Int}[(F_)^{((c_.)*((a_.) + (b_.)*(x_)))^n}, x\_Symbol] \rightarrow \text{Simp}[(F^{(c*(a + b*x))})^n/(b*c*n*\text{Log}[F]), x] /; \text{FreeQ}\{F, a, b, c, n\}, x]$

#### Rule 2176

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_)))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[((c + d*x)^m*(b*F^(g*(e + f*x)))^n)/(f*g*n*Log[F]),
x] - Dist[(d*m)/(f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] && !UseGamma == True
```

### Rubi steps

$$\begin{aligned}
\int F^{c(a+bx)} (d + ex + fx^2) dx &= \int (dF^{c(a+bx)} + eF^{c(a+bx)}x + fF^{c(a+bx)}x^2) dx \\
&= d \int F^{c(a+bx)} dx + e \int F^{c(a+bx)}x dx + f \int F^{c(a+bx)}x^2 dx \\
&= \frac{dF^{c(a+bx)}}{bc \log(F)} + \frac{eF^{c(a+bx)}x}{bc \log(F)} + \frac{fF^{c(a+bx)}x^2}{bc \log(F)} - \frac{e \int F^{c(a+bx)} dx}{bc \log(F)} - \frac{(2f) \int F^{c(a+bx)}x dx}{bc \log(F)} \\
&= -\frac{eF^{c(a+bx)}}{b^2c^2 \log^2(F)} - \frac{2fF^{c(a+bx)}x}{b^2c^2 \log^2(F)} + \frac{dF^{c(a+bx)}}{bc \log(F)} + \frac{eF^{c(a+bx)}x}{bc \log(F)} + \frac{fF^{c(a+bx)}x^2}{bc \log(F)} + \frac{(2f) \int F^{c(a+bx)} dx}{b^2c^2 \log^2(F)} \\
&= \frac{2fF^{c(a+bx)}}{b^3c^3 \log^3(F)} - \frac{eF^{c(a+bx)}}{b^2c^2 \log^2(F)} - \frac{2fF^{c(a+bx)}x}{b^2c^2 \log^2(F)} + \frac{dF^{c(a+bx)}}{bc \log(F)} + \frac{eF^{c(a+bx)}x}{bc \log(F)} + \frac{fF^{c(a+bx)}x^2}{bc \log(F)}
\end{aligned}$$

**Mathematica [A]** time = 0.0749048, size = 56, normalized size = 0.41

$$\frac{F^{c(a+bx)} (b^2c^2 \log^2(F)(d + x(e + fx)) - bc \log(F)(e + 2fx) + 2f)}{b^3c^3 \log^3(F)}$$

Antiderivative was successfully verified.

```
[In] Integrate[F^(c*(a + b*x))*(d + e*x + f*x^2),x]
```

```
[Out] (F^(c*(a + b*x))*(2*f - b*c*(e + 2*f*x)*Log[F] + b^2*c^2*(d + x*(e + f*x))*
Log[F]^2))/(b^3*c^3*Log[F]^3)
```

**Maple [A]** time = 0.003, size = 80, normalized size = 0.6

$$\frac{(fx^2b^2c^2 (\ln(F))^2 + (\ln(F))^2 b^2c^2ex + b^2c^2 (\ln(F))^2 d - 2 \ln(F) bcfx - \ln(F) bce + 2f) F^{c(bx+a)}}{b^3c^3 (\ln(F))^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(F^(c*(b*x+a))*(f*x^2+e*x+d),x)`

[Out]  $(f*x^2*b^2*c^2*\ln(F)^2+\ln(F)^2*b^2*c^2*e*x+b^2*c^2*\ln(F)^2*d-2*\ln(F)*b*c*f*x-\ln(F)*b*c*e+2*f)*F^(c*(b*x+a))/b^3/c^3/\ln(F)^3$

**Maxima [A]** time = 1.1881, size = 158, normalized size = 1.17

$$\frac{F^{bcx+ac}d}{bc \log(F)} + \frac{(F^{ac}bcx \log(F) - F^{ac})F^{bcx}e}{b^2c^2 \log(F)^2} + \frac{(F^{ac}b^2c^2x^2 \log(F)^2 - 2F^{ac}bcx \log(F) + 2F^{ac})F^{bcx}f}{b^3c^3 \log(F)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(c*(b*x+a))*(f*x^2+e*x+d),x, algorithm="maxima")`

[Out]  $F^{(b*c*x + a*c)}*d/(b*c*\log(F)) + (F^{(a*c)}*b*c*x*\log(F) - F^{(a*c)})*F^{(b*c*x)}*e/(b^2*c^2*\log(F)^2) + (F^{(a*c)}*b^2*c^2*x^2*\log(F)^2 - 2*F^{(a*c)}*b*c*x*\log(F) + 2*F^{(a*c)})*F^{(b*c*x)}*f/(b^3*c^3*\log(F)^3)$

**Fricas [A]** time = 1.51259, size = 167, normalized size = 1.24

$$\frac{((b^2c^2fx^2 + b^2c^2ex + b^2c^2d) \log(F)^2 - (2bcfx + bce) \log(F) + 2f)F^{bcx+ac}}{b^3c^3 \log(F)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(c*(b*x+a))*(f*x^2+e*x+d),x, algorithm="fricas")`

[Out]  $((b^2*c^2*f*x^2 + b^2*c^2*e*x + b^2*c^2*d)*\log(F)^2 - (2*b*c*f*x + b*c*e)*\log(F) + 2*f)*F^{(b*c*x + a*c)}/(b^3*c^3*\log(F)^3)$

**Sympy [A]** time = 0.155854, size = 116, normalized size = 0.86

$$\begin{cases} \frac{F^{c(a+bx)}(b^2c^2d \log(F)^2 + b^2c^2ex \log(F)^2 + b^2c^2fx^2 \log(F)^2 - bce \log(F) - 2bcfx \log(F) + 2f)}{b^3c^3 \log(F)^3} & \text{for } b^3c^3 \log(F)^3 \neq 0 \\ dx + \frac{ex^2}{2} + \frac{fx^3}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(c*(b*x+a))*(f*x**2+e*x+d),x)
```

```
[Out] Piecewise((F**(c*(a + b*x))*(b**2*c**2*d*log(F)**2 + b**2*c**2*e*x*log(F)**2 + b**2*c**2*f*x**2*log(F)**2 - b*c*e*log(F) - 2*b*c*f*x*log(F) + 2*f)/(b**3*c**3*log(F)**3), Ne(b**3*c**3*log(F)**3, 0)), (d*x + e*x**2/2 + f*x**3/3, True))
```

**Giac [C]** time = 1.3837, size = 3362, normalized size = 24.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] (2*((pi*b^2*c^2*log(abs(F))*sgn(F) - pi*b^2*c^2*log(abs(F)))*(pi*b*c*x*sgn(F) - pi*b*c*x)/((pi^2*b^2*c^2*sgn(F) - pi^2*b^2*c^2 + 2*b^2*c^2*log(abs(F))^2)^2 + 4*(pi*b^2*c^2*log(abs(F))*sgn(F) - pi*b^2*c^2*log(abs(F)))^2) + (pi^2*b^2*c^2*sgn(F) - pi^2*b^2*c^2 + 2*b^2*c^2*log(abs(F))^2)*(b*c*x*log(abs(F)) - 1)/((pi^2*b^2*c^2*sgn(F) - pi^2*b^2*c^2 + 2*b^2*c^2*log(abs(F))^2)^2 + 4*(pi*b^2*c^2*log(abs(F))*sgn(F) - pi*b^2*c^2*log(abs(F)))^2))*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c) + ((pi^2*b^2*c^2*sgn(F) - pi^2*b^2*c^2 + 2*b^2*c^2*log(abs(F))^2)*(pi*b*c*x*sgn(F) - pi*b*c*x)/((pi^2*b^2*c^2*sgn(F) - pi^2*b^2*c^2 + 2*b^2*c^2*log(abs(F))^2)^2 + 4*(pi*b^2*c^2*log(abs(F))*sgn(F) - pi*b^2*c^2*log(abs(F)))^2) - 4*(pi*b^2*c^2*log(abs(F))*sgn(F) - pi*b^2*c^2*log(abs(F)))*(b*c*x*log(abs(F)) - 1)/((pi^2*b^2*c^2*sgn(F) - pi^2*b^2*c^2 + 2*b^2*c^2*log(abs(F))^2)^2 + 4*(pi*b^2*c^2*log(abs(F))*sgn(F) - pi*b^2*c^2*log(abs(F)))^2))*sin(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F)) + 1) - 1/2*I*((2*pi*b*c*x*sgn(F) - 2*pi*b*c*x - 4*I*b*c*x*log(abs(F)) + 4*I)*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*a*c)/(2*pi^2*b^2*c^2*sgn(F) + 4*I*pi*b^2*c^2*log(abs(F))*sgn(F) - 2*pi^2*b^2*c^2 - 4*I*pi*b^2*c^2*log(abs(F)) + 4*b^2*c^2*log(abs(F))^2) + (2*pi*b*c*x*sgn(F) - 2*pi*b*c*x + 4*I*b*c*x*log(abs(F)) - 4*I)*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c)/(2*pi^2*b^2*c^2*sgn(F) - 4*I*pi*b^2*c^2*log(abs(F))*sgn(F) - 2*pi^2*b^2*c^2 + 4*I*pi*b^2*c^2*log(abs(F)) + 4*b^2*c^2*log(abs(F))^2))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F)) + 1) + (((pi^2*b^2*c^2*f*x^2*sgn(F) - pi^2*b^2*c^2*f*x^2 + 2*b^2*c^2*f*x^2*log(abs(F))^2 + pi^2*b^2*c^2*d*sgn(F) - pi^2*b^2*c^2*d + 2*b^2*c^2*d*log(abs(F))^2 - 4*b*c*f*x*log(abs(F)) + 4*f)*(3*pi^2*b^3*c^3*log(abs(F))*sgn(F) - 3*pi^2*b^3*c^3*log(abs(F)) + 2*b^3*c^3*log(abs(F)))^3)/((pi^3*b^3*c^3*sgn(F) - 3*pi*b^3*c^3*log(abs(F))^2*sgn(F) - pi^3*b^3*c^3
```



$$\begin{aligned}
& c^3 + 3\pi b^3 c^3 \log(\text{abs}(F))^2 + (3\pi^2 b^3 c^3 \log(\text{abs}(F)) \text{sgn}(F) - \\
& 3\pi^2 b^3 c^3 \log(\text{abs}(F)) + 2b^3 c^3 \log(\text{abs}(F))^3)^2 - 2(\pi^3 b^3 c^3 \text{sgn}(F) - 3\pi b^3 c^3 \log(\text{abs}(F))^2 \text{sgn}(F) - \pi^3 b^3 c^3 + 3\pi b^3 c^3 \log(\text{abs}(F))^2) * (\pi b^2 c^2 f x^2 \log(\text{abs}(F)) \text{sgn}(F) - \pi b^2 c^2 f x^2 \log(\text{abs}(F)) + \pi b^2 c^2 d \log(\text{abs}(F)) \text{sgn}(F) - \pi b^2 c^2 d \log(\text{abs}(F)) - \pi b^2 c^2 f x \text{sgn}(F) + \pi b^2 c^2 f x) / ((\pi^3 b^3 c^3 \text{sgn}(F) - 3\pi b^3 c^3 \log(\text{abs}(F))^2 \text{sgn}(F) - \pi^3 b^3 c^3 + 3\pi b^3 c^3 \log(\text{abs}(F))^2) \text{sgn}(F) - \pi^3 b^3 c^3 + 3\pi b^3 c^3 \log(\text{abs}(F))^2)^2 + (3\pi^2 b^3 c^3 \log(\text{abs}(F)) \text{sgn}(F) - 3\pi^2 b^3 c^3 \log(\text{abs}(F)) + 2b^3 c^3 \log(\text{abs}(F))^3)^2) * \cos(-1/2 \pi b^2 c^2 f x \text{sgn}(F) + 1/2 \pi b^2 c^2 f x - 1/2 \pi a^2 c^2 \text{sgn}(F) + 1/2 \pi a^2 c^2) + ((\pi^3 b^3 c^3 \text{sgn}(F) - 3\pi b^3 c^3 \log(\text{abs}(F))^2 \text{sgn}(F) - \pi^3 b^3 c^3 + 3\pi b^3 c^3 \log(\text{abs}(F))^2) * (\pi^2 b^2 c^2 f x^2 \text{sgn}(F) - \pi^2 b^2 c^2 f x^2 + 2b^2 c^2 f x^2 \log(\text{abs}(F))^2 + \pi^2 b^2 c^2 d \text{sgn}(F) - \pi^2 b^2 c^2 d + 2b^2 c^2 d \log(\text{abs}(F))^2 - 4b^2 c^2 f x \log(\text{abs}(F)) + 4f) / ((\pi^3 b^3 c^3 \text{sgn}(F) - 3\pi b^3 c^3 \log(\text{abs}(F))^2 \text{sgn}(F) - \pi^3 b^3 c^3 + 3\pi b^3 c^3 \log(\text{abs}(F))^2)^2 + (3\pi^2 b^3 c^3 \log(\text{abs}(F)) \text{sgn}(F) - 3\pi^2 b^3 c^3 \log(\text{abs}(F)) + 2b^3 c^3 \log(\text{abs}(F))^3)^2) + 2(3\pi^2 b^3 c^3 \log(\text{abs}(F)) \text{sgn}(F) - 3\pi^2 b^3 c^3 \log(\text{abs}(F)) + 2b^3 c^3 \log(\text{abs}(F))^3) * (\pi b^2 c^2 f x^2 \log(\text{abs}(F)) \text{sgn}(F) - \pi b^2 c^2 f x^2 \log(\text{abs}(F)) + \pi b^2 c^2 d \log(\text{abs}(F)) \text{sgn}(F) - \pi b^2 c^2 d \log(\text{abs}(F)) - \pi b^2 c^2 f x \text{sgn}(F) + \pi b^2 c^2 f x) / ((\pi^3 b^3 c^3 \text{sgn}(F) - 3\pi b^3 c^3 \log(\text{abs}(F))^2 \text{sgn}(F) - \pi^3 b^3 c^3 + 3\pi b^3 c^3 \log(\text{abs}(F))^2)^2 + (3\pi^2 b^3 c^3 \log(\text{abs}(F)) \text{sgn}(F) - 3\pi^2 b^3 c^3 \log(\text{abs}(F)) + 2b^3 c^3 \log(\text{abs}(F))^3)^2) * \sin(-1/2 \pi b^2 c^2 f x \text{sgn}(F) + 1/2 \pi b^2 c^2 f x - 1/2 \pi a^2 c^2 \text{sgn}(F) + 1/2 \pi a^2 c^2) * e^{(b^2 c^2 f x \log(\text{abs}(F)) + a^2 c^2 \log(\text{abs}(F)))} + 1/2 I * ((4I \pi^2 b^2 c^2 f x^2 \text{sgn}(F) - 8\pi b^2 c^2 f x^2 \log(\text{abs}(F)) \text{sgn}(F) - 4I \pi^2 b^2 c^2 f x^2 + 8\pi b^2 c^2 f x^2 \log(\text{abs}(F)) + 8I b^2 c^2 f x^2 \log(\text{abs}(F))^2 + 4I \pi^2 b^2 c^2 d \text{sgn}(F) - 8\pi b^2 c^2 d \log(\text{abs}(F)) \text{sgn}(F) - 4I \pi^2 b^2 c^2 d + 8\pi b^2 c^2 d \log(\text{abs}(F)) + 8I b^2 c^2 d \log(\text{abs}(F))^2 + 8\pi b^2 c^2 f x \text{sgn}(F) - 8\pi b^2 c^2 f x - 16I b^2 c^2 f x \log(\text{abs}(F)) + 16I f) * e^{(1/2 I \pi b^2 c^2 f x \text{sgn}(F) - 1/2 I \pi b^2 c^2 f x + 1/2 I \pi a^2 c^2 \text{sgn}(F) - 1/2 I \pi a^2 c^2) / (-4I \pi^3 b^3 c^3 \text{sgn}(F) + 12\pi^2 b^3 c^3 \log(\text{abs}(F)) \text{sgn}(F) + 12I \pi b^3 c^3 \log(\text{abs}(F))^2 \text{sgn}(F) + 4I \pi^3 b^3 c^3 - 12\pi^2 b^3 c^3 \log(\text{abs}(F)) - 12I \pi b^3 c^3 \log(\text{abs}(F))^2 + 8b^3 c^3 \log(\text{abs}(F))^3) - (4I \pi^2 b^2 c^2 f x^2 \text{sgn}(F) + 8\pi b^2 c^2 f x^2 \log(\text{abs}(F)) \text{sgn}(F) - 4I \pi^2 b^2 c^2 f x^2 - 8\pi b^2 c^2 f x^2 \log(\text{abs}(F)) + 8I b^2 c^2 f x^2 \log(\text{abs}(F))^2 + 4I \pi^2 b^2 c^2 d \text{sgn}(F) + 8\pi b^2 c^2 d \log(\text{abs}(F)) \text{sgn}(F) - 4I \pi^2 b^2 c^2 d - 8\pi b^2 c^2 d \log(\text{abs}(F)) + 8I b^2 c^2 d \log(\text{abs}(F))^2 - 8\pi b^2 c^2 f x \text{sgn}(F) + 8\pi b^2 c^2 f x - 16I b^2 c^2 f x \log(\text{abs}(F)) + 16I f) * e^{(-1/2 I \pi b^2 c^2 f x \text{sgn}(F) + 1/2 I \pi b^2 c^2 f x - 1/2 I \pi a^2 c^2 \text{sgn}(F) + 1/2 I \pi a^2 c^2) / (4I \pi^3 b^3 c^3 \text{sgn}(F) + 12\pi^2 b^3 c^3 \log(\text{abs}(F)) \text{sgn}(F) - 12I \pi b^3 c^3 \log(\text{abs}(F))^2 \text{sgn}(F) - 4I \pi^3 b^3 c^3 - 12\pi^2 b^3 c^3 \log(\text{abs}(F)) + 12I \pi b^3 c^3 \log(\text{abs}(F))^2 + 8b^3 c^3 \log(\text{abs}(F))^3) * e^{(b^2 c^2 f x \log(\text{abs}(F)) + a^2 c^2 \log(\text{abs}(F)))}
\end{aligned}$$

### 3.53 $\int F^{c(a+bx)} (d + ex + fx^2 + gx^3) dx$

**Optimal.** Leaf size=229

$$-\frac{eF^{c(a+bx)}}{b^2c^2 \log^2(F)} - \frac{2fxF^{c(a+bx)}}{b^2c^2 \log^2(F)} + \frac{2fF^{c(a+bx)}}{b^3c^3 \log^3(F)} - \frac{3gx^2F^{c(a+bx)}}{b^2c^2 \log^2(F)} + \frac{6gxF^{c(a+bx)}}{b^3c^3 \log^3(F)} - \frac{6gF^{c(a+bx)}}{b^4c^4 \log^4(F)} + \frac{dF^{c(a+bx)}}{bc \log(F)} + \frac{exF^{c(a+bx)}}{bc \log(F)} + \frac{fx^2F^{c(a+bx)}}{bc \log(F)} + \frac{gx^3F^{c(a+bx)}}{bc \log(F)}$$

[Out]  $(-6F^{c(a+bx)}g)/(b^4c^4 \log^4[F]) + (2fF^{c(a+bx)})/(b^3c^3 \log^3[F]) + (6F^{c(a+bx)}gx)/(b^3c^3 \log^3[F]) - (eF^{c(a+bx)})/(b^2c^2 \log^2[F]) - (2fF^{c(a+bx)}x)/(b^2c^2 \log^2[F]) - (3F^{c(a+bx)}gx^2)/(b^2c^2 \log^2[F]) + (dF^{c(a+bx)})/(bc \log[F]) + (eF^{c(a+bx)}x)/(bc \log[F]) + (fF^{c(a+bx)}x^2)/(bc \log[F]) + (F^{c(a+bx)}gx^3)/(bc \log[F])$

**Rubi [A]** time = 0.191302, antiderivative size = 229, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 3, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.12$ , Rules used = {2196, 2194, 2176}

$$-\frac{eF^{c(a+bx)}}{b^2c^2 \log^2(F)} - \frac{2fxF^{c(a+bx)}}{b^2c^2 \log^2(F)} + \frac{2fF^{c(a+bx)}}{b^3c^3 \log^3(F)} - \frac{3gx^2F^{c(a+bx)}}{b^2c^2 \log^2(F)} + \frac{6gxF^{c(a+bx)}}{b^3c^3 \log^3(F)} - \frac{6gF^{c(a+bx)}}{b^4c^4 \log^4(F)} + \frac{dF^{c(a+bx)}}{bc \log(F)} + \frac{exF^{c(a+bx)}}{bc \log(F)} + \frac{fx^2F^{c(a+bx)}}{bc \log(F)} + \frac{gx^3F^{c(a+bx)}}{bc \log(F)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[F^{c(a+bx)}(d + ex + fx^2 + gx^3), x]$

[Out]  $(-6F^{c(a+bx)}g)/(b^4c^4 \log^4[F]) + (2fF^{c(a+bx)})/(b^3c^3 \log^3[F]) + (6F^{c(a+bx)}gx)/(b^3c^3 \log^3[F]) - (eF^{c(a+bx)})/(b^2c^2 \log^2[F]) - (2fF^{c(a+bx)}x)/(b^2c^2 \log^2[F]) - (3F^{c(a+bx)}gx^2)/(b^2c^2 \log^2[F]) + (dF^{c(a+bx)})/(bc \log[F]) + (eF^{c(a+bx)}x)/(bc \log[F]) + (fF^{c(a+bx)}x^2)/(bc \log[F]) + (F^{c(a+bx)}gx^3)/(bc \log[F])$

#### Rule 2196

$\text{Int}[(F_)^{((c_.)*(v_))*(u_)}, x\_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[F^{c \text{ExpandToSum}[v, x]}, u, x], x] /;$  FreeQ[{F, c}, x] && PolynomialQ[u, x] && LinearQ[v, x] && !\$UseGamma === True

#### Rule 2194

$\text{Int}[(F_)^{((c_.)*((a_.) + (b_.)*(x_)))^{(n_.)}, x\_Symbol] \rightarrow \text{Simp}[(F^{c(a+bx)})^n/(b*c*n*\log[F]), x] /;$  FreeQ[{F, a, b, c, n}, x]

Rule 2176

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[((c + d*x)^m*(b*F^(g*(e + f*x)))^n)/(f*g*n*Log[F]),
x] - Dist[(d*m)/(f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] && !$UseGamma === True
```

Rubi steps

$$\begin{aligned}
\int F^{c(ax)}(d + ex + fx^2 + gx^3) dx &= \int (dF^{c(ax)} + eF^{c(ax)}x + fF^{c(ax)}x^2 + F^{c(ax)}gx^3) dx \\
&= d \int F^{c(ax)} dx + e \int F^{c(ax)}x dx + f \int F^{c(ax)}x^2 dx + g \int F^{c(ax)}x^3 dx \\
&= \frac{dF^{c(ax)}}{bc \log(F)} + \frac{eF^{c(ax)}x}{bc \log(F)} + \frac{fF^{c(ax)}x^2}{bc \log(F)} + \frac{F^{c(ax)}gx^3}{bc \log(F)} - \frac{e \int F^{c(ax)} dx}{bc \log(F)} - \frac{(2f) \int F^{c(ax)}x dx}{bc \log(F)} \\
&= -\frac{eF^{c(ax)}}{b^2c^2 \log^2(F)} - \frac{2fF^{c(ax)}x}{b^2c^2 \log^2(F)} - \frac{3F^{c(ax)}gx^2}{b^2c^2 \log^2(F)} + \frac{dF^{c(ax)}}{bc \log(F)} + \frac{eF^{c(ax)}x}{bc \log(F)} + \frac{fF^{c(ax)}x^2}{bc \log(F)} \\
&= \frac{2fF^{c(ax)}}{b^3c^3 \log^3(F)} + \frac{6F^{c(ax)}gx}{b^3c^3 \log^3(F)} - \frac{eF^{c(ax)}}{b^2c^2 \log^2(F)} - \frac{2fF^{c(ax)}x}{b^2c^2 \log^2(F)} - \frac{3F^{c(ax)}gx^2}{b^2c^2 \log^2(F)} + \frac{dF^{c(ax)}}{bc \log(F)} \\
&= -\frac{6F^{c(ax)}g}{b^4c^4 \log^4(F)} + \frac{2fF^{c(ax)}}{b^3c^3 \log^3(F)} + \frac{6F^{c(ax)}gx}{b^3c^3 \log^3(F)} - \frac{eF^{c(ax)}}{b^2c^2 \log^2(F)} - \frac{2fF^{c(ax)}x}{b^2c^2 \log^2(F)} - \frac{3F^{c(ax)}gx^2}{b^2c^2 \log^2(F)}
\end{aligned}$$

**Mathematica [A]** time = 0.110618, size = 84, normalized size = 0.37

$$\frac{F^{c(ax)}(b^3c^3 \log^3(F)(d + x(e + x(f + gx))) - b^2c^2 \log^2(F)(e + x(2f + 3gx)) + 2bc \log(F)(f + 3gx) - 6g)}{b^4c^4 \log^4(F)}$$

Antiderivative was successfully verified.

[In] Integrate[F^(c\*(a + b\*x))\*(d + e\*x + f\*x^2 + g\*x^3), x]

[Out] (F^(c\*(a + b\*x))\*(-6\*g + 2\*b\*c\*(f + 3\*g\*x)\*Log[F] - b^2\*c^2\*(e + x\*(2\*f + 3\*g\*x))\*Log[F]^2 + b^3\*c^3\*(d + x\*(e + x\*(f + g\*x)))\*Log[F]^3)/(b^4\*c^4\*Log[F]^4)

**Maple [A]** time = 0.004, size = 138, normalized size = 0.6

$$\frac{(gx^3b^3c^3(\ln(F))^3 + (\ln(F))^3b^3c^3fx^2 + (\ln(F))^3b^3c^3ex + b^3c^3(\ln(F))^3d - 3(\ln(F))^2b^2c^2gx^2 - 2(\ln(F))^2b^2c^2fx - b^2c^2e)}{b^4c^4(\ln(F))^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(F^(c*(b*x+a))*(g*x^3+f*x^2+e*x+d),x)`

[Out]  $(g*x^3*b^3*c^3*\ln(F)^3+\ln(F)^3*b^3*c^3*f*x^2+\ln(F)^3*b^3*c^3*e*x+b^3*c^3*\ln(F)^3*d-3*\ln(F)^2*b^2*c^2*g*x^2-2*\ln(F)^2*b^2*c^2*f*x-b^2*c^2*\ln(F)^2*e+6*\ln(F)*b*c*g*x+2*f*b*c*\ln(F)-6*g)*F^(c*(b*x+a))/b^4/c^4/\ln(F)^4$

**Maxima [A]** time = 1.13206, size = 262, normalized size = 1.14

$$\frac{F^{bcx+ac}d}{bc \log(F)} + \frac{(F^{ac}bcx \log(F) - F^{ac})F^{bcx}e}{b^2c^2 \log(F)^2} + \frac{(F^{ac}b^2c^2x^2 \log(F)^2 - 2F^{ac}bcx \log(F) + 2F^{ac})F^{bcx}f}{b^3c^3 \log(F)^3} + \frac{(F^{ac}b^3c^3x^3 \log(F)^3 - 3F^{ac}b^2c^2x^2 \log(F)^2 + 6F^{ac}bcx \log(F) - 6F^{ac})F^{bcx}g}{b^4c^4 \log(F)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(c*(b*x+a))*(g*x^3+f*x^2+e*x+d),x, algorithm="maxima")`

[Out]  $F^{(b*c*x + a*c)*d/(b*c*\log(F))} + (F^{(a*c)*b*c*x*\log(F)} - F^{(a*c)})*F^{(b*c*x)*e/(b^2*c^2*\log(F)^2)} + (F^{(a*c)*b^2*c^2*x^2*\log(F)^2} - 2*F^{(a*c)*b*c*x*\log(F)} + 2*F^{(a*c)})*F^{(b*c*x)*f/(b^3*c^3*\log(F)^3)} + (F^{(a*c)*b^3*c^3*x^3*\log(F)^3} - 3*F^{(a*c)*b^2*c^2*x^2*\log(F)^2} + 6*F^{(a*c)*b*c*x*\log(F)} - 6*F^{(a*c)})*F^{(b*c*x)*g/(b^4*c^4*\log(F)^4)}$

**Fricas [A]** time = 1.51232, size = 269, normalized size = 1.17

$$\frac{((b^3c^3gx^3 + b^3c^3fx^2 + b^3c^3ex + b^3c^3d) \log(F)^3 - (3b^2c^2gx^2 + 2b^2c^2fx + b^2c^2e) \log(F)^2 + 2(3bcgx + bcf) \log(F) - 6g) F^{(b*c*x + a*c)}}{b^4c^4 \log(F)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(c*(b*x+a))*(g*x^3+f*x^2+e*x+d),x, algorithm="fricas")`

[Out]  $((b^3*c^3*g*x^3 + b^3*c^3*f*x^2 + b^3*c^3*e*x + b^3*c^3*d)*\log(F)^3 - (3*b^2*c^2*g*x^2 + 2*b^2*c^2*f*x + b^2*c^2*e)*\log(F)^2 + 2*(3*b*c*g*x + b*c*f)*\log(F) - 6*g)*F^{(b*c*x + a*c)}/(b^4*c^4*\log(F)^4)$

**Sympy [A]** time = 0.184252, size = 190, normalized size = 0.83

$$\left\{ \begin{array}{l} \frac{F^{c(a+bx)}(b^3c^3d \log(F)^3 + b^3c^3ex \log(F)^3 + b^3c^3fx^2 \log(F)^3 + b^3c^3gx^3 \log(F)^3 - b^2c^2e \log(F)^2 - 2b^2c^2fx \log(F)^2 - 3b^2c^2gx^2 \log(F)^2 + 2bcf \log(F) + 6bcgx \log(F) - 6}{b^4c^4 \log(F)^4} \\ dx + \frac{cx^2}{2} + \frac{fx^3}{3} + \frac{gx^4}{4} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(c\*(b\*x+a))\*(g\*x\*\*3+f\*x\*\*2+e\*x+d), x)

[Out] Piecewise((F\*\*(c\*(a + b\*x))\*(b\*\*3\*c\*\*3\*d\*log(F)\*\*3 + b\*\*3\*c\*\*3\*e\*x\*log(F)\*\*3 + b\*\*3\*c\*\*3\*f\*x\*\*2\*log(F)\*\*3 + b\*\*3\*c\*\*3\*g\*x\*\*3\*log(F)\*\*3 - b\*\*2\*c\*\*2\*e\*log(F)\*\*2 - 2\*b\*\*2\*c\*\*2\*f\*x\*log(F)\*\*2 - 3\*b\*\*2\*c\*\*2\*g\*x\*\*2\*log(F)\*\*2 + 2\*b\*c\*\*f\*log(F) + 6\*b\*c\*g\*x\*log(F) - 6\*g)/(b\*\*4\*c\*\*4\*log(F)\*\*4), Ne(b\*\*4\*c\*\*4\*log(F)\*\*4, 0)), (d\*x + e\*x\*\*2/2 + f\*x\*\*3/3 + g\*x\*\*4/4, True))

**Giac [C]** time = 1.41631, size = 5787, normalized size = 25.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(c\*(b\*x+a))\*(g\*x^3+f\*x^2+e\*x+d), x, algorithm="giac")

[Out] (2\*((pi\*b^2\*c^2\*log(abs(F))\*sgn(F) - pi\*b^2\*c^2\*log(abs(F)))\*(pi\*b\*c\*x\*sgn(F) - pi\*b\*c\*x)/((pi^2\*b^2\*c^2\*sgn(F) - pi^2\*b^2\*c^2 + 2\*b^2\*c^2\*log(abs(F))^2)^2 + 4\*(pi\*b^2\*c^2\*log(abs(F))\*sgn(F) - pi\*b^2\*c^2\*log(abs(F)))^2) + (pi^2\*b^2\*c^2\*sgn(F) - pi^2\*b^2\*c^2 + 2\*b^2\*c^2\*log(abs(F))^2)\*(b\*c\*x\*log(abs(F)) - 1)/((pi^2\*b^2\*c^2\*sgn(F) - pi^2\*b^2\*c^2 + 2\*b^2\*c^2\*log(abs(F))^2)^2 + 4\*(pi\*b^2\*c^2\*log(abs(F))\*sgn(F) - pi\*b^2\*c^2\*log(abs(F)))^2)\*cos(-1/2\*pi\*b\*c\*x\*sgn(F) + 1/2\*pi\*b\*c\*x - 1/2\*pi\*a\*c\*sgn(F) + 1/2\*pi\*a\*c) + ((pi^2\*b^2\*c^2\*sgn(F) - pi^2\*b^2\*c^2 + 2\*b^2\*c^2\*log(abs(F))^2)\*(pi\*b\*c\*x\*sgn(F) - pi\*b\*c\*x)/((pi^2\*b^2\*c^2\*sgn(F) - pi^2\*b^2\*c^2 + 2\*b^2\*c^2\*log(abs(F))^2)^2 + 4\*(pi\*b^2\*c^2\*log(abs(F))\*sgn(F) - pi\*b^2\*c^2\*log(abs(F)))^2) - 4\*(pi\*b^2\*c^2\*log(abs(F))\*sgn(F) - pi\*b^2\*c^2\*log(abs(F)))\*(b\*c\*x\*log(abs(F)) - 1)/((pi^2\*b^2\*c^2\*sgn(F) - pi^2\*b^2\*c^2 + 2\*b^2\*c^2\*log(abs(F))^2)^2 + 4\*(pi\*b^2\*c^2\*log(abs(F))\*sgn(F) - pi\*b^2\*c^2\*log(abs(F)))^2))\*sin(-1/2\*pi\*b\*c\*x\*sgn(F) + 1/2\*pi\*b\*c\*x - 1/2\*pi\*a\*c\*sgn(F) + 1/2\*pi\*a\*c))\*e^(b\*c\*x\*log(abs(F)) + a\*c\*log(abs(F)) + 1) - 1/2\*I\*((2\*pi\*b\*c\*x\*sgn(F) - 2\*pi\*b\*c\*x - 4\*I\*b\*c\*x\*log(abs(F)) + 4\*I)\*e^(1/2\*I\*pi\*b\*c\*x\*sgn(F) - 1/2\*I\*pi\*b\*c\*x + 1/2\*I\*pi\*a\*c\*sgn(F) - 1/2\*I\*pi\*a\*c)/(2\*pi^2\*b^2\*c^2\*sgn(F) + 4\*I\*pi\*b^2\*c^2\*log(abs(F))

$$\begin{aligned}
&))\operatorname{sgn}(F) - 2\pi^2 b^2 c^2 - 4I\pi b^2 c^2 \log(\operatorname{abs}(F)) + 4b^2 c^2 \log(\operatorname{abs}(F))^2 + (2\pi b c x \operatorname{sgn}(F) - 2\pi b c x + 4I b c x \log(\operatorname{abs}(F)) - 4I) e^{(-1/2 I \pi b c x \operatorname{sgn}(F) + 1/2 I \pi b c x - 1/2 I \pi a c \operatorname{sgn}(F) + 1/2 I \pi i a c)} / (2\pi^2 b^2 c^2 \operatorname{sgn}(F) - 4I \pi b^2 c^2 \log(\operatorname{abs}(F)) \operatorname{sgn}(F) - 2\pi^2 b^2 c^2 + 4I \pi b^2 c^2 \log(\operatorname{abs}(F)) + 4b^2 c^2 \log(\operatorname{abs}(F))^2) e^{(b c x \log(\operatorname{abs}(F)) + a c \log(\operatorname{abs}(F)) + 1) - (((3\pi^2 b^3 c^3 g x^3 \log(\operatorname{abs}(F)) \operatorname{sgn}(F) - 3\pi^2 b^3 c^3 g x^3 \log(\operatorname{abs}(F)) + 2b^3 c^3 g x^3 \log(\operatorname{abs}(F))^3 + 3\pi^2 b^2 b^3 c^3 f x^2 \log(\operatorname{abs}(F)) \operatorname{sgn}(F) - 3\pi^2 b^2 b^3 c^3 f x^2 \log(\operatorname{abs}(F)) + 2b^2 b^3 c^3 f x^2 \log(\operatorname{abs}(F))^3 + 3\pi^2 b^2 b^3 c^3 d \log(\operatorname{abs}(F)) \operatorname{sgn}(F) - 3\pi^2 b^2 b^3 c^3 d \log(\operatorname{abs}(F)) + 2b^2 b^3 c^3 d \log(\operatorname{abs}(F))^3 - 3\pi^2 b^2 c^2 g x^2 \operatorname{sgn}(F) + 3\pi^2 b^2 c^2 g x^2 - 6b^2 c^2 g x^2 \log(\operatorname{abs}(F))^2 - 2\pi^2 b^2 c^2 f x \operatorname{sgn}(F) + 2\pi^2 b^2 c^2 f x - 4b^2 c^2 f x \log(\operatorname{abs}(F))^2 + 12b c g x \log(\operatorname{abs}(F)) + 4b c f \log(\operatorname{abs}(F)) - 12g) (\pi^4 b^4 c^4 \operatorname{sgn}(F) - 6\pi^2 b^4 c^4 \log(\operatorname{abs}(F))^2 \operatorname{sgn}(F) - \pi^4 b^4 c^4 + 6\pi^2 b^4 c^4 \log(\operatorname{abs}(F))^2 - 2b^4 c^4 \log(\operatorname{abs}(F))^4) / ((\pi^4 b^4 c^4 \operatorname{sgn}(F) - 6\pi^2 b^4 c^4 \log(\operatorname{abs}(F))^2 \operatorname{sgn}(F) - \pi^4 b^4 c^4 + 6\pi^2 b^4 c^4 \log(\operatorname{abs}(F))^2 - 2b^4 c^4 \log(\operatorname{abs}(F))^4)^2 + 16(\pi^3 b^4 c^4 \log(\operatorname{abs}(F)) \operatorname{sgn}(F) - \pi b^4 c^4 \log(\operatorname{abs}(F))^3 \operatorname{sgn}(F) - \pi^3 b^4 c^4 \log(\operatorname{abs}(F)) + \pi b^4 c^4 \log(\operatorname{abs}(F))^3)^2 - 4(\pi^3 b^3 c^3 g x^3 \operatorname{sgn}(F) - 3\pi b^3 c^3 g x^3 \log(\operatorname{abs}(F))^2 \operatorname{sgn}(F) - \pi^3 b^3 c^3 g x^3 + 3\pi b^3 c^3 g x^3 \log(\operatorname{abs}(F))^2 + \pi^3 b^3 c^3 f x^2 \operatorname{sgn}(F) - 3\pi b^3 c^3 f x^2 \log(\operatorname{abs}(F))^2 \operatorname{sgn}(F) - \pi^3 b^3 c^3 f x^2 + 3\pi b^3 c^3 f x^2 \log(\operatorname{abs}(F))^2 + \pi^3 b^3 c^3 d \operatorname{sgn}(F) - 3\pi b^3 c^3 d \log(\operatorname{abs}(F))^2 \operatorname{sgn}(F) - \pi^3 b^3 c^3 d + 3\pi b^3 c^3 d \log(\operatorname{abs}(F))^2 + 6\pi b^2 c^2 g x^2 \log(\operatorname{abs}(F)) \operatorname{sgn}(F) - 6\pi b^2 c^2 g x^2 \log(\operatorname{abs}(F)) + 4\pi b^2 c^2 f x \log(\operatorname{abs}(F)) \operatorname{sgn}(F) - 4\pi b^2 c^2 f x \log(\operatorname{abs}(F)) - 6\pi b c g x \operatorname{sgn}(F) + 6\pi b c g x - 2\pi b c f \operatorname{sgn}(F) + 2\pi b c f) (\pi^3 b^4 c^4 \log(\operatorname{abs}(F)) \operatorname{sgn}(F) - \pi b^4 c^4 \log(\operatorname{abs}(F))^3 \operatorname{sgn}(F) - \pi^3 b^4 c^4 \log(\operatorname{abs}(F)) + \pi b^4 c^4 \log(\operatorname{abs}(F))^3) / ((\pi^4 b^4 c^4 \operatorname{sgn}(F) - 6\pi^2 b^4 c^4 \log(\operatorname{abs}(F))^2 \operatorname{sgn}(F) - \pi^4 b^4 c^4 + 6\pi^2 b^4 c^4 \log(\operatorname{abs}(F))^2 - 2b^4 c^4 \log(\operatorname{abs}(F))^4)^2 + 16(\pi^3 b^4 c^4 \log(\operatorname{abs}(F)) \operatorname{sgn}(F) - \pi b^4 c^4 \log(\operatorname{abs}(F))^3 \operatorname{sgn}(F) - \pi^3 b^4 c^4 \log(\operatorname{abs}(F)) + \pi b^4 c^4 \log(\operatorname{abs}(F))^3)^2) \cos(-1/2 \pi b c x \operatorname{sgn}(F) + 1/2 \pi b c x - 1/2 \pi a c \operatorname{sgn}(F) + 1/2 \pi i a c) - ((\pi^3 b^3 c^3 g x^3 \operatorname{sgn}(F) - 3\pi b^3 c^3 g x^3 \log(\operatorname{abs}(F))^2 \operatorname{sgn}(F) - \pi^3 b^3 c^3 g x^3 + 3\pi b^3 c^3 g x^3 \log(\operatorname{abs}(F))^2 + \pi^3 b^3 c^3 f x^2 \operatorname{sgn}(F) - 3\pi b^3 c^3 f x^2 \log(\operatorname{abs}(F))^2 \operatorname{sgn}(F) - \pi^3 b^3 c^3 f x^2 + 3\pi b^3 c^3 f x^2 \log(\operatorname{abs}(F))^2 + \pi^3 b^3 c^3 d \operatorname{sgn}(F) - 3\pi b^3 c^3 d \log(\operatorname{abs}(F))^2 \operatorname{sgn}(F) - \pi^3 b^3 c^3 d + 3\pi b^2 c^2 g x^2 \log(\operatorname{abs}(F)) \operatorname{sgn}(F) - 6\pi b^2 c^2 g x^2 \log(\operatorname{abs}(F)) + 4\pi b^2 c^2 f x \log(\operatorname{abs}(F)) \operatorname{sgn}(F) - 4\pi b^2 c^2 f x \log(\operatorname{abs}(F)) - 6\pi b c g x \operatorname{sgn}(F) + 6\pi b c g x - 2\pi b c f \operatorname{sgn}(F) + 2\pi b c f) (\pi^4 b^4 c^4 \operatorname{sgn}(F) - 6\pi^2 b^4 c^4 \log(\operatorname{abs}(F))^2 \operatorname{sgn}(F) - \pi^4 b^4 c^4 + 6\pi^2 b^4 c^4 \log(\operatorname{abs}(F))^2 - 2b^4 c^4 \log(\operatorname{abs}(F))^4) / ((\pi^4 b^4 c^4 \operatorname{sgn}(F) - 6\pi^2 b^4 c^4 \log(\operatorname{abs}(F))^2 \operatorname{sgn}(F) - \pi^4 b^4 c^4 + 6\pi^2 b^4 c^4 \log(\operatorname{abs}(F))^2 - 2b^4 c^4 \log(\operatorname{abs}(F))^4)^2 + 16(\pi^3 b^4 c^4 \log(\operatorname{abs}(F)) \operatorname{sgn}(F) - \pi b^4 c^4 \log(\operatorname{abs}(F))^3 \operatorname{sgn}(F) - \pi^3 b^4 c^4 \log(\operatorname{abs}(F)) + \pi b^4 c^4 \log(\operatorname{abs}(F))^3)^2) + 4(3\pi^2 b^3 c^3 g
\end{aligned}$$

$$\begin{aligned}
& *x^3 \log(\text{abs}(F)) * \text{sgn}(F) - 3\pi^2 b^3 c^3 g x^3 \log(\text{abs}(F)) + 2b^3 c^3 g x^3 \\
& 3 \log(\text{abs}(F))^3 + 3\pi^2 b^3 c^3 f x^2 \log(\text{abs}(F)) * \text{sgn}(F) - 3\pi^2 b^3 c^3 f \\
& f x^2 \log(\text{abs}(F)) + 2b^3 c^3 f x^2 \log(\text{abs}(F))^3 + 3\pi^2 b^3 c^3 d \log(\text{abs}(F)) * \text{sgn}(F) - 3\pi^2 b^3 c^3 d \log(\text{abs}(F))^3 - 3 \\
& \pi^2 b^2 c^2 g x^2 * \text{sgn}(F) + 3\pi^2 b^2 c^2 g x^2 - 6b^2 c^2 g x^2 \log(\text{abs}(F))^2 - 2\pi^2 b^2 c^2 f x * \text{sgn}(F) + 2\pi^2 b^2 c^2 f x - 4b^2 c^2 f x \log \\
& (\text{abs}(F))^2 + 12b^2 c^2 g x \log(\text{abs}(F)) + 4b^2 c^2 f \log(\text{abs}(F)) - 12g * (\pi^3 b^4 \\
& c^4 \log(\text{abs}(F)) * \text{sgn}(F) - \pi b^4 c^4 \log(\text{abs}(F))^3 * \text{sgn}(F) - \pi^3 b^4 c^4 \log \\
& (\text{abs}(F)) + \pi b^4 c^4 \log(\text{abs}(F))^3) / ((\pi^4 b^4 c^4 * \text{sgn}(F) - 6\pi^2 b^4 c^4 \\
& 4 \log(\text{abs}(F))^2 * \text{sgn}(F) - \pi^4 b^4 c^4 + 6\pi^2 b^4 c^4 \log(\text{abs}(F))^2 - 2b^4 \\
& 4 c^4 \log(\text{abs}(F))^4)^2 + 16 * (\pi^3 b^4 c^4 \log(\text{abs}(F)) * \text{sgn}(F) - \pi b^4 c^4 \log \\
& (\text{abs}(F))^3 * \text{sgn}(F) - \pi^3 b^4 c^4 \log(\text{abs}(F)) + \pi b^4 c^4 \log(\text{abs}(F))^3)^2) * \sin(-1/2 * \pi * b * c * x * \text{sgn}(F) + 1/2 * \pi * b * c * x - 1/2 * \pi * a * c * \text{sgn}(F) + 1/2 * \pi * a * c) * e^{(b * c * x * \log(\text{abs}(F)) + a * c * \log(\text{abs}(F)))} - 1/2 * I * ((8\pi^3 b^3 c^3 g x^3 * \text{sgn}(F) + 24 * I * \pi^2 b^3 c^3 g x^3 \log(\text{abs}(F)) * \text{sgn}(F) - 24 * \pi b^3 c^3 g x^3 \log(\text{abs}(F))^2 * \text{sgn}(F) - 8\pi^3 b^3 c^3 g x^3 - 24 * I * \pi^2 b^3 c^3 g x^3 \log(\text{abs}(F)) + 24 * \pi b^3 c^3 g x^3 \log(\text{abs}(F))^2 + 16 * I * b^3 c^3 g x^3 \log(\text{abs}(F))^3 + 8\pi^3 b^3 c^3 f x^2 * \text{sgn}(F) + 24 * I * \pi^2 b^3 c^3 f x^2 \log(\text{abs}(F)) * \text{sgn}(F) - 24 * \pi b^3 c^3 f x^2 \log(\text{abs}(F))^2 * \text{sgn}(F) - 8\pi^3 b^3 c^3 f x^2 - 24 * I * \pi^2 b^3 c^3 f x^2 \log(\text{abs}(F)) + 24 * \pi b^3 c^3 f x^2 \log(\text{abs}(F))^2 + 16 * I * b^3 c^3 f x^2 \log(\text{abs}(F))^3 + 8\pi^3 b^3 c^3 d * \text{sgn}(F) + 24 * I * \pi^2 b^3 c^3 d \log(\text{abs}(F)) * \text{sgn}(F) - 24 * \pi b^3 c^3 d \log(\text{abs}(F))^2 * \text{sgn}(F) - 8\pi^3 b^3 c^3 d - 24 * I * \pi^2 b^3 c^3 d \log(\text{abs}(F)) + 24 * \pi b^3 c^3 d \log(\text{abs}(F))^2 + 16 * I * b^3 c^3 d \log(\text{abs}(F))^3 - 24 * I * \pi^2 b^2 c^2 g x^2 * \text{sgn}(F) + 48 * \pi b^2 c^2 g x^2 \log(\text{abs}(F)) * \text{sgn}(F) + 24 * I * \pi^2 b^2 c^2 g x^2 - 48 * \pi b^2 c^2 g x^2 \log(\text{abs}(F)) - 48 * I * b^2 c^2 g x^2 \log(\text{abs}(F))^2 - 16 * I * \pi^2 b^2 c^2 f x * \text{sgn}(F) + 32 * \pi b^2 c^2 f x \log(\text{abs}(F)) * \text{sgn}(F) + 16 * I * \pi^2 b^2 c^2 f x - 32 * \pi b^2 c^2 f x \log(\text{abs}(F)) - 32 * I * b^2 c^2 f x \log(\text{abs}(F))^2 - 48 * \pi b^2 c^2 g x * \text{sgn}(F) + 48 * \pi b^2 c^2 g x + 96 * I * b^2 c^2 g x \log(\text{abs}(F)) - 16 * \pi b^2 c^2 f * \text{sgn}(F) + 16 * \pi b^2 c^2 f + 32 * I * b^2 c^2 f \log(\text{abs}(F)) - 96 * I * g) * e^{(1/2 * I * \pi * b * c * x * \text{sgn}(F) - 1/2 * I * \pi * b * c * x + 1/2 * I * \pi * a * c * \text{sgn}(F) - 1/2 * I * \pi * a * c) / (8\pi^4 b^4 c^4 * \text{sgn}(F) + 32 * I * \pi^3 b^4 c^4 \log(\text{abs}(F)) * \text{sgn}(F) - 48 * \pi^2 b^4 c^4 \log(\text{abs}(F))^2 * \text{sgn}(F) - 32 * I * \pi b^4 c^4 \log(\text{abs}(F))^3 * \text{sgn}(F) - 8\pi^4 b^4 c^4 - 32 * I * \pi^3 b^4 c^4 \log(\text{abs}(F)) + 48 * \pi^2 b^4 c^4 \log(\text{abs}(F))^2 + 32 * I * \pi b^4 c^4 \log(\text{abs}(F))^3 - 16 * b^4 c^4 \log(\text{abs}(F))^4) + (8\pi^3 b^3 c^3 g x^3 * \text{sgn}(F) - 24 * I * \pi^2 b^3 c^3 g x^3 \log(\text{abs}(F)) * \text{sgn}(F) - 24 * \pi b^3 c^3 g x^3 \log(\text{abs}(F))^2 * \text{sgn}(F) - 8\pi^3 b^3 c^3 g x^3 + 24 * I * \pi^2 b^3 c^3 g x^3 \log(\text{abs}(F)) + 24 * \pi b^3 c^3 g x^3 \log(\text{abs}(F))^2 - 16 * I * b^3 c^3 g x^3 \log(\text{abs}(F))^3 + 8\pi^3 b^3 c^3 f x^2 * \text{sgn}(F) - 24 * I * \pi^2 b^3 c^3 f x^2 \log(\text{abs}(F)) * \text{sgn}(F) - 24 * \pi b^3 c^3 f x^2 \log(\text{abs}(F))^2 * \text{sgn}(F) - 8\pi^3 b^3 c^3 f x^2 + 24 * I * \pi^2 b^3 c^3 f x^2 \log(\text{abs}(F)) + 24 * \pi b^3 c^3 f x^2 \log(\text{abs}(F))^2 - 16 * I * b^3 c^3 f x^2 \log(\text{abs}(F))^3 + 8\pi^3 b^3 c^3 d * \text{sgn}(F) - 24 * I * \pi^2 b^3 c^3 d \log(\text{abs}(F)) * \text{sgn}(F) - 24 * \pi b^3 c^3 d \log(\text{abs}(F))^2 * \text{sgn}(F) - 8\pi^3 b^3 c^3 d + 24 * I * \pi^2 b^3 c^3 d \log(\text{abs}(F)) + 24 * \pi b^3 c^3 d \log(\text{abs}(F))^2 - 16 * I * b^3 c^3 d \log(\text{abs}(F))^3 + 24 * I * \pi^2 b^2 c^2 g x^2 * \text{sgn}(F) + 48 * \pi b^2 c^2 g x^2 \log(\text{abs}(F)) * \text{sgn}(F) - 24
\end{aligned}$$

$$\begin{aligned}
& *I\pi^2b^2c^2g^x^2 - 48\pi b^2c^2g^x^2\log(\text{abs}(F)) + 48Ib^2c^2g^x^2 \\
& 2\log(\text{abs}(F))^2 + 16I\pi^2b^2c^2f^x\text{sgn}(F) + 32\pi b^2c^2f^x\log(\text{abs}(F)) \\
& \text{sgn}(F) - 16I\pi^2b^2c^2f^x - 32\pi b^2c^2f^x\log(\text{abs}(F)) + 32Ib^2c^2f^x \\
& \log(\text{abs}(F))^2 - 48\pi b^2c^2g^x\text{sgn}(F) + 48\pi b^2c^2g^x - 96Ib^2c^2g^x \\
& \log(\text{abs}(F)) - 16\pi b^2c^2f^x\text{sgn}(F) + 16\pi b^2c^2f^x - 32Ib^2c^2f^x\log(\text{abs}(F)) \\
& + 96I\pi g^x e^{(-1/2I\pi b^2c^2f^x\text{sgn}(F) + 1/2I\pi b^2c^2f^x - 1/2I\pi a^2c^2\text{sgn}(F) \\
& + 1/2I\pi a^2c^2)} / (8\pi^4b^4c^4\text{sgn}(F) - 32I\pi^3b^4c^4\log(\text{abs}(F))\text{sgn}(F) \\
& - 48\pi^2b^4c^4\log(\text{abs}(F))^2\text{sgn}(F) + 32I\pi b^4c^4\log(\text{abs}(F))^3\text{sgn}(F) \\
& - 8\pi^4b^4c^4 + 32I\pi^3b^4c^4\log(\text{abs}(F)) + 48\pi^2b^4c^4\log(\text{abs}(F))^2 \\
& - 32I\pi b^4c^4\log(\text{abs}(F))^3 - 16b^4c^4\log(\text{abs}(F))^4) * e^{(b^2c^2f^x\log(\text{abs}(F)) + a^2c^2\log(\text{abs}(F)))}
\end{aligned}$$



### 3.54 $\int F^{c(a+bx)} (d + ex + fx^2 + gx^3 + hx^4) dx$

**Optimal.** Leaf size=348

$$-\frac{eF^{c(a+bx)}}{b^2c^2 \log^2(F)} - \frac{2fxF^{c(a+bx)}}{b^2c^2 \log^2(F)} + \frac{2fF^{c(a+bx)}}{b^3c^3 \log^3(F)} - \frac{3gx^2F^{c(a+bx)}}{b^2c^2 \log^2(F)} + \frac{6gxF^{c(a+bx)}}{b^3c^3 \log^3(F)} - \frac{6gF^{c(a+bx)}}{b^4c^4 \log^4(F)} - \frac{4hx^3F^{c(a+bx)}}{b^2c^2 \log^2(F)} + \frac{12hx^2F^{c(a+bx)}}{b^3c^3 \log^3(F)}$$

[Out]  $(24F^{c(a+bx)}h)/(b^5c^5 \log^5[F]) - (6F^{c(a+bx)}g)/(b^4c^4 \log^4[F]) - (24F^{c(a+bx)}hx)/(b^4c^4 \log^4[F]) + (2fF^{c(a+bx)}x)/(b^3c^3 \log^3[F]) + (6F^{c(a+bx)}gx)/(b^3c^3 \log^3[F]) + (12F^{c(a+bx)}hx^2)/(b^3c^3 \log^3[F]) - (eF^{c(a+bx)})/(b^2c^2 \log^2[F]) - (2fF^{c(a+bx)}x)/(b^2c^2 \log^2[F]) - (3F^{c(a+bx)}gx^2)/(b^2c^2 \log^2[F]) - (4F^{c(a+bx)}hx^3)/(b^2c^2 \log^2[F]) + (dF^{c(a+bx)})/(b^2c^2 \log^2[F]) + (eF^{c(a+bx)}x)/(b^2c^2 \log^2[F]) + (fF^{c(a+bx)}x^2)/(b^2c^2 \log^2[F]) + (F^{c(a+bx)}gx^3)/(b^2c^2 \log^2[F]) + (F^{c(a+bx)}hx^4)/(b^2c^2 \log^2[F])$

**Rubi [A]** time = 0.312263, antiderivative size = 348, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 3, integrand size = 30,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$ , Rules used = {2196, 2194, 2176}

$$-\frac{eF^{c(a+bx)}}{b^2c^2 \log^2(F)} - \frac{2fxF^{c(a+bx)}}{b^2c^2 \log^2(F)} + \frac{2fF^{c(a+bx)}}{b^3c^3 \log^3(F)} - \frac{3gx^2F^{c(a+bx)}}{b^2c^2 \log^2(F)} + \frac{6gxF^{c(a+bx)}}{b^3c^3 \log^3(F)} - \frac{6gF^{c(a+bx)}}{b^4c^4 \log^4(F)} - \frac{4hx^3F^{c(a+bx)}}{b^2c^2 \log^2(F)} + \frac{12hx^2F^{c(a+bx)}}{b^3c^3 \log^3(F)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[F^{c(a+bx)}(d + ex + fx^2 + gx^3 + hx^4), x]$

[Out]  $(24F^{c(a+bx)}h)/(b^5c^5 \log^5[F]) - (6F^{c(a+bx)}g)/(b^4c^4 \log^4[F]) - (24F^{c(a+bx)}hx)/(b^4c^4 \log^4[F]) + (2fF^{c(a+bx)}x)/(b^3c^3 \log^3[F]) + (6F^{c(a+bx)}gx)/(b^3c^3 \log^3[F]) + (12F^{c(a+bx)}hx^2)/(b^3c^3 \log^3[F]) - (eF^{c(a+bx)})/(b^2c^2 \log^2[F]) - (2fF^{c(a+bx)}x)/(b^2c^2 \log^2[F]) - (3F^{c(a+bx)}gx^2)/(b^2c^2 \log^2[F]) - (4F^{c(a+bx)}hx^3)/(b^2c^2 \log^2[F]) + (dF^{c(a+bx)})/(b^2c^2 \log^2[F]) + (eF^{c(a+bx)}x)/(b^2c^2 \log^2[F]) + (fF^{c(a+bx)}x^2)/(b^2c^2 \log^2[F]) + (F^{c(a+bx)}gx^3)/(b^2c^2 \log^2[F]) + (F^{c(a+bx)}hx^4)/(b^2c^2 \log^2[F])$

#### Rule 2196

$\text{Int}[(F_.)^{c(v_)}(u_), x\_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[F^{c(\text{ExpandToSum}[v, x])}, u, x], x] /; \text{FreeQ}[\{F, c\}, x] \ \&\& \ \text{PolynomialQ}[u, x] \ \&\& \ \text{LinearQ}[v,$

x] && !\$UseGamma === True

### Rule 2194

Int[((F\_)^((c\_)\*(a\_) + (b\_)\*(x\_)))^(n\_), x\_Symbol] := Simp[(F^(c\*(a + b\*x)))^n/(b\*c\*n\*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]

### Rule 2176

Int[((b\_)\*(F\_)^((g\_)\*(e\_) + (f\_)\*(x\_)))^(n\_)\*((c\_) + (d\_)\*(x\_))^(m\_), x\_Symbol] := Simp[((c + d\*x)^m\*(b\*F^(g\*(e + f\*x)))^n)/(f\*g\*n\*Log[F]), x] - Dist[(d\*m)/(f\*g\*n\*Log[F]), Int[(c + d\*x)^(m - 1)\*(b\*F^(g\*(e + f\*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2\*m] && !\$UseGamma === True

### Rubi steps

$$\begin{aligned}
 \int F^{c(ax)} (d + ex + fx^2 + gx^3 + hx^4) dx &= \int (dF^{c(ax)} + eF^{c(ax)}x + fF^{c(ax)}x^2 + F^{c(ax)}gx^3 + F^{c(ax)}hx^4) dx \\
 &= d \int F^{c(ax)} dx + e \int F^{c(ax)}x dx + f \int F^{c(ax)}x^2 dx + g \int F^{c(ax)}x^3 dx + h \int F^{c(ax)}x^4 dx \\
 &= \frac{dF^{c(ax)}}{bc \log(F)} + \frac{eF^{c(ax)}x}{bc \log(F)} + \frac{fF^{c(ax)}x^2}{bc \log(F)} + \frac{F^{c(ax)}gx^3}{bc \log(F)} + \frac{F^{c(ax)}hx^4}{bc \log(F)} - \frac{e \int F^{c(ax)} dx}{bc \log(F)} \\
 &= -\frac{eF^{c(ax)}}{b^2c^2 \log^2(F)} - \frac{2fF^{c(ax)}x}{b^2c^2 \log^2(F)} - \frac{3F^{c(ax)}gx^2}{b^2c^2 \log^2(F)} - \frac{4F^{c(ax)}hx^3}{b^2c^2 \log^2(F)} + \frac{dF^{c(ax)}}{bc \log(F)} + \frac{e \int F^{c(ax)} dx}{bc \log(F)} \\
 &= \frac{2fF^{c(ax)}}{b^3c^3 \log^3(F)} + \frac{6F^{c(ax)}gx}{b^3c^3 \log^3(F)} + \frac{12F^{c(ax)}hx^2}{b^3c^3 \log^3(F)} - \frac{eF^{c(ax)}}{b^2c^2 \log^2(F)} - \frac{2fF^{c(ax)}x}{b^2c^2 \log^2(F)} \\
 &= -\frac{6F^{c(ax)}g}{b^4c^4 \log^4(F)} - \frac{24F^{c(ax)}hx}{b^4c^4 \log^4(F)} + \frac{2fF^{c(ax)}}{b^3c^3 \log^3(F)} + \frac{6F^{c(ax)}gx}{b^3c^3 \log^3(F)} + \frac{12F^{c(ax)}hx^2}{b^3c^3 \log^3(F)} \\
 &= \frac{24F^{c(ax)}h}{b^5c^5 \log^5(F)} - \frac{6F^{c(ax)}g}{b^4c^4 \log^4(F)} - \frac{24F^{c(ax)}hx}{b^4c^4 \log^4(F)} + \frac{2fF^{c(ax)}}{b^3c^3 \log^3(F)} + \frac{6F^{c(ax)}gx}{b^3c^3 \log^3(F)} + \frac{12F^{c(ax)}hx^2}{b^3c^3 \log^3(F)}
 \end{aligned}$$

**Mathematica [A]** time = 0.161942, size = 117, normalized size = 0.34

$$\frac{F^{c(ax)} (b^4c^4 \log^4(F)(d + x(e + x(f + x(g + hx)))) - b^3c^3 \log^3(F)(e + x(2f + 3gx + 4hx^2))) + 2b^2c^2 \log^2(F)(f + 3x(g + 2fx^2))}{b^5c^5 \log^5(F)}$$

Antiderivative was successfully verified.

[In] Integrate[F^(c\*(a + b\*x))\*(d + e\*x + f\*x^2 + g\*x^3 + h\*x^4), x]

```
[Out] (F^(c*(a + b*x))*(24*h - 6*b*c*(g + 4*h*x)*Log[F] + 2*b^2*c^2*(f + 3*x*(g + 2*h*x))*Log[F]^2 - b^3*c^3*(e + x*(2*f + 3*g*x + 4*h*x^2))*Log[F]^3 + b^4*c^4*(d + x*(e + x*(f + x*(g + h*x))))*Log[F]^4))/b^5*c^5*Log[F]^5)
```

**Maple [A]** time = 0.005, size = 212, normalized size = 0.6

$$\frac{(hx^4b^4c^4(\ln(F))^4 + (\ln(F))^4b^4c^4gx^3 + (\ln(F))^4b^4c^4fx^2 + (\ln(F))^4b^4c^4ex + (\ln(F))^4b^4c^4d - 4(\ln(F))^3b^3c^3hx^3 - 3(\ln(F))^3b^3c^3fx^2 - 2(\ln(F))^3b^3c^3ex - 2(\ln(F))^3b^3c^3d - 4(\ln(F))^2b^2c^2hx^4 - 4(\ln(F))^2b^2c^2gx^3 - 4(\ln(F))^2b^2c^2fx^2 - 4(\ln(F))^2b^2c^2ex - 4(\ln(F))^2b^2c^2d - 4(\ln(F))b^3c^3hx^3 - 4(\ln(F))b^3c^3fx^2 - 4(\ln(F))b^3c^3ex - 4(\ln(F))b^3c^3d - 4b^4c^4hx^4 - 4b^4c^4gx^3 - 4b^4c^4fx^2 - 4b^4c^4ex - 4b^4c^4d)}{b^5c^5(\ln(F))^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(b*x+a))*(h*x^4+g*x^3+f*x^2+e*x+d), x)
```

```
[Out] (h*x^4*b^4*c^4*ln(F)^4+ln(F)^4*b^4*c^4*g*x^3+ln(F)^4*b^4*c^4*f*x^2+ln(F)^4*b^4*c^4*e*x+ln(F)^4*b^4*c^4*d-4*ln(F)^3*b^3*c^3*h*x^3-3*ln(F)^3*b^3*c^3*g*x^2-2*ln(F)^3*b^3*c^3*f*x-ln(F)^3*b^3*c^3*e+12*ln(F)^2*b^2*c^2*h*x^2+6*ln(F)^2*b^2*c^2*g*x+2*b^2*c^2*ln(F)^2*f-24*ln(F)*b*c*h*x-6*g*b*c*ln(F)+24*h)*F^(c*(b*x+a))/b^5/c^5/ln(F)^5
```

**Maxima [A]** time = 1.06509, size = 393, normalized size = 1.13

$$\frac{F^{bcx+ac}d}{bc \log(F)} + \frac{(F^{ac}bcx \log(F) - F^{ac})F^{bcx}e}{b^2c^2 \log(F)^2} + \frac{(F^{ac}b^2c^2x^2 \log(F)^2 - 2F^{ac}bcx \log(F) + 2F^{ac})F^{bcx}f}{b^3c^3 \log(F)^3} + \frac{(F^{ac}b^3c^3x^3 \log(F)^3 - 3(F^{ac}b^2c^2x^2 \log(F)^2 - 2F^{ac}bcx \log(F) + 2F^{ac})F^{bcx}f + (F^{ac}b^2c^2x^2 \log(F)^2 - 2F^{ac}bcx \log(F) + 2F^{ac})F^{bcx}e - F^{ac}b^2c^2x^2 \log(F)^2 + 6F^{ac}bcx \log(F) - 6F^{ac})F^{bcx}g}{b^4c^4 \log(F)^4} + \frac{(F^{ac}b^4c^4x^4 \log(F)^4 - 4F^{ac}b^3c^3x^3 \log(F)^3 + 12F^{ac}b^2c^2x^2 \log(F)^2 - 24F^{ac}bcx \log(F) + 24F^{ac})F^{bcx}h}{b^5c^5 \log(F)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*(h*x^4+g*x^3+f*x^2+e*x+d), x, algorithm="maxima")
```

```
[Out] F^(b*c*x + a*c)*d/(b*c*log(F)) + (F^(a*c)*b*c*x*log(F) - F^(a*c))*F^(b*c*x)*e/(b^2*c^2*log(F)^2) + (F^(a*c)*b^2*c^2*x^2*log(F)^2 - 2*F^(a*c)*b*c*x*log(F) + 2*F^(a*c))*F^(b*c*x)*f/(b^3*c^3*log(F)^3) + (F^(a*c)*b^3*c^3*x^3*log(F)^3 - 3*F^(a*c)*b^2*c^2*x^2*log(F)^2 + 6*F^(a*c)*b*c*x*log(F) - 6*F^(a*c))*F^(b*c*x)*g/(b^4*c^4*log(F)^4) + (F^(a*c)*b^4*c^4*x^4*log(F)^4 - 4*F^(a*c)*b^3*c^3*x^3*log(F)^3 + 12*F^(a*c)*b^2*c^2*x^2*log(F)^2 - 24*F^(a*c)*b*c*x*log(F) + 24*F^(a*c))*F^(b*c*x)*h/(b^5*c^5*log(F)^5)
```

**Fricas [A]** time = 1.56461, size = 396, normalized size = 1.14

$$\frac{\left( (b^4 c^4 h x^4 + b^4 c^4 g x^3 + b^4 c^4 f x^2 + b^4 c^4 e x + b^4 c^4 d) \log(F)^4 - (4 b^3 c^3 h x^3 + 3 b^3 c^3 g x^2 + 2 b^3 c^3 f x + b^3 c^3 e) \log(F)^3 + 2 (6 b^2 c^2 h x^2 + 5 b^2 c^2 g x + 4 b^2 c^2 f) \log(F)^2 - 6 (4 b c h x + b c g) \log(F) + 24 h \right) F^{(b c x + a c)}}{b^5 c^5 \log(F)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(c\*(b\*x+a))\*(h\*x^4+g\*x^3+f\*x^2+e\*x+d),x, algorithm="fricas")

[Out] ((b^4\*c^4\*h\*x^4 + b^4\*c^4\*g\*x^3 + b^4\*c^4\*f\*x^2 + b^4\*c^4\*e\*x + b^4\*c^4\*d)\*log(F)^4 - (4\*b^3\*c^3\*h\*x^3 + 3\*b^3\*c^3\*g\*x^2 + 2\*b^3\*c^3\*f\*x + b^3\*c^3\*e)\*log(F)^3 + 2\*(6\*b^2\*c^2\*h\*x^2 + 3\*b^2\*c^2\*g\*x + b^2\*c^2\*f)\*log(F)^2 - 6\*(4\*b\*c\*h\*x + b\*c\*g)\*log(F) + 24\*h)\*F^(b\*c\*x + a\*c)/(b^5\*c^5\*log(F)^5)

**Sympy [A]** time = 0.216861, size = 284, normalized size = 0.82

$$\left\{ \begin{array}{l} \frac{F^{c(a+bx)}(b^4 c^4 d \log(F)^4 + b^4 c^4 e x \log(F)^4 + b^4 c^4 f x^2 \log(F)^4 + b^4 c^4 g x^3 \log(F)^4 + b^4 c^4 h x^4 \log(F)^4 - b^3 c^3 e \log(F)^3 - 2 b^3 c^3 f x \log(F)^3 - 3 b^3 c^3 g x^2 \log(F)^3 - 4 b^3 c^3 h x^3 \log(F)^3 - 6 b^2 c^2 h x^2 \log(F)^2 - 5 b^2 c^2 g x \log(F)^2 - 4 b^2 c^2 f \log(F)^2 - 6 b c h x \log(F) - 6 b c g \log(F) + 24 h)}{b^5 c^5 \log(F)^5} \\ dx + \frac{ex^2}{2} + \frac{fx^3}{3} + \frac{gx^4}{4} + \frac{hx^5}{5} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(c\*(b\*x+a))\*(h\*x\*\*4+g\*x\*\*3+f\*x\*\*2+e\*x+d),x)

[Out] Piecewise((F\*\*(c\*(a + b\*x))\*(b\*\*4\*c\*\*4\*d\*log(F)\*\*4 + b\*\*4\*c\*\*4\*e\*x\*log(F)\*\*4 + b\*\*4\*c\*\*4\*f\*x\*\*2\*log(F)\*\*4 + b\*\*4\*c\*\*4\*g\*x\*\*3\*log(F)\*\*4 + b\*\*4\*c\*\*4\*h\*x\*\*4\*log(F)\*\*4 - b\*\*3\*c\*\*3\*e\*log(F)\*\*3 - 2\*b\*\*3\*c\*\*3\*f\*x\*log(F)\*\*3 - 3\*b\*\*3\*c\*\*3\*g\*x\*\*2\*log(F)\*\*3 - 4\*b\*\*3\*c\*\*3\*h\*x\*\*3\*log(F)\*\*3 + 2\*b\*\*2\*c\*\*2\*f\*log(F)\*\*2 + 6\*b\*\*2\*c\*\*2\*g\*x\*log(F)\*\*2 + 12\*b\*\*2\*c\*\*2\*h\*x\*\*2\*log(F)\*\*2 - 6\*b\*c\*g\*log(F) - 24\*b\*c\*h\*x\*log(F) + 24\*h)/(b\*\*5\*c\*\*5\*log(F)\*\*5), Ne(b\*\*5\*c\*\*5\*log(F)\*\*5, 0)), (d\*x + e\*x\*\*2/2 + f\*x\*\*3/3 + g\*x\*\*4/4 + h\*x\*\*5/5, True))

**Giac [C]** time = 1.51238, size = 10024, normalized size = 28.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(c\*(b\*x+a))\*(h\*x^4+g\*x^3+f\*x^2+e\*x+d),x, algorithm="giac")

```

[Out] (2*((pi*b^2*c^2*log(abs(F))*sgn(F) - pi*b^2*c^2*log(abs(F)))*(pi*b*c*x*sgn(
F) - pi*b*c*x)/((pi^2*b^2*c^2*sgn(F) - pi^2*b^2*c^2 + 2*b^2*c^2*log(abs(F))
^2)^2 + 4*(pi*b^2*c^2*log(abs(F))*sgn(F) - pi*b^2*c^2*log(abs(F)))^2) + (pi
^2*b^2*c^2*sgn(F) - pi^2*b^2*c^2 + 2*b^2*c^2*log(abs(F))^2)*(b*c*x*log(abs(
F)) - 1)/((pi^2*b^2*c^2*sgn(F) - pi^2*b^2*c^2 + 2*b^2*c^2*log(abs(F))^2)^2
+ 4*(pi*b^2*c^2*log(abs(F))*sgn(F) - pi*b^2*c^2*log(abs(F)))^2))*cos(-1/2*pi
i*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c) + ((pi^2*b^
2*c^2*sgn(F) - pi^2*b^2*c^2 + 2*b^2*c^2*log(abs(F))^2)*(pi*b*c*x*sgn(F) - p
i*b*c*x)/((pi^2*b^2*c^2*sgn(F) - pi^2*b^2*c^2 + 2*b^2*c^2*log(abs(F))^2)^2
+ 4*(pi*b^2*c^2*log(abs(F))*sgn(F) - pi*b^2*c^2*log(abs(F)))^2) - 4*(pi*b^2
*c^2*log(abs(F))*sgn(F) - pi*b^2*c^2*log(abs(F)))*(b*c*x*log(abs(F)) - 1)/((
pi^2*b^2*c^2*sgn(F) - pi^2*b^2*c^2 + 2*b^2*c^2*log(abs(F))^2)^2 + 4*(pi*b^
2*c^2*log(abs(F))*sgn(F) - pi*b^2*c^2*log(abs(F)))^2))*sin(-1/2*pi*b*c*x*sg
n(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c))*e^(b*c*x*log(abs(F))
+ a*c*log(abs(F)) + 1) - 1/2*I*((2*pi*b*c*x*sgn(F) - 2*pi*b*c*x - 4*I*b*c*
x*log(abs(F)) + 4*I)*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a
*c*sgn(F) - 1/2*I*pi*a*c)/(2*pi^2*b^2*c^2*sgn(F) + 4*I*pi*b^2*c^2*log(abs(F
))*sgn(F) - 2*pi^2*b^2*c^2 - 4*I*pi*b^2*c^2*log(abs(F)) + 4*b^2*c^2*log(abs
(F))^2) + (2*pi*b*c*x*sgn(F) - 2*pi*b*c*x + 4*I*b*c*x*log(abs(F)) - 4*I)*e^
(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a
*c)/(2*pi^2*b^2*c^2*sgn(F) - 4*I*pi*b^2*c^2*log(abs(F))*sgn(F) - 2*pi^2*b^2
*c^2 + 4*I*pi*b^2*c^2*log(abs(F)) + 4*b^2*c^2*log(abs(F))^2))*e^(b*c*x*log(
abs(F)) + a*c*log(abs(F)) + 1) - (((4*pi^3*b^4*c^4*h*x^4*log(abs(F))*sgn(F)
- 4*pi*b^4*c^4*h*x^4*log(abs(F))^3*sgn(F) - 4*pi^3*b^4*c^4*h*x^4*log(abs(F
)) + 4*pi*b^4*c^4*h*x^4*log(abs(F))^3 + 4*pi^3*b^4*c^4*g*x^3*log(abs(F))*sg
n(F) - 4*pi*b^4*c^4*g*x^3*log(abs(F))^3*sgn(F) - 4*pi^3*b^4*c^4*g*x^3*log(a
bs(F)) + 4*pi*b^4*c^4*g*x^3*log(abs(F))^3 + 4*pi^3*b^4*c^4*f*x^2*log(abs(F
))*sgn(F) - 4*pi*b^4*c^4*f*x^2*log(abs(F))^3*sgn(F) - 4*pi^3*b^4*c^4*f*x^2*l
og(abs(F)) + 4*pi*b^4*c^4*f*x^2*log(abs(F))^3 - 4*pi^3*b^3*c^3*h*x^3*sgn(F)
+ 4*pi^3*b^4*c^4*d*log(abs(F))*sgn(F) + 12*pi*b^3*c^3*h*x^3*log(abs(F))^2*
sgn(F) - 4*pi*b^4*c^4*d*log(abs(F))^3*sgn(F) + 4*pi^3*b^3*c^3*h*x^3 - 4*pi^
3*b^4*c^4*d*log(abs(F)) - 12*pi*b^3*c^3*h*x^3*log(abs(F))^2 + 4*pi*b^4*c^4
d*log(abs(F))^3 - 3*pi^3*b^3*c^3*g*x^2*sgn(F) + 9*pi*b^3*c^3*g*x^2*log(abs(
F))^2*sgn(F) + 3*pi^3*b^3*c^3*g*x^2 - 9*pi*b^3*c^3*g*x^2*log(abs(F))^2 - 2*
pi^3*b^3*c^3*f*x*sgn(F) + 6*pi*b^3*c^3*f*x*log(abs(F))^2*sgn(F) + 2*pi^3*b^
3*c^3*f*x - 6*pi*b^3*c^3*f*x*log(abs(F))^2 - 24*pi*b^2*c^2*h*x^2*log(abs(F
))*sgn(F) + 24*pi*b^2*c^2*h*x^2*log(abs(F)) - 12*pi*b^2*c^2*g*x*log(abs(F))*
sgn(F) + 12*pi*b^2*c^2*g*x*log(abs(F)) - 4*pi*b^2*c^2*f*log(abs(F))*sgn(F)
+ 4*pi*b^2*c^2*f*log(abs(F)) + 24*pi*b*c*h*x*sgn(F) - 24*pi*b*c*h*x + 6*pi*
b*c*g*sgn(F) - 6*pi*b*c*g)*(pi^5*b^5*c^5*sgn(F) - 10*pi^3*b^5*c^5*log(abs(F
))^2*sgn(F) + 5*pi*b^5*c^5*log(abs(F))^4*sgn(F) - pi^5*b^5*c^5 + 10*pi^3*b^
5*c^5*log(abs(F))^2 - 5*pi*b^5*c^5*log(abs(F))^4)/((pi^5*b^5*c^5*sgn(F) - 1
0*pi^3*b^5*c^5*log(abs(F))^2*sgn(F) + 5*pi*b^5*c^5*log(abs(F))^4*sgn(F) - p
i^5*b^5*c^5 + 10*pi^3*b^5*c^5*log(abs(F))^2 - 5*pi*b^5*c^5*log(abs(F))^4)^2
+ (5*pi^4*b^5*c^5*log(abs(F))*sgn(F) - 10*pi^2*b^5*c^5*log(abs(F))^3*sgn(F)

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$$\begin{aligned}
& ) - 5\pi^4 b^5 c^5 \log(\text{abs}(F)) + 10\pi^2 b^5 c^5 \log(\text{abs}(F))^3 - 2b^5 c^5 \log(\text{abs}(F))^5)^2 - (\pi^4 b^4 c^4 h x^4 \text{sgn}(F) - 6\pi^2 b^4 c^4 h x^4 \log(\text{abs}(F))^2 \text{sgn}(F) - \pi^4 b^4 c^4 h x^4 + 6\pi^2 b^4 c^4 h x^4 \log(\text{abs}(F))^2 - 2b^4 c^4 h x^4 \log(\text{abs}(F))^4 + \pi^4 b^4 c^4 g x^3 \text{sgn}(F) - 6\pi^2 b^4 c^4 g x^3 \log(\text{abs}(F))^2 \text{sgn}(F) - \pi^4 b^4 c^4 g x^3 + 6\pi^2 b^4 c^4 g x^3 \log(\text{abs}(F))^2 - 2b^4 c^4 g x^3 \log(\text{abs}(F))^4 + \pi^4 b^4 c^4 f x^2 \text{sgn}(F) - 6\pi^2 b^4 c^4 f x^2 \log(\text{abs}(F))^2 \text{sgn}(F) - \pi^4 b^4 c^4 f x^2 + 6\pi^2 b^4 c^4 f x^2 \log(\text{abs}(F))^2 - 2b^4 c^4 f x^2 \log(\text{abs}(F))^4 + \pi^4 b^4 c^4 d \text{sgn}(F) + 12\pi^2 b^3 c^3 h x^3 \log(\text{abs}(F)) \text{sgn}(F) - 6\pi^2 b^4 c^4 d \log(\text{abs}(F))^2 \text{sgn}(F) - \pi^4 b^4 c^4 d - 12\pi^2 b^3 c^3 h x^3 \log(\text{abs}(F)) + 6\pi^2 b^4 c^4 d \log(\text{abs}(F))^2 + 8b^3 c^3 h x^3 \log(\text{abs}(F))^3 - 2b^4 c^4 d \log(\text{abs}(F))^4 + 9\pi^2 b^3 c^3 g x^2 \log(\text{abs}(F)) \text{sgn}(F) - 9\pi^2 b^3 c^3 g x^2 \log(\text{abs}(F)) + 6b^3 c^3 g x^2 \log(\text{abs}(F))^3 + 6\pi^2 b^3 c^3 f x \log(\text{abs}(F)) \text{sgn}(F) - 6\pi^2 b^3 c^3 f x \log(\text{abs}(F)) + 4b^3 c^3 f x \log(\text{abs}(F))^3 - 12\pi^2 b^2 c^2 h x^2 \text{sgn}(F) + 12\pi^2 b^2 c^2 h x^2 - 24b^2 c^2 h x^2 \log(\text{abs}(F))^2 - 6\pi^2 b^2 c^2 g x \text{sgn}(F) + 6\pi^2 b^2 c^2 g x - 12b^2 c^2 g x \log(\text{abs}(F))^2 - 2\pi^2 b^2 c^2 f \text{sgn}(F) + 2\pi^2 b^2 c^2 f - 4b^2 c^2 f \log(\text{abs}(F))^2 + 48b c h x \log(\text{abs}(F)) + 12b c g \log(\text{abs}(F)) - 48h) * (5\pi^4 b^5 c^5 \log(\text{abs}(F)) \text{sgn}(F) - 10\pi^2 b^5 c^5 \log(\text{abs}(F))^3 \text{sgn}(F) - 5\pi^4 b^5 c^5 \log(\text{abs}(F)) + 10\pi^2 b^5 c^5 \log(\text{abs}(F))^3 - 2b^5 c^5 \log(\text{abs}(F))^5) / ((\pi^5 b^5 c^5 \text{sgn}(F) - 10\pi^3 b^5 c^5 \log(\text{abs}(F))^2 \text{sgn}(F) + 5\pi b^5 c^5 \log(\text{abs}(F))^4 \text{sgn}(F) - \pi^5 b^5 c^5 + 10\pi^3 b^5 c^5 \log(\text{abs}(F))^2 - 5\pi b^5 c^5 \log(\text{abs}(F))^4)^2 + (5\pi^4 b^5 c^5 \log(\text{abs}(F)) \text{sgn}(F) - 10\pi^2 b^5 c^5 \log(\text{abs}(F))^3 \text{sgn}(F) - 5\pi^4 b^5 c^5 \log(\text{abs}(F)) + 10\pi^2 b^5 c^5 \log(\text{abs}(F))^3 - 2b^5 c^5 \log(\text{abs}(F))^5)^2) * \cos(-1/2\pi b c x \text{sgn}(F) + 1/2\pi b c x - 1/2\pi a c \text{sgn}(F) + 1/2\pi a c) - ((\pi^4 b^4 c^4 h x^4 \text{sgn}(F) - 6\pi^2 b^4 c^4 h x^4 \log(\text{abs}(F))^2 \text{sgn}(F) - \pi^4 b^4 c^4 h x^4 + 6\pi^2 b^4 c^4 h x^4 \log(\text{abs}(F))^2 - 2b^4 c^4 h x^4 \log(\text{abs}(F))^4 + \pi^4 b^4 c^4 g x^3 \text{sgn}(F) - 6\pi^2 b^4 c^4 g x^3 \log(\text{abs}(F))^2 \text{sgn}(F) - \pi^4 b^4 c^4 g x^3 + 6\pi^2 b^4 c^4 g x^3 \log(\text{abs}(F))^2 - 2b^4 c^4 g x^3 \log(\text{abs}(F))^4 + \pi^4 b^4 c^4 f x^2 \text{sgn}(F) - 6\pi^2 b^4 c^4 f x^2 \log(\text{abs}(F))^2 \text{sgn}(F) - \pi^4 b^4 c^4 f x^2 + 6\pi^2 b^4 c^4 f x^2 \log(\text{abs}(F))^2 - 2b^4 c^4 f x^2 \log(\text{abs}(F))^4 + \pi^4 b^4 c^4 d \text{sgn}(F) + 12\pi^2 b^3 c^3 h x^3 \log(\text{abs}(F)) \text{sgn}(F) - 6\pi^2 b^4 c^4 d \log(\text{abs}(F))^2 \text{sgn}(F) - \pi^4 b^4 c^4 d - 12\pi^2 b^3 c^3 h x^3 \log(\text{abs}(F)) + 6\pi^2 b^4 c^4 d \log(\text{abs}(F))^2 + 8b^3 c^3 h x^3 \log(\text{abs}(F))^3 - 2b^4 c^4 d \log(\text{abs}(F))^4 + 9\pi^2 b^3 c^3 g x^2 \log(\text{abs}(F)) \text{sgn}(F) - 9\pi^2 b^3 c^3 g x^2 \log(\text{abs}(F)) + 6b^3 c^3 g x^2 \log(\text{abs}(F))^3 + 6\pi^2 b^3 c^3 f x \log(\text{abs}(F)) \text{sgn}(F) - 6\pi^2 b^3 c^3 f x \log(\text{abs}(F)) + 4b^3 c^3 f x \log(\text{abs}(F))^3 - 12\pi^2 b^2 c^2 h x^2 \text{sgn}(F) + 12\pi^2 b^2 c^2 h x^2 - 24b^2 c^2 h x^2 \log(\text{abs}(F))^2 - 6\pi^2 b^2 c^2 g x \text{sgn}(F) + 6\pi^2 b^2 c^2 g x - 12b^2 c^2 g x \log(\text{abs}(F))^2 - 2\pi^2 b^2 c^2 f \text{sgn}(F) + 2\pi^2 b^2 c^2 f - 4b^2 c^2 f \log(\text{abs}(F))^2 + 48b c h x \log(\text{abs}(F)) + 12b c g \log(\text{abs}(F)) - 48h) * (\pi^5 b^5 c^5 \text{sgn}(F) - 10\pi^3 b^5 c^5 \log(\text{abs}(F))^2 \text{sgn}(F) + 5\pi b^5 c^5 \log(\text{abs}(F))^4 \text{sgn}(F) - \pi^5 b^5 c^5 + 10\pi^3 b^5 c^5 \log(\text{abs}(F))^2 - 5\pi b^5 c^5 \log(\text{abs}(F))^4) / ((\pi^5 b^5 c^5 \text{sgn}(F) - 10\pi^3 b^5 c^5 \log(\text{abs}(F))^2 \text{sgn}(F) + 5\pi b^5 c^5 \log(\text{abs}(F))^4 \text{sgn}(F) - \pi^5 b^5 c^5 + 10\pi^3 b^5 c^5 \log(\text{abs}(F))^2 - 5\pi b^5 c^5 \log(\text{abs}(F))^4)
\end{aligned}$$

$$\begin{aligned}
& 3*b^5*c^5*\log(\text{abs}(F))^2*\text{sgn}(F) + 5*\pi*b^5*c^5*\log(\text{abs}(F))^4*\text{sgn}(F) - \pi^5*b^5*c^5 \\
& + 10*\pi^3*b^5*c^5*\log(\text{abs}(F))^2 - 5*\pi*b^5*c^5*\log(\text{abs}(F))^4)^2 + (5 \\
& *\pi^4*b^5*c^5*\log(\text{abs}(F))*\text{sgn}(F) - 10*\pi^2*b^5*c^5*\log(\text{abs}(F))^3*\text{sgn}(F) - 5 \\
& *\pi^4*b^5*c^5*\log(\text{abs}(F)) + 10*\pi^2*b^5*c^5*\log(\text{abs}(F))^3 - 2*b^5*c^5*\log(\text{abs}(F))^5)^2 \\
& + (4*\pi^3*b^4*c^4*h*x^4*\log(\text{abs}(F))*\text{sgn}(F) - 4*\pi*b^4*c^4*h*x^4 \\
& *4*\log(\text{abs}(F))^3*\text{sgn}(F) - 4*\pi^3*b^4*c^4*h*x^4*\log(\text{abs}(F)) + 4*\pi*b^4*c^4*h*x^4 \\
& *4*\log(\text{abs}(F))^3 + 4*\pi^3*b^4*c^4*g*x^3*\log(\text{abs}(F))*\text{sgn}(F) - 4*\pi*b^4*c^4 \\
& *g*x^3*\log(\text{abs}(F))^3*\text{sgn}(F) - 4*\pi^3*b^4*c^4*g*x^3*\log(\text{abs}(F)) + 4*\pi*b^4*c^4 \\
& *g*x^3*\log(\text{abs}(F))^3 + 4*\pi^3*b^4*c^4*f*x^2*\log(\text{abs}(F))*\text{sgn}(F) - 4*\pi*b^4*c^4 \\
& *f*x^2*\log(\text{abs}(F))^3*\text{sgn}(F) - 4*\pi^3*b^4*c^4*f*x^2*\log(\text{abs}(F)) + 4*\pi*b^4 \\
& *c^4*f*x^2*\log(\text{abs}(F))^3 - 4*\pi^3*b^3*c^3*h*x^3*\text{sgn}(F) + 4*\pi^3*b^4*c^4*d \\
& *4*\log(\text{abs}(F))*\text{sgn}(F) + 12*\pi*b^3*c^3*h*x^3*\log(\text{abs}(F))^2*\text{sgn}(F) - 4*\pi*b^4*c^4 \\
& *d*\log(\text{abs}(F))^3*\text{sgn}(F) + 4*\pi^3*b^3*c^3*h*x^3 - 4*\pi^3*b^4*c^4*d*\log(\text{abs}(F)) \\
& - 12*\pi*b^3*c^3*h*x^3*\log(\text{abs}(F))^2 + 4*\pi*b^4*c^4*d*\log(\text{abs}(F))^3 - 3*\pi^3 \\
& *b^3*c^3*g*x^2*\text{sgn}(F) + 9*\pi*b^3*c^3*g*x^2*\log(\text{abs}(F))^2*\text{sgn}(F) + 3*\pi^3 \\
& *b^3*c^3*g*x^2 - 9*\pi*b^3*c^3*g*x^2*\log(\text{abs}(F))^2 - 2*\pi^3*b^3*c^3*f*x*\text{sgn} \\
& (F) + 6*\pi*b^3*c^3*f*x*\log(\text{abs}(F))^2*\text{sgn}(F) + 2*\pi^3*b^3*c^3*f*x - 6*\pi*b^3 \\
& *c^3*f*x*\log(\text{abs}(F))^2 - 24*\pi*b^2*c^2*h*x^2*\log(\text{abs}(F))*\text{sgn}(F) + 24*\pi*b^2 \\
& *c^2*h*x^2*\log(\text{abs}(F)) - 12*\pi*b^2*c^2*g*x*\log(\text{abs}(F))*\text{sgn}(F) + 12*\pi*b^2*c^2 \\
& *g*x*\log(\text{abs}(F)) - 4*\pi*b^2*c^2*f*\log(\text{abs}(F))*\text{sgn}(F) + 4*\pi*b^2*c^2*f*\log \\
& (\text{abs}(F)) + 24*\pi*b*c*h*x*\text{sgn}(F) - 24*\pi*b*c*h*x + 6*\pi*b*c*g*\text{sgn}(F) - 6*\pi \\
& *b*c*g)*(5*\pi^4*b^5*c^5*\log(\text{abs}(F))*\text{sgn}(F) - 10*\pi^2*b^5*c^5*\log(\text{abs}(F))^3*s \\
& \text{gn}(F) - 5*\pi^4*b^5*c^5*\log(\text{abs}(F)) + 10*\pi^2*b^5*c^5*\log(\text{abs}(F))^3 - 2*b^5*c^5 \\
& *c^5*\log(\text{abs}(F))^5)/((\pi^5*b^5*c^5*\text{sgn}(F) - 10*\pi^3*b^5*c^5*\log(\text{abs}(F))^2*\text{sg} \\
& \text{n}(F) + 5*\pi*b^5*c^5*\log(\text{abs}(F))^4*\text{sgn}(F) - \pi^5*b^5*c^5 + 10*\pi^3*b^5*c^5*\log \\
& (\text{abs}(F))^2 - 5*\pi*b^5*c^5*\log(\text{abs}(F))^4)^2 + (5*\pi^4*b^5*c^5*\log(\text{abs}(F))* \\
& \text{sgn}(F) - 10*\pi^2*b^5*c^5*\log(\text{abs}(F))^3*\text{sgn}(F) - 5*\pi^4*b^5*c^5*\log(\text{abs}(F)) \\
& + 10*\pi^2*b^5*c^5*\log(\text{abs}(F))^3 - 2*b^5*c^5*\log(\text{abs}(F))^5)^2)*\sin(-1/2*\pi* \\
& b*c*x*\text{sgn}(F) + 1/2*\pi*b*c*x - 1/2*\pi*a*c*\text{sgn}(F) + 1/2*\pi*a*c)) * e^{(b*c*x*\log \\
& (\text{abs}(F)) + a*c*\log(\text{abs}(F))) + 1/2*I*((-16*I*\pi^4*b^4*c^4*h*x^4*\text{sgn}(F) + 64* \\
& \pi^3*b^4*c^4*h*x^4*\log(\text{abs}(F))*\text{sgn}(F) + 96*I*\pi^2*b^4*c^4*h*x^4*\log(\text{abs}(F)) \\
& ^2*\text{sgn}(F) - 64*\pi*b^4*c^4*h*x^4*\log(\text{abs}(F))^3*\text{sgn}(F) + 16*I*\pi^4*b^4*c^4*h* \\
& x^4 - 64*\pi^3*b^4*c^4*h*x^4*\log(\text{abs}(F)) - 96*I*\pi^2*b^4*c^4*h*x^4*\log(\text{abs}(F)) \\
& )^2 + 64*\pi*b^4*c^4*h*x^4*\log(\text{abs}(F))^3 + 32*I*b^4*c^4*h*x^4*\log(\text{abs}(F))^4 \\
& - 16*I*\pi^4*b^4*c^4*g*x^3*\text{sgn}(F) + 64*\pi^3*b^4*c^4*g*x^3*\log(\text{abs}(F))*\text{sgn}(F) \\
& ) + 96*I*\pi^2*b^4*c^4*g*x^3*\log(\text{abs}(F))^2*\text{sgn}(F) - 64*\pi*b^4*c^4*g*x^3*\log \\
& (\text{abs}(F))^3*\text{sgn}(F) + 16*I*\pi^4*b^4*c^4*g*x^3 - 64*\pi^3*b^4*c^4*g*x^3*\log(\text{abs}(F)) \\
& - 96*I*\pi^2*b^4*c^4*g*x^3*\log(\text{abs}(F))^2 + 64*\pi*b^4*c^4*g*x^3*\log(\text{abs}(F)) \\
& )^3 + 32*I*b^4*c^4*g*x^3*\log(\text{abs}(F))^4 - 16*I*\pi^4*b^4*c^4*f*x^2*\text{sgn}(F) + \\
& 64*\pi^3*b^4*c^4*f*x^2*\log(\text{abs}(F))*\text{sgn}(F) + 96*I*\pi^2*b^4*c^4*f*x^2*\log(\text{abs}(F)) \\
& )^2*\text{sgn}(F) - 64*\pi*b^4*c^4*f*x^2*\log(\text{abs}(F))^3*\text{sgn}(F) + 16*I*\pi^4*b^4*c^4 \\
& *f*x^2 - 64*\pi^3*b^4*c^4*f*x^2*\log(\text{abs}(F)) - 96*I*\pi^2*b^4*c^4*f*x^2*\log(\text{abs}(F)) \\
& )^2 + 64*\pi*b^4*c^4*f*x^2*\log(\text{abs}(F))^3 + 32*I*b^4*c^4*f*x^2*\log(\text{abs}(F)) \\
& )^4 - 16*I*\pi^4*b^4*c^4*d*\text{sgn}(F) - 64*\pi^3*b^3*c^3*h*x^3*\text{sgn}(F) + 64*\pi^3*b^3 \\
& *c^3*h*x^3*\log(\text{abs}(F))*\text{sgn}(F) - 192*I*\pi^2*b^3*c^3*h*x^3*\log(\text{abs}(F))*\text{sgn}(F) +
\end{aligned}$$

$$\begin{aligned}
& 96*I*\pi^2*b^4*c^4*d*\log(\text{abs}(F))^2*\text{sgn}(F) + 192*\pi*b^3*c^3*h*x^3*\log(\text{abs}(F))^2*\text{sgn}(F) - 64*\pi*b^4*c^4*d*\log(\text{abs}(F))^3*\text{sgn}(F) + 16*I*\pi^4*b^4*c^4*d + \\
& 4*\pi^3*b^3*c^3*h*x^3 - 64*\pi^3*b^4*c^4*d*\log(\text{abs}(F)) + 192*I*\pi^2*b^3*c^3*h*x^3*\log(\text{abs}(F)) - 96*I*\pi^2*b^4*c^4*d*\log(\text{abs}(F))^2 - 192*\pi*b^3*c^3*h*x^3 \\
& *\log(\text{abs}(F))^2 + 64*\pi*b^4*c^4*d*\log(\text{abs}(F))^3 - 128*I*b^3*c^3*h*x^3*\log(\text{abs}(F))^3 + 32*I*b^4*c^4*d*\log(\text{abs}(F))^4 - 48*\pi^3*b^3*c^3*g*x^2*\text{sgn}(F) - 144 \\
& *I*\pi^2*b^3*c^3*g*x^2*\log(\text{abs}(F))*\text{sgn}(F) + 144*\pi*b^3*c^3*g*x^2*\log(\text{abs}(F))^2*\text{sgn}(F) + 48*\pi^3*b^3*c^3*g*x^2 + 144*I*\pi^2*b^3*c^3*g*x^2*\log(\text{abs}(F)) - \\
& 144*\pi*b^3*c^3*g*x^2*\log(\text{abs}(F))^2 - 96*I*b^3*c^3*g*x^2*\log(\text{abs}(F))^3 - 32*\pi^3*b^3*c^3*f*x*\text{sgn}(F) - 96*I*\pi^2*b^3*c^3*f*x*\log(\text{abs}(F))*\text{sgn}(F) + 96*\pi*b^3*c^3 \\
& *f*x*\log(\text{abs}(F))^2*\text{sgn}(F) + 32*\pi^3*b^3*c^3*f*x + 96*I*\pi^2*b^3*c^3*f*x*\log(\text{abs}(F)) - 96*\pi*b^3*c^3*f*x*\log(\text{abs}(F))^2 - 64*I*b^3*c^3*f*x*\log(\text{abs}(F))^3 + \\
& 192*I*\pi^2*b^2*c^2*h*x^2*\text{sgn}(F) - 384*\pi*b^2*c^2*h*x^2*\log(\text{abs}(F))*\text{sgn}(F) - 192*I*\pi^2*b^2*c^2*h*x^2 + 384*\pi*b^2*c^2*h*x^2*\log(\text{abs}(F)) + 38 \\
& 4*I*b^2*c^2*h*x^2*\log(\text{abs}(F))^2 + 96*I*\pi^2*b^2*c^2*g*x*\text{sgn}(F) - 192*\pi*b^2*c^2*g*x*\log(\text{abs}(F))*\text{sgn}(F) - 96*I*\pi^2*b^2*c^2*g*x + 192*\pi*b^2*c^2*g*x*\log(\text{abs}(F)) + \\
& 192*I*b^2*c^2*g*x*\log(\text{abs}(F))^2 + 32*I*\pi^2*b^2*c^2*f*\text{sgn}(F) - 64*\pi*b^2*c^2*f*\log(\text{abs}(F))*\text{sgn}(F) - 32*I*\pi^2*b^2*c^2*f + 64*\pi*b^2*c^2*f*\log(\text{abs}(F)) + \\
& 64*I*b^2*c^2*f*\log(\text{abs}(F))^2 + 384*\pi*b*c*h*x*\text{sgn}(F) - 384*\pi*b*c*h*x - 768*I*b*c*h*x*\log(\text{abs}(F)) + 96*\pi*b*c*g*\text{sgn}(F) - 96*\pi*b*c*g - 1 \\
& 92*I*b*c*g*\log(\text{abs}(F)) + 768*I*h)*e^{(1/2*I*\pi*b*c*x*\text{sgn}(F) - 1/2*I*\pi*b*c*x + 1/2*I*\pi*a*c*\text{sgn}(F) - 1/2*I*\pi*a*c)/(16*I*\pi^5*b^5*c^5*\text{sgn}(F) - 80*\pi^4*b^5*c^5*\log(\text{abs}(F))*\text{sgn}(F) - 160*I*\pi^3*b^5*c^5*\log(\text{abs}(F))^2*\text{sgn}(F) + 160*\pi^2*b^5*c^5*\log(\text{abs}(F))^3*\text{sgn}(F) + 80*I*\pi*b^5*c^5*\log(\text{abs}(F))^4*\text{sgn}(F) - 16*I*\pi^5*b^5*c^5 + 80*\pi^4*b^5*c^5*\log(\text{abs}(F)) + 160*I*\pi^3*b^5*c^5*\log(\text{abs}(F))^2 - 160*\pi^2*b^5*c^5*\log(\text{abs}(F))^3 - 80*I*\pi*b^5*c^5*\log(\text{abs}(F))^4 + 32*b^5*c^5*\log(\text{abs}(F))^5} - (-16*I*\pi^4*b^4*c^4*h*x^4*\text{sgn}(F) - 64*\pi^3*b^4*c^4*h*x^4*\log(\text{abs}(F))*\text{sgn}(F) + 96*I*\pi^2*b^4*c^4*h*x^4*\log(\text{abs}(F))^2*\text{sgn}(F) + 64*\pi*b^4*c^4*h*x^4*\log(\text{abs}(F))^3*\text{sgn}(F) + 16*I*\pi^4*b^4*c^4*h*x^4 + 64*\pi^3*b^4*c^4*h*x^4*\log(\text{abs}(F)) - 96*I*\pi^2*b^4*c^4*h*x^4*\log(\text{abs}(F))^2 - 64*\pi*b^4*c^4*h*x^4*\log(\text{abs}(F))^3 + 32*I*b^4*c^4*h*x^4*\log(\text{abs}(F))^4 - 16*I*\pi^4*b^4*c^4*g*x^3*\text{sgn}(F) - 64*\pi^3*b^4*c^4*g*x^3*\log(\text{abs}(F))*\text{sgn}(F) + 96*I*\pi^2*b^4*c^4*g*x^3*\log(\text{abs}(F))^2*\text{sgn}(F) + 64*\pi*b^4*c^4*g*x^3*\log(\text{abs}(F))^3*\text{sgn}(F) + 16*I*\pi^4*b^4*c^4*g*x^3 + 64*\pi^3*b^4*c^4*g*x^3*\log(\text{abs}(F)) - 96*I*\pi^2*b^4*c^4*g*x^3*\log(\text{abs}(F))^2 - 64*\pi*b^4*c^4*g*x^3*\log(\text{abs}(F))^3 + 32*I*b^4*c^4*g*x^3*\log(\text{abs}(F))^4 - 16*I*\pi^4*b^4*c^4*f*x^2*\text{sgn}(F) - 64*\pi^3*b^4*c^4*f*x^2*\log(\text{abs}(F))*\text{sgn}(F) + 96*I*\pi^2*b^4*c^4*f*x^2*\log(\text{abs}(F))^2*\text{sgn}(F) + 64*\pi*b^4*c^4*f*x^2*\log(\text{abs}(F))^3*\text{sgn}(F) + 16*I*\pi^4*b^4*c^4*f*x^2 + 64*\pi^3*b^4*c^4*f*x^2*\log(\text{abs}(F)) - 96*I*\pi^2*b^4*c^4*f*x^2*\log(\text{abs}(F))^2 - 64*\pi*b^4*c^4*f*x^2*\log(\text{abs}(F))^3 + 32*I*b^4*c^4*f*x^2*\log(\text{abs}(F))^4 - 16*I*\pi^4*b^4*c^4*d*\text{sgn}(F) + 64*\pi^3*b^3*c^3*h*x^3*\text{sgn}(F) - 64*\pi^3*b^4*c^4*d*\log(\text{abs}(F))*\text{sgn}(F) - 192*I*\pi^2*b^3*c^3*h*x^3*\log(\text{abs}(F))*\text{sgn}(F) + 96*I*\pi^2*b^4*c^4*d*\log(\text{abs}(F))^2*\text{sgn}(F) - 192*\pi*b^3*c^3*h*x^3*\log(\text{abs}(F))^2*\text{sgn}(F) + 64*\pi*b^4*c^4*d*\log(\text{abs}(F))^3*\text{sgn}(F) + 16*I*\pi^4*b^4*c^4*d - 64*\pi^3*b^3*c^3*h*x^3 + 64*\pi^3*b^4*c^4*d*\log(\text{abs}(F)) + 192*I*\pi^2*b^3*c^3*h*x^3*\log(
\end{aligned}$$



$$\begin{aligned}
& \text{abs}(F)) - 96*I*\pi^2*b^4*c^4*d*\log(\text{abs}(F))^2 + 192*\pi*b^3*c^3*h*x^3*\log(\text{abs}(F))^2 - 64*\pi*b^4*c^4*d*\log(\text{abs}(F))^3 - 128*I*b^3*c^3*h*x^3*\log(\text{abs}(F))^3 + \\
& 32*I*b^4*c^4*d*\log(\text{abs}(F))^4 + 48*\pi^3*b^3*c^3*g*x^2*\text{sgn}(F) - 144*I*\pi^2*b^3*c^3*g*x^2*\log(\text{abs}(F))*\text{sgn}(F) - 144*\pi*b^3*c^3*g*x^2*\log(\text{abs}(F))^2*\text{sgn}(F) \\
& - 48*\pi^3*b^3*c^3*g*x^2 + 144*I*\pi^2*b^3*c^3*g*x^2*\log(\text{abs}(F)) + 144*\pi*b^3*c^3*g*x^2*\log(\text{abs}(F))^2 - 96*I*b^3*c^3*g*x^2*\log(\text{abs}(F))^3 + 32*\pi^3*b^3*c^3*f*x*\text{sgn}(F) - 96*I*\pi^2*b^3*c^3*f*x*\log(\text{abs}(F))*\text{sgn}(F) - 96*\pi*b^3*c^3*f*x*\log(\text{abs}(F))^2*\text{sgn}(F) - 32*\pi^3*b^3*c^3*f*x + 96*I*\pi^2*b^3*c^3*f*x*\log(\text{abs}(F)) + 96*\pi*b^3*c^3*f*x*\log(\text{abs}(F))^2 - 64*I*b^3*c^3*f*x*\log(\text{abs}(F))^3 + \\
& 192*I*\pi^2*b^2*c^2*h*x^2*\text{sgn}(F) + 384*\pi*b^2*c^2*h*x^2*\log(\text{abs}(F))*\text{sgn}(F) - 192*I*\pi^2*b^2*c^2*h*x^2 - 384*\pi*b^2*c^2*h*x^2*\log(\text{abs}(F)) + 384*I*b^2*c^2*h*x^2*\log(\text{abs}(F))^2 + 96*I*\pi^2*b^2*c^2*g*x*\text{sgn}(F) + 192*\pi*b^2*c^2*g*x*\log(\text{abs}(F))*\text{sgn}(F) - 96*I*\pi^2*b^2*c^2*g*x - 192*\pi*b^2*c^2*g*x*\log(\text{abs}(F)) + 192*I*b^2*c^2*g*x*\log(\text{abs}(F))^2 + 32*I*\pi^2*b^2*c^2*f*\text{sgn}(F) + 64*\pi*b^2*c^2*f*\log(\text{abs}(F))*\text{sgn}(F) - 32*I*\pi^2*b^2*c^2*f - 64*\pi*b^2*c^2*f*\log(\text{abs}(F)) + 64*I*b^2*c^2*f*\log(\text{abs}(F))^2 - 384*\pi*b*c*h*x*\text{sgn}(F) + 384*\pi*b*c*h*x - 768*I*b*c*h*x*\log(\text{abs}(F)) - 96*\pi*b*c*g*\text{sgn}(F) + 96*\pi*b*c*g - 192*I*b*c*g*\log(\text{abs}(F)) + 768*I*h)*e^{(-1/2*I*\pi*b*c*x*\text{sgn}(F) + 1/2*I*\pi*b*c*x - 1/2*I*\pi*a*c*\text{sgn}(F) + 1/2*I*\pi*a*c)/(-16*I*\pi^5*b^5*c^5*\text{sgn}(F) - 80*\pi^4*b^5*c^5*\log(\text{abs}(F))*\text{sgn}(F) + 160*I*\pi^3*b^5*c^5*\log(\text{abs}(F))^2*\text{sgn}(F) + 160*\pi^2*b^5*c^5*\log(\text{abs}(F))^3*\text{sgn}(F) - 80*I*\pi*b^5*c^5*\log(\text{abs}(F))^4*\text{sgn}(F) + 16*I*\pi^5*b^5*c^5 + 80*\pi^4*b^5*c^5*\log(\text{abs}(F)) - 160*I*\pi^3*b^5*c^5*\log(\text{abs}(F))^2 - 160*\pi^2*b^5*c^5*\log(\text{abs}(F))^3 + 80*I*\pi*b^5*c^5*\log(\text{abs}(F))^4 + 32*b^5*c^5*\log(\text{abs}(F))^5)}*e^{(b*c*x*\log(\text{abs}(F)) + a*c*\log(\text{abs}(F)))}
\end{aligned}$$

### 3.55 $\int e^{-a-bx} x^m (a + bx)^3 dx$

**Optimal.** Leaf size=116

$$\frac{a^3 e^{-a} x^m (bx)^{-m} \Gamma(m+1, bx)}{b} - \frac{3a^2 e^{-a} x^m (bx)^{-m} \Gamma(m+2, bx)}{b} - \frac{3ae^{-a} x^m (bx)^{-m} \Gamma(m+3, bx)}{b} - \frac{e^{-a} x^{m+3}}{b}$$

[Out]  $-\left(\frac{a^3 x^m \Gamma[1+m, b x]}{b E^a (b x)^m}\right) - \left(\frac{3 a^2 x^m \Gamma[2+m, b x]}{b E^a (b x)^m}\right) - \left(\frac{3 a x^m \Gamma[3+m, b x]}{b E^a (b x)^m}\right) - \left(\frac{x^{m+3} \Gamma[4+m, b x]}{b E^a (b x)^m}\right)$

**Rubi [A]** time = 0.174801, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 2, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$ , Rules used = {2199, 2181}

$$\frac{a^3 e^{-a} x^m (bx)^{-m} \Gamma(m+1, bx)}{b} - \frac{3a^2 e^{-a} x^m (bx)^{-m} \Gamma(m+2, bx)}{b} - \frac{3ae^{-a} x^m (bx)^{-m} \Gamma(m+3, bx)}{b} - \frac{e^{-a} x^{m+3}}{b}$$

Antiderivative was successfully verified.

[In] Int[E^(-a - b\*x)\*x^m\*(a + b\*x)^3,x]

[Out]  $-\left(\frac{a^3 x^m \Gamma[1+m, b x]}{b E^a (b x)^m}\right) - \left(\frac{3 a^2 x^m \Gamma[2+m, b x]}{b E^a (b x)^m}\right) - \left(\frac{3 a x^m \Gamma[3+m, b x]}{b E^a (b x)^m}\right) - \left(\frac{x^{m+3} \Gamma[4+m, b x]}{b E^a (b x)^m}\right)$

#### Rule 2199

Int[(F\_)^((c\_.)\*(v\_.))\*(u\_)^(m\_.)\*(w\_), x\_Symbol] := Int[ExpandIntegrand[F^(c\*ExpandToSum[v, x]), w\*NormalizePowerOfLinear[u, x]^m, x] /; FreeQ[{F, c}, x] && PolynomialQ[w, x] && LinearQ[v, x] && PowerOfLinearQ[u, x] && IntegerQ[m] && !\$UseGamma == True

#### Rule 2181

Int[(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_)))\*((c\_.) + (d\_.)\*(x\_))^(m\_), x\_Symbol] := -Simp[(F^(g\*(e - (c\*f)/d))\*(c + d\*x)^FracPart[m]\*Gamma[m + 1, (-((f\*g\*Log[F])/d))\*(c + d\*x])]/(d\*(-((f\*g\*Log[F])/d))^(IntPart[m] + 1)\*(-((f\*g\*Log[F])\*(c + d\*x)/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !IntegerQ[m]

#### Rubi steps

$$\begin{aligned} \int e^{-a-bx} x^m (a+bx)^3 dx &= \int (a^3 e^{-a-bx} x^m + 3a^2 b e^{-a-bx} x^{1+m} + 3ab^2 e^{-a-bx} x^{2+m} + b^3 e^{-a-bx} x^{3+m}) dx \\ &= a^3 \int e^{-a-bx} x^m dx + (3a^2 b) \int e^{-a-bx} x^{1+m} dx + (3ab^2) \int e^{-a-bx} x^{2+m} dx + b^3 \int e^{-a-bx} x^{3+m} dx \\ &= -\frac{a^3 e^{-a} x^m (bx)^{-m} \Gamma(1+m, bx)}{b} - \frac{3a^2 e^{-a} x^m (bx)^{-m} \Gamma(2+m, bx)}{b} - \frac{3a e^{-a} x^m (bx)^{-m} \Gamma(3+m, bx)}{b} \end{aligned}$$

**Mathematica [A]** time = 0.0568417, size = 61, normalized size = 0.53

$$\frac{e^{-a} x^m (bx)^{-m} (a^3 \text{Gamma}(m+1, bx) + 3a^2 \text{Gamma}(m+2, bx) + 3a \text{Gamma}(m+3, bx) + \text{Gamma}(m+4, bx))}{b}$$

Antiderivative was successfully verified.

[In] Integrate[E^(-a - b\*x)\*x^m\*(a + b\*x)^3, x]

[Out] -((x^m\*(a^3\*Gamma[1 + m, b\*x] + 3\*a^2\*Gamma[2 + m, b\*x] + 3\*a\*Gamma[3 + m, b\*x] + Gamma[4 + m, b\*x]))/(b\*E^a\*(b\*x)^m))

**Maple [C]** time = 0.08, size = 334, normalized size = 2.9

$$b^{-m-1} e^{-a} \left( x^m b^m (m^2 + 5m + 6) (bx)^{-\frac{m}{2}} e^{-\frac{bx}{2}} M_{\frac{m}{2}, \frac{m}{2} + \frac{1}{2}}(bx) - x^m b^m (b^2 x^2 + bmx + 3bx + m^2 + 5m + 6) (bx)^{-\frac{m}{2}} e^{-\frac{bx}{2}} M_{\frac{m}{2}, \frac{m}{2} + \frac{1}{2}}(bx) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(-b\*x-a)\*x^m\*(b\*x+a)^3, x)

[Out] b^(-m-1)\*exp(-a)\*(x^m\*b^m\*(m^2+5\*m+6)\*(b\*x)^(-1/2\*m)\*exp(-1/2\*b\*x)\*WhittakerM(1/2\*m, 1/2\*m+1/2, b\*x) - x^m\*b^m\*(b^2\*x^2+b\*m\*x+3\*b\*x+m^2+5\*m+6)\*(b\*x)^(-1/2\*m)\*exp(-1/2\*b\*x)\*WhittakerM(1/2\*m+1, 1/2\*m+1/2, b\*x))+3\*b^(-m-1)\*exp(-a)\*a\*(x^m\*b^m\*(2+m)\*(b\*x)^(-1/2\*m)\*exp(-1/2\*b\*x)\*WhittakerM(1/2\*m, 1/2\*m+1/2, b\*x) - x^m\*b^m\*(b\*x+m+2)\*(b\*x)^(-1/2\*m)\*exp(-1/2\*b\*x)\*WhittakerM(1/2\*m+1, 1/2\*m+1/2, b\*x))+3\*b^(-m-1)\*exp(-a)\*a^2\*(x^m\*b^m\*(b\*x)^(-1/2\*m)\*exp(-1/2\*b\*x)\*WhittakerM(1/2\*m, 1/2\*m+1/2, b\*x)+1/(2+m)\*x^m\*b^m\*(-2-m)\*(b\*x)^(-1/2\*m)\*exp(-1/2\*b\*x)\*WhittakerM(1/2\*m+1, 1/2\*m+1/2, b\*x))+exp(-a-1/2\*b\*x)/b\*a^3/(1+m)\*x^m\*(b\*x)^(-1/2\*m)\*WhittakerM(1/2\*m, 1/2\*m+1/2, b\*x)

**Maxima [A]** time = 1.23557, size = 166, normalized size = 1.43

$$-(bx)^{-m-4} b^3 x^{m+4} e^{(-a)} \Gamma(m+4, bx) - 3 (bx)^{-m-3} a b^2 x^{m+3} e^{(-a)} \Gamma(m+3, bx) - 3 (bx)^{-m-2} a^2 b x^{m+2} e^{(-a)} \Gamma(m+2, bx) - (bx)^{-m-1} a^3 x^{m+1} e^{(-a)} \Gamma(m+1, bx)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(-b\*x-a)\*x^m\*(b\*x+a)^3,x, algorithm="maxima")

[Out]  $-(b*x)^{-m-4} * b^3 * x^{m+4} * e^{-a} * \text{gamma}(m+4, b*x) - 3 * (b*x)^{-m-3} * a * b^2 * x^{m+3} * e^{-a} * \text{gamma}(m+3, b*x) - 3 * (b*x)^{-m-2} * a^2 * b * x^{m+2} * e^{-a} * \text{gamma}(m+2, b*x) - (b*x)^{-m-1} * a^3 * x^{m+1} * e^{-a} * \text{gamma}(m+1, b*x)$

**Fricas [A]** time = 1.56245, size = 297, normalized size = 2.56

$$\frac{(b^3 x^3 + (3(a+1)b^2 + b^2 m)x^2 + ((3a+5)bm + bm^2 + 3(a^2 + 2a + 2)b)x)x^m e^{-bx-a} + (a^3 + 3(a+2)m^2 + m^3 + 3a^2 + 3a + 6)e^{-m \log(b) - a} \text{gamma}(m+1, b*x))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(-b\*x-a)\*x^m\*(b\*x+a)^3,x, algorithm="fricas")

[Out]  $-((b^3 * x^3 + (3 * (a + 1) * b^2 + b^2 * m) * x^2 + ((3 * a + 5) * b * m + b * m^2 + 3 * (a^2 + 2 * a + 2) * b) * x) * x^m * e^{-b * x - a} + (a^3 + 3 * (a + 2) * m^2 + m^3 + 3 * a^2 + 3 * a + 6) * e^{-m * \log(b) - a} * \text{gamma}(m + 1, b * x)) / b$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(-b\*x-a)\*x\*\*m\*(b\*x+a)\*\*3,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx + a)^3 x^m e^{(-bx-a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(-b*x-a)*x^m*(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^3*x^m*e^(-b*x - a), x)
```

### 3.56 $\int e^{-a-bx} x^3 (a + bx)^3 dx$

**Optimal.** Leaf size=397

$$\frac{3a^3 x^2 e^{-a-bx}}{b^2} - \frac{36a^2 x^2 e^{-a-bx}}{b^2} - \frac{6a^3 x e^{-a-bx}}{b^3} - \frac{72a^2 x e^{-a-bx}}{b^3} - \frac{6a^3 e^{-a-bx}}{b^4} - \frac{72a^2 e^{-a-bx}}{b^4} - 3a^2 x^4 e^{-a-bx} - \frac{a^3 x^3 e^{-a-bx}}{b} - \frac{12a^2 x^2 e^{-a-bx}}{b^2}$$

[Out]  $(-720 * E^{(-a - b * x)}) / b^4 - (360 * a * E^{(-a - b * x)}) / b^4 - (72 * a^2 * E^{(-a - b * x)}) / b^4 - (6 * a^3 * E^{(-a - b * x)}) / b^4 - (720 * E^{(-a - b * x)} * x) / b^3 - (360 * a * E^{(-a - b * x)} * x) / b^3 - (72 * a^2 * E^{(-a - b * x)} * x) / b^3 - (6 * a^3 * E^{(-a - b * x)} * x) / b^3 - (360 * E^{(-a - b * x)} * x^2) / b^2 - (180 * a * E^{(-a - b * x)} * x^2) / b^2 - (36 * a^2 * E^{(-a - b * x)} * x^2) / b^2 - (3 * a^3 * E^{(-a - b * x)} * x^2) / b^2 - (120 * E^{(-a - b * x)} * x^3) / b - (60 * a * E^{(-a - b * x)} * x^3) / b - (12 * a^2 * E^{(-a - b * x)} * x^3) / b - (a^3 * E^{(-a - b * x)} * x^3) / b - 30 * E^{(-a - b * x)} * x^4 - 15 * a * E^{(-a - b * x)} * x^4 - 3 * a^2 * E^{(-a - b * x)} * x^4 - 6 * b * E^{(-a - b * x)} * x^5 - 3 * a * b * E^{(-a - b * x)} * x^5 - b^2 * E^{(-a - b * x)} * x^6$

**Rubi [A]** time = 0.516056, antiderivative size = 397, normalized size of antiderivative = 1., number of steps used = 24, number of rules used = 3, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {2196, 2176, 2194}

$$\frac{3a^3 x^2 e^{-a-bx}}{b^2} - \frac{36a^2 x^2 e^{-a-bx}}{b^2} - \frac{6a^3 x e^{-a-bx}}{b^3} - \frac{72a^2 x e^{-a-bx}}{b^3} - \frac{6a^3 e^{-a-bx}}{b^4} - \frac{72a^2 e^{-a-bx}}{b^4} - 3a^2 x^4 e^{-a-bx} - \frac{a^3 x^3 e^{-a-bx}}{b} - \frac{12a^2 x^2 e^{-a-bx}}{b^2}$$

Antiderivative was successfully verified.

[In] Int[E<sup>(-a - b\*x)</sup>\*x<sup>3</sup>\*(a + b\*x)<sup>3</sup>, x]

[Out]  $(-720 * E^{(-a - b * x)}) / b^4 - (360 * a * E^{(-a - b * x)}) / b^4 - (72 * a^2 * E^{(-a - b * x)}) / b^4 - (6 * a^3 * E^{(-a - b * x)}) / b^4 - (720 * E^{(-a - b * x)} * x) / b^3 - (360 * a * E^{(-a - b * x)} * x) / b^3 - (72 * a^2 * E^{(-a - b * x)} * x) / b^3 - (6 * a^3 * E^{(-a - b * x)} * x) / b^3 - (360 * E^{(-a - b * x)} * x^2) / b^2 - (180 * a * E^{(-a - b * x)} * x^2) / b^2 - (36 * a^2 * E^{(-a - b * x)} * x^2) / b^2 - (3 * a^3 * E^{(-a - b * x)} * x^2) / b^2 - (120 * E^{(-a - b * x)} * x^3) / b - (60 * a * E^{(-a - b * x)} * x^3) / b - (12 * a^2 * E^{(-a - b * x)} * x^3) / b - (a^3 * E^{(-a - b * x)} * x^3) / b - 30 * E^{(-a - b * x)} * x^4 - 15 * a * E^{(-a - b * x)} * x^4 - 3 * a^2 * E^{(-a - b * x)} * x^4 - 6 * b * E^{(-a - b * x)} * x^5 - 3 * a * b * E^{(-a - b * x)} * x^5 - b^2 * E^{(-a - b * x)} * x^6$

#### Rule 2196

Int[(F\_)^((c\_.)\*(v\_))\*(u\_), x\_Symbol] :> Int[ExpandIntegrand[F^(c\*ExpandToSum[v, x]), u, x], x] /; FreeQ[{F, c}, x] && PolynomialQ[u, x] && LinearQ[v, x] && !\$UseGamma === True

Rule 2176

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_)))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] := Simp[((c + d*x)^m*(b*F^(g*(e + f*x)))^n)/(f*g*n*Log[F]),
x] - Dist[(d*m)/(f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] && !$UseGamma === True
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_)))^(n_.), x_Symbol] := Simp[(F^(c*(a +
b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned}
\int e^{-a-bx}x^3(a+bx)^3 dx &= \int (a^3e^{-a-bx}x^3 + 3a^2be^{-a-bx}x^4 + 3ab^2e^{-a-bx}x^5 + b^3e^{-a-bx}x^6) dx \\
&= a^3 \int e^{-a-bx}x^3 dx + (3a^2b) \int e^{-a-bx}x^4 dx + (3ab^2) \int e^{-a-bx}x^5 dx + b^3 \int e^{-a-bx}x^6 dx \\
&= -\frac{a^3e^{-a-bx}x^3}{b} - 3a^2e^{-a-bx}x^4 - 3abe^{-a-bx}x^5 - b^2e^{-a-bx}x^6 + (12a^2) \int e^{-a-bx}x^3 dx + \frac{(3a^3) \int e^{-a-bx}x^3 dx}{b} \\
&= -\frac{3a^3e^{-a-bx}x^2}{b^2} - \frac{12a^2e^{-a-bx}x^3}{b} - \frac{a^3e^{-a-bx}x^3}{b} - 15ae^{-a-bx}x^4 - 3a^2e^{-a-bx}x^4 - 6be^{-a-bx}x^5 - 3abe^{-a-bx}x^6 \\
&= -\frac{6a^3e^{-a-bx}x}{b^3} - \frac{36a^2e^{-a-bx}x^2}{b^2} - \frac{3a^3e^{-a-bx}x^2}{b^2} - \frac{60ae^{-a-bx}x^3}{b} - \frac{12a^2e^{-a-bx}x^3}{b} - \frac{a^3e^{-a-bx}x^3}{b} - 30e^{-a-bx}x^4 \\
&= -\frac{6a^3e^{-a-bx}}{b^4} - \frac{72a^2e^{-a-bx}x}{b^3} - \frac{6a^3e^{-a-bx}x}{b^3} - \frac{180ae^{-a-bx}x^2}{b^2} - \frac{36a^2e^{-a-bx}x^2}{b^2} - \frac{3a^3e^{-a-bx}x^2}{b^2} - \frac{120ae^{-a-bx}x^3}{b} \\
&= -\frac{72a^2e^{-a-bx}}{b^4} - \frac{6a^3e^{-a-bx}}{b^4} - \frac{360ae^{-a-bx}x}{b^3} - \frac{72a^2e^{-a-bx}x}{b^3} - \frac{6a^3e^{-a-bx}x}{b^3} - \frac{360e^{-a-bx}x^2}{b^2} - \frac{180ae^{-a-bx}x^3}{b} \\
&= -\frac{360ae^{-a-bx}}{b^4} - \frac{72a^2e^{-a-bx}}{b^4} - \frac{6a^3e^{-a-bx}}{b^4} - \frac{720e^{-a-bx}x}{b^3} - \frac{360ae^{-a-bx}x}{b^3} - \frac{72a^2e^{-a-bx}x}{b^3} - \frac{6a^3e^{-a-bx}x^2}{b^2} \\
&= -\frac{720e^{-a-bx}}{b^4} - \frac{360ae^{-a-bx}}{b^4} - \frac{72a^2e^{-a-bx}}{b^4} - \frac{6a^3e^{-a-bx}}{b^4} - \frac{720e^{-a-bx}x}{b^3} - \frac{360ae^{-a-bx}x}{b^3} - \frac{72a^2e^{-a-bx}x^2}{b^2} - \frac{6a^3e^{-a-bx}x^3}{b}
\end{aligned}$$

**Mathematica [A]** time = 0.323201, size = 121, normalized size = 0.3

$$e^{-a-bx} \left( -\frac{3(a^3 + 12a^2 + 60a + 120)x^2}{b^2} - \frac{6(a^3 + 12a^2 + 60a + 120)x}{b^3} - \frac{6(a^3 + 12a^2 + 60a + 120)}{b^4} - \frac{(a^3 + 12a^2 + 60a + 120)}{b} \right)$$

Antiderivative was successfully verified.

[In] Integrate[E^(-a - b\*x)\*x^3\*(a + b\*x)^3,x]

[Out]  $E^{-a - b*x} * \left( \frac{-6*(120 + 60*a + 12*a^2 + a^3)}{b^4} - \frac{6*(120 + 60*a + 12*a^2 + a^3)*x}{b^3} - \frac{3*(120 + 60*a + 12*a^2 + a^3)*x^2}{b^2} - \frac{((120 + 60*a + 12*a^2 + a^3)*x^3)}{b} - 3*(10 + 5*a + a^2)*x^4 - 3*(2 + a)*b*x^5 - b^2*x^6 \right)$

**Maple [A]** time = 0.004, size = 182, normalized size = 0.5

$$\frac{(b^6 x^6 + 3 b^5 x^5 a + 3 a^2 b^4 x^4 + 6 b^5 x^5 + a^3 b^3 x^3 + 15 a b^4 x^4 + 12 a^2 b^3 x^3 + 30 x^4 b^4 + 3 a^3 b^2 x^2 + 60 a b^3 x^3 + 36 a^2 b^2 x^2 + 120 a^3 b x + 180 a^3 + 360 a^2 b x + 720 a^2 + 720 b x + 360 a + 720) \exp(-b*x-a)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(-b\*x-a)\*x^3\*(b\*x+a)^3,x)

[Out]  $-(b^6*x^6 + 3*a*b^5*x^5 + 3*a^2*b^4*x^4 + 6*b^5*x^5 + a^3*b^3*x^3 + 15*a*b^4*x^4 + 12*a^2*b^3*x^3 + 30*b^4*x^4 + 3*a^3*b^2*x^2 + 60*a*b^3*x^3 + 36*a^2*b^2*x^2 + 120*b^3*x^3 + 6*a^3*b*x + 180*a*b^2*x^2 + 72*a^2*b*x + 360*b^2*x^2 + 6*a^3 + 360*a*b*x + 72*a^2 + 720*b*x + 360*a + 720) * \exp(-b*x-a) / b^4$

**Maxima [A]** time = 1.06296, size = 265, normalized size = 0.67

$$\frac{(b^3 x^3 + 3 b^2 x^2 + 6 b x + 6) a^3 e^{-b x - a}}{b^4} - \frac{3 (b^4 x^4 + 4 b^3 x^3 + 12 b^2 x^2 + 24 b x + 24) a^2 e^{-b x - a}}{b^4} - \frac{3 (b^5 x^5 + 5 b^4 x^4 + 20 b^3 x^3 + 30 b^2 x^2 + 24 b x + 24) a e^{-b x - a}}{b^4} - \frac{(b^6 x^6 + 6 b^5 x^5 + 30 b^4 x^4 + 120 b^3 x^3 + 360 b^2 x^2 + 720 b x + 720) e^{-b x - a}}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(-b\*x-a)\*x^3\*(b\*x+a)^3,x, algorithm="maxima")

[Out]  $-(b^3*x^3 + 3*b^2*x^2 + 6*b*x + 6)*a^3*e^{-b*x - a}/b^4 - 3*(b^4*x^4 + 4*b^3*x^3 + 12*b^2*x^2 + 24*b*x + 24)*a^2*e^{-b*x - a}/b^4 - 3*(b^5*x^5 + 5*b^4*x^4 + 20*b^3*x^3 + 60*b^2*x^2 + 120*b*x + 120)*a*e^{-b*x - a}/b^4 - (b^6*x^6 + 6*b^5*x^5 + 30*b^4*x^4 + 120*b^3*x^3 + 360*b^2*x^2 + 720*b*x + 720)*e^{-b*x - a}/b^4$

**Fricas [A]** time = 1.47151, size = 301, normalized size = 0.76

$$\frac{(b^6 x^6 + 3(a+2)b^5 x^5 + 3(a^2 + 5a + 10)b^4 x^4 + (a^3 + 12a^2 + 60a + 120)b^3 x^3 + 3(a^3 + 12a^2 + 60a + 120)b^2 x^2 + 6a^3 + 36a^2 + 60a + 120)b x + 36a^3 + 360a^2 + 720a + 720) \exp(-b*x-a)}{b^4}$$



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(-b\*x-a)\*x^3\*(b\*x+a)^3,x, algorithm="fricas")

[Out]  $-(b^6*x^6 + 3*(a + 2)*b^5*x^5 + 3*(a^2 + 5*a + 10)*b^4*x^4 + (a^3 + 12*a^2 + 60*a + 120)*b^3*x^3 + 3*(a^3 + 12*a^2 + 60*a + 120)*b^2*x^2 + 6*a^3 + 6*(a^3 + 12*a^2 + 60*a + 120)*b*x + 72*a^2 + 360*a + 720)*e^{(-b*x - a)}/b^4$

**Sympy [A]** time = 0.192338, size = 236, normalized size = 0.59

$$\left\{ \frac{(-a^3b^3x^3 - 3a^3b^2x^2 - 6a^3bx - 6a^3 - 3a^2b^4x^4 - 12a^2b^3x^3 - 36a^2b^2x^2 - 72a^2bx - 72a^2 - 3ab^5x^5 - 15ab^4x^4 - 60ab^3x^3 - 180ab^2x^2 - 360abx - 360a - b^6x^6 - 6b^5x^5 - 30b^4x^4 - 120b^3x^3 - 360b^2x^2 - 720bx - 720)*e^{(-a - b*x)}/b^4, \frac{a^3x^4}{4} + \frac{3a^2bx^5}{5} + \frac{ab^2x^6}{2} + \frac{b^3x^7}{7} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(-b\*x-a)\*x\*\*3\*(b\*x+a)\*\*3,x)

[Out] Piecewise((( -a\*\*3\*b\*\*3\*x\*\*3 - 3\*a\*\*3\*b\*\*2\*x\*\*2 - 6\*a\*\*3\*b\*x - 6\*a\*\*3 - 3\*a\*\*2\*b\*\*4\*x\*\*4 - 12\*a\*\*2\*b\*\*3\*x\*\*3 - 36\*a\*\*2\*b\*\*2\*x\*\*2 - 72\*a\*\*2\*b\*x - 72\*a\*\*2 - 3\*a\*b\*\*5\*x\*\*5 - 15\*a\*b\*\*4\*x\*\*4 - 60\*a\*b\*\*3\*x\*\*3 - 180\*a\*b\*\*2\*x\*\*2 - 360\*a\*b\*x - 360\*a - b\*\*6\*x\*\*6 - 6\*b\*\*5\*x\*\*5 - 30\*b\*\*4\*x\*\*4 - 120\*b\*\*3\*x\*\*3 - 360\*b\*\*2\*x\*\*2 - 720\*b\*x - 720)\*exp(-a - b\*x)/b\*\*4, Ne(b\*\*4, 0)), (a\*\*3\*x\*\*4/4 + 3\*a\*\*2\*b\*x\*\*5/5 + a\*b\*\*2\*x\*\*6/2 + b\*\*3\*x\*\*7/7, True))

**Giac [A]** time = 1.25142, size = 273, normalized size = 0.69

$$\frac{(b^9x^6 + 3ab^8x^5 + 3a^2b^7x^4 + 6b^8x^5 + a^3b^6x^3 + 15ab^7x^4 + 12a^2b^6x^3 + 30b^7x^4 + 3a^3b^5x^2 + 60ab^6x^3 + 36a^2b^5x^2 + 120a^3b^4x + 180a^2b^5x^2 + 72a^2b^4x + 360ab^5x^2 + 6a^3b^3 + 360a^2b^4x + 72a^2b^3 + 720b^4x + 360a^2b^3 + 720ab^3)*e^{(-b*x - a)}/b^7$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(-b\*x-a)\*x^3\*(b\*x+a)^3,x, algorithm="giac")

[Out]  $-(b^9*x^6 + 3*a*b^8*x^5 + 3*a^2*b^7*x^4 + 6*b^8*x^5 + a^3*b^6*x^3 + 15*a*b^7*x^4 + 12*a^2*b^6*x^3 + 30*b^7*x^4 + 3*a^3*b^5*x^2 + 60*a*b^6*x^3 + 36*a^2*b^5*x^2 + 120*b^6*x^3 + 6*a^3*b^4*x + 180*a*b^5*x^2 + 72*a^2*b^4*x + 360*b^5*x^2 + 6*a^3*b^3 + 360*a*b^4*x + 72*a^2*b^3 + 720*b^4*x + 360*a*b^3 + 720*b^3)*e^{(-b*x - a)}/b^7$

### 3.57 $\int e^{-a-bx} x^2 (a + bx)^3 dx$

**Optimal.** Leaf size=318

$$-\frac{2a^3 x e^{-a-bx}}{b^2} - \frac{18a^2 x e^{-a-bx}}{b^2} - \frac{2a^3 e^{-a-bx}}{b^3} - \frac{18a^2 e^{-a-bx}}{b^3} - 3a^2 x^3 e^{-a-bx} - \frac{a^3 x^2 e^{-a-bx}}{b} - \frac{9a^2 x^2 e^{-a-bx}}{b} - b^2 x^5 e^{-a-bx} - \frac{72ax e^{-a-bx}}{b^2}$$

[Out]  $(-120 * E^{-a - b * x}) / b^3 - (72 * a * E^{-a - b * x}) / b^3 - (18 * a^2 * E^{-a - b * x}) / b^3 - (2 * a^3 * E^{-a - b * x}) / b^3 - (120 * E^{-a - b * x} * x) / b^2 - (72 * a * E^{-a - b * x} * x) / b^2 - (18 * a^2 * E^{-a - b * x} * x) / b^2 - (2 * a^3 * E^{-a - b * x} * x) / b^2 - (60 * E^{-a - b * x} * x^2) / b - (36 * a * E^{-a - b * x} * x^2) / b - (9 * a^2 * E^{-a - b * x} * x^2) / b - (a^3 * E^{-a - b * x} * x^2) / b - 20 * E^{-a - b * x} * x^3 - 12 * a * E^{-a - b * x} * x^3 - 3 * a^2 * E^{-a - b * x} * x^3 - 5 * b * E^{-a - b * x} * x^4 - 3 * a * b * E^{-a - b * x} * x^4 - b^2 * E^{-a - b * x} * x^5$

**Rubi [A]** time = 0.412468, antiderivative size = 318, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 3, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {2196, 2176, 2194}

$$-\frac{2a^3 x e^{-a-bx}}{b^2} - \frac{18a^2 x e^{-a-bx}}{b^2} - \frac{2a^3 e^{-a-bx}}{b^3} - \frac{18a^2 e^{-a-bx}}{b^3} - 3a^2 x^3 e^{-a-bx} - \frac{a^3 x^2 e^{-a-bx}}{b} - \frac{9a^2 x^2 e^{-a-bx}}{b} - b^2 x^5 e^{-a-bx} - \frac{72ax e^{-a-bx}}{b^2}$$

Antiderivative was successfully verified.

[In] Int[E^{-a - b \* x} \* x^2 \* (a + b \* x)^3, x]

[Out]  $(-120 * E^{-a - b * x}) / b^3 - (72 * a * E^{-a - b * x}) / b^3 - (18 * a^2 * E^{-a - b * x}) / b^3 - (2 * a^3 * E^{-a - b * x}) / b^3 - (120 * E^{-a - b * x} * x) / b^2 - (72 * a * E^{-a - b * x} * x) / b^2 - (18 * a^2 * E^{-a - b * x} * x) / b^2 - (2 * a^3 * E^{-a - b * x} * x) / b^2 - (60 * E^{-a - b * x} * x^2) / b - (36 * a * E^{-a - b * x} * x^2) / b - (9 * a^2 * E^{-a - b * x} * x^2) / b - (a^3 * E^{-a - b * x} * x^2) / b - 20 * E^{-a - b * x} * x^3 - 12 * a * E^{-a - b * x} * x^3 - 3 * a^2 * E^{-a - b * x} * x^3 - 5 * b * E^{-a - b * x} * x^4 - 3 * a * b * E^{-a - b * x} * x^4 - b^2 * E^{-a - b * x} * x^5$

#### Rule 2196

Int[(F\_)^((c\_.)\*(v\_))\*(u\_), x\_Symbol] :> Int[ExpandIntegrand[F^(c\*ExpandToSum[v, x]), u, x], x] /; FreeQ[{F, c}, x] && PolynomialQ[u, x] && LinearQ[v, x] && !\$UseGamma === True

#### Rule 2176

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] := Simp[((c + d*x)^m*(b*F^(g*(e + f*x)))^n)/(f*g*n*Log[F]),
x] - Dist[(d*m)/(f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] && !$UseGamma == True
```

### Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] := Simp[(F^(c*(a +
b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

### Rubi steps

$$\begin{aligned}
\int e^{-a-bx}x^2(a+bx)^3 dx &= \int (a^3e^{-a-bx}x^2 + 3a^2be^{-a-bx}x^3 + 3ab^2e^{-a-bx}x^4 + b^3e^{-a-bx}x^5) dx \\
&= a^3 \int e^{-a-bx}x^2 dx + (3a^2b) \int e^{-a-bx}x^3 dx + (3ab^2) \int e^{-a-bx}x^4 dx + b^3 \int e^{-a-bx}x^5 dx \\
&= -\frac{a^3e^{-a-bx}x^2}{b} - 3a^2e^{-a-bx}x^3 - 3abe^{-a-bx}x^4 - b^2e^{-a-bx}x^5 + (9a^2) \int e^{-a-bx}x^2 dx + \frac{(2a^3) \int e^{-a-bx}x^2 dx}{b} \\
&= -\frac{2a^3e^{-a-bx}x}{b^2} - \frac{9a^2e^{-a-bx}x^2}{b} - \frac{a^3e^{-a-bx}x^2}{b} - 12ae^{-a-bx}x^3 - 3a^2e^{-a-bx}x^3 - 5be^{-a-bx}x^4 - 3abe^{-a-bx}x^4 \\
&= -\frac{2a^3e^{-a-bx}}{b^3} - \frac{18a^2e^{-a-bx}}{b^2} - \frac{2a^3e^{-a-bx}}{b^2} - \frac{36ae^{-a-bx}x^2}{b} - \frac{9a^2e^{-a-bx}x^2}{b} - \frac{a^3e^{-a-bx}x^2}{b} - 20e^{-a-bx}x^3 \\
&= -\frac{18a^2e^{-a-bx}}{b^3} - \frac{2a^3e^{-a-bx}}{b^3} - \frac{72ae^{-a-bx}x}{b^2} - \frac{18a^2e^{-a-bx}x}{b^2} - \frac{2a^3e^{-a-bx}x}{b^2} - \frac{60e^{-a-bx}x^2}{b} - \frac{36ae^{-a-bx}x^2}{b} \\
&= -\frac{72ae^{-a-bx}}{b^3} - \frac{18a^2e^{-a-bx}}{b^3} - \frac{2a^3e^{-a-bx}}{b^3} - \frac{120e^{-a-bx}x}{b^2} - \frac{72ae^{-a-bx}x}{b^2} - \frac{18a^2e^{-a-bx}x}{b^2} - \frac{2a^3e^{-a-bx}x}{b^2} \\
&= -\frac{120e^{-a-bx}}{b^3} - \frac{72ae^{-a-bx}}{b^3} - \frac{18a^2e^{-a-bx}}{b^3} - \frac{2a^3e^{-a-bx}}{b^3} - \frac{120e^{-a-bx}x}{b^2} - \frac{72ae^{-a-bx}x}{b^2} - \frac{18a^2e^{-a-bx}x}{b^2}
\end{aligned}$$

**Mathematica [A]** time = 0.198432, size = 130, normalized size = 0.41

$$e^{-bx} \left( -\frac{2(a^3 + 9a^2 + 36a + 60)e^{-ax}}{b^2} - \frac{2(a^3 + 9a^2 + 36a + 60)e^{-a}}{b^3} - \frac{(a^3 + 9a^2 + 36a + 60)e^{-ax^2}}{b} - (3a^2 + 12a + 20)e^{-bx} \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(-a - b*x)*x^2*(a + b*x)^3, x]
```

```
[Out] ((-2*(60 + 36*a + 9*a^2 + a^3))/(b^3*E^a) - (2*(60 + 36*a + 9*a^2 + a^3)*x)
/(b^2*E^a) - ((60 + 36*a + 9*a^2 + a^3)*x^2)/(b*E^a) - ((20 + 12*a + 3*a^2)
```

$$*x^3)/E^a - ((5 + 3*a)*b*x^4)/E^a - (b^2*x^5)/E^a)/E^{(b*x)}$$

**Maple [A]** time = 0.005, size = 143, normalized size = 0.5

$$\frac{(b^5x^5 + 3b^4x^4a + 3a^2b^3x^3 + 5b^4x^4 + a^3b^2x^2 + 12ab^3x^3 + 9a^2b^2x^2 + 20x^3b^3 + 2a^3bx + 36ab^2x^2 + 18a^2bx + 60b^2x^2 + 120b^3x^3 + 120a^2b^2x^2 + 120ab^3x^3 + 120a^3b^2x^2 + 120a^4bx + 120a^5)/b^3}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(-b\*x-a)\*x^2\*(b\*x+a)^3,x)

[Out]  $-(b^5x^5 + 3a*b^4x^4 + 3a^2*b^3x^3 + 5b^4x^4 + a^3*b^2x^2 + 12a*b^3x^3 + 9a^2*b^2x^2 + 20*b^3x^3 + 2a^3*b*x + 36a*b^2x^2 + 18a^2*b*x + 60*b^2x^2 + 2a^3 + 72*a*b*x + 18a^2 + 120*b*x + 72a + 120) \exp(-b*x-a) / b^3$

**Maxima [A]** time = 1.05629, size = 221, normalized size = 0.69

$$\frac{(b^2x^2 + 2bx + 2)a^3e^{(-bx-a)}}{b^3} - \frac{3(b^3x^3 + 3b^2x^2 + 6bx + 6)a^2e^{(-bx-a)}}{b^3} - \frac{3(b^4x^4 + 4b^3x^3 + 12b^2x^2 + 24bx + 24)ae^{(-bx-a)}}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(-b\*x-a)\*x^2\*(b\*x+a)^3,x, algorithm="maxima")

[Out]  $-(b^2x^2 + 2b*x + 2)*a^3*e^{(-b*x - a)}/b^3 - 3*(b^3*x^3 + 3*b^2*x^2 + 6*b*x + 6)*a^2*e^{(-b*x - a)}/b^3 - 3*(b^4*x^4 + 4*b^3*x^3 + 12*b^2*x^2 + 24*b*x + 24)*a*e^{(-b*x - a)}/b^3 - (b^5*x^5 + 5*b^4*x^4 + 20*b^3*x^3 + 60*b^2*x^2 + 120*b*x + 120)*e^{(-b*x - a)}/b^3$

**Fricas [A]** time = 1.52069, size = 242, normalized size = 0.76

$$\frac{(b^5x^5 + (3a + 5)b^4x^4 + (3a^2 + 12a + 20)b^3x^3 + (a^3 + 9a^2 + 36a + 60)b^2x^2 + 2a^3 + 2(a^3 + 9a^2 + 36a + 60)bx + 18a^3 + 120b^3x^3 + 120a^2b^2x^2 + 120ab^3x^3 + 120a^3b^2x^2 + 120a^4bx + 120a^5)/b^3}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(-b\*x-a)\*x^2\*(b\*x+a)^3,x, algorithm="fricas")

[Out]  $-(b^5x^5 + (3a + 5)b^4x^4 + (3a^2 + 12a + 20)b^3x^3 + (a^3 + 9a^2 + 36a + 60)b^2x^2 + 2a^3 + 2(a^3 + 9a^2 + 36a + 60)bx + 18a^2 + 72a + 120)e^{(-bx - a)}/b^3$

**Sympy [A]** time = 0.175937, size = 196, normalized size = 0.62

$$\left\{ \frac{(-a^3b^2x^2 - 2a^3bx - 2a^3 - 3a^2b^3x^3 - 9a^2b^2x^2 - 18a^2bx - 18a^2 - 3ab^4x^4 - 12ab^3x^3 - 36ab^2x^2 - 72abx - 72a - b^5x^5 - 5b^4x^4 - 20b^3x^3 - 60b^2x^2 - 120bx - 120)e^{-a-bx}}{b^3}, \frac{a^3x^3}{3} + \frac{3a^2bx^4}{4} + \frac{3ab^2x^5}{5} + \frac{b^3x^6}{6} \right.$$

for  $b^3$   
other

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(-b\*x-a)\*x\*\*2\*(b\*x+a)\*\*3,x)

[Out] Piecewise((( -a\*\*3\*b\*\*2\*x\*\*2 - 2\*a\*\*3\*b\*x - 2\*a\*\*3 - 3\*a\*\*2\*b\*\*3\*x\*\*3 - 9\*a\*\*2\*b\*\*2\*x\*\*2 - 18\*a\*\*2\*b\*x - 18\*a\*\*2 - 3\*a\*b\*\*4\*x\*\*4 - 12\*a\*b\*\*3\*x\*\*3 - 36\*a\*b\*\*2\*x\*\*2 - 72\*a\*b\*x - 72\*a - b\*\*5\*x\*\*5 - 5\*b\*\*4\*x\*\*4 - 20\*b\*\*3\*x\*\*3 - 60\*b\*\*2\*x\*\*2 - 120\*b\*x - 120)\*exp(-a - b\*x)/b\*\*3, Ne(b\*\*3, 0)), (a\*\*3\*x\*\*3/3 + 3\*a\*\*2\*b\*x\*\*4/4 + 3\*a\*b\*\*2\*x\*\*5/5 + b\*\*3\*x\*\*6/6, True))

**Giac [A]** time = 1.36125, size = 220, normalized size = 0.69

$$\frac{(b^8x^5 + 3ab^7x^4 + 3a^2b^6x^3 + 5b^7x^4 + a^3b^5x^2 + 12ab^6x^3 + 9a^2b^5x^2 + 20b^6x^3 + 2a^3b^4x + 36ab^5x^2 + 18a^2b^4x + 60b^5x^2 + 120ab^4x + 72a^2b^3 + 120b^4x + 72a^3b^3 + 120b^3)e^{(-bx - a)}/b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(-b\*x-a)\*x^2\*(b\*x+a)^3,x, algorithm="giac")

[Out]  $-(b^8x^5 + 3a^2b^6x^3 + 5b^7x^4 + a^3b^5x^2 + 12a^2b^6x^3 + 9a^2b^5x^2 + 20b^6x^3 + 2a^3b^4x + 36a^2b^5x^2 + 18a^2b^4x + 60b^5x^2 + 2a^3b^3 + 72a^2b^4x + 18a^2b^3 + 120b^4x + 72a^3b^3 + 120b^3)e^{(-bx - a)}/b^6$

### 3.58 $\int e^{-a-bx} x(a+bx)^3 dx$

**Optimal.** Leaf size=184

$$-\frac{e^{-a-bx}(a+bx)^4}{b^2} + \frac{ae^{-a-bx}(a+bx)^3}{b^2} - \frac{4e^{-a-bx}(a+bx)^3}{b^2} + \frac{3ae^{-a-bx}(a+bx)^2}{b^2} - \frac{12e^{-a-bx}(a+bx)^2}{b^2} + \frac{6ae^{-a-bx}(a+bx)}{b^2} - \frac{2e^{-a-bx}}{b^2}$$

[Out]  $(-24 * E^{-a - b * x}) / b^2 + (6 * a * E^{-a - b * x}) / b^2 - (24 * E^{-a - b * x} * (a + b * x)) / b^2 + (6 * a * E^{-a - b * x} * (a + b * x)) / b^2 - (12 * E^{-a - b * x} * (a + b * x)^2) / b^2 + (3 * a * E^{-a - b * x} * (a + b * x)^2) / b^2 - (4 * E^{-a - b * x} * (a + b * x)^3) / b^2 + (a * E^{-a - b * x} * (a + b * x)^3) / b^2 - (E^{-a - b * x} * (a + b * x)^4) / b^2$

**Rubi [A]** time = 0.241916, antiderivative size = 184, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 3, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$ , Rules used = {2196, 2176, 2194}

$$-\frac{e^{-a-bx}(a+bx)^4}{b^2} + \frac{ae^{-a-bx}(a+bx)^3}{b^2} - \frac{4e^{-a-bx}(a+bx)^3}{b^2} + \frac{3ae^{-a-bx}(a+bx)^2}{b^2} - \frac{12e^{-a-bx}(a+bx)^2}{b^2} + \frac{6ae^{-a-bx}(a+bx)}{b^2} - \frac{2e^{-a-bx}}{b^2}$$

Antiderivative was successfully verified.

[In] Int[E<sup>-(a + b\*x)</sup>\*x\*(a + b\*x)<sup>3</sup>, x]

[Out]  $(-24 * E^{-a - b * x}) / b^2 + (6 * a * E^{-a - b * x}) / b^2 - (24 * E^{-a - b * x} * (a + b * x)) / b^2 + (6 * a * E^{-a - b * x} * (a + b * x)) / b^2 - (12 * E^{-a - b * x} * (a + b * x)^2) / b^2 + (3 * a * E^{-a - b * x} * (a + b * x)^2) / b^2 - (4 * E^{-a - b * x} * (a + b * x)^3) / b^2 + (a * E^{-a - b * x} * (a + b * x)^3) / b^2 - (E^{-a - b * x} * (a + b * x)^4) / b^2$

#### Rule 2196

Int[(F\_)^((c\_)\*(v\_))\*(u\_), x\_Symbol] :> Int[ExpandIntegrand[F^(c\*ExpandToSum[v, x]), u, x], x] /; FreeQ[{F, c}, x] && PolynomialQ[u, x] && LinearQ[v, x] && !\$UseGamma === True

#### Rule 2176

Int[((b\_)\*(F\_)^((g\_)\*((e\_)+(f\_)\*(x\_))))^(n\_)\*((c\_)+(d\_)\*(x\_))^(m\_), x\_Symbol] :> Simp[((c + d\*x)^m\*(b\*F^(g\*(e + f\*x)))^n)/(f\*g\*n\*Log[F]), x] - Dist[(d\*m)/(f\*g\*n\*Log[F]), Int[(c + d\*x)^(m - 1)\*(b\*F^(g\*(e + f\*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2\*m] && !\$UseGamma === True

Rule 2194

`Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] := Simp[(F^(c*(a + b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]`

Rubi steps

$$\begin{aligned}
 \int e^{-a-bx} x(a+bx)^3 dx &= \int \left( -\frac{ae^{-a-bx}(a+bx)^3}{b} + \frac{e^{-a-bx}(a+bx)^4}{b} \right) dx \\
 &= \frac{\int e^{-a-bx}(a+bx)^4 dx}{b} - \frac{a \int e^{-a-bx}(a+bx)^3 dx}{b} \\
 &= \frac{ae^{-a-bx}(a+bx)^3}{b^2} - \frac{e^{-a-bx}(a+bx)^4}{b^2} + \frac{4 \int e^{-a-bx}(a+bx)^3 dx}{b} - \frac{(3a) \int e^{-a-bx}(a+bx)^2 dx}{b} \\
 &= \frac{3ae^{-a-bx}(a+bx)^2}{b^2} - \frac{4e^{-a-bx}(a+bx)^3}{b^2} + \frac{ae^{-a-bx}(a+bx)^3}{b^2} - \frac{e^{-a-bx}(a+bx)^4}{b^2} + \frac{12 \int e^{-a-bx}(a+bx) dx}{b} \\
 &= \frac{6ae^{-a-bx}(a+bx)}{b^2} - \frac{12e^{-a-bx}(a+bx)^2}{b^2} + \frac{3ae^{-a-bx}(a+bx)^2}{b^2} - \frac{4e^{-a-bx}(a+bx)^3}{b^2} + \frac{ae^{-a-bx}(a+bx)}{b^2} \\
 &= \frac{6ae^{-a-bx}}{b^2} - \frac{24e^{-a-bx}(a+bx)}{b^2} + \frac{6ae^{-a-bx}(a+bx)}{b^2} - \frac{12e^{-a-bx}(a+bx)^2}{b^2} + \frac{3ae^{-a-bx}(a+bx)^2}{b^2} - \frac{4e^{-a-bx}(a+bx)^3}{b^2} \\
 &= -\frac{24e^{-a-bx}}{b^2} + \frac{6ae^{-a-bx}}{b^2} - \frac{24e^{-a-bx}(a+bx)}{b^2} + \frac{6ae^{-a-bx}(a+bx)}{b^2} - \frac{12e^{-a-bx}(a+bx)^2}{b^2} + \frac{3ae^{-a-bx}(a+bx)^2}{b^2}
 \end{aligned}$$

**Mathematica [A]** time = 0.121952, size = 96, normalized size = 0.52

$$\frac{e^{-a-bx} \left( -3a^2 (b^2 x^2 + 2bx + 2) - a^3 (bx + 1) - 3a (b^3 x^3 + 3b^2 x^2 + 6bx + 6) - b^4 x^4 - 4b^3 x^3 - 12b^2 x^2 - 24bx - 24 \right)}{b^2}$$

Antiderivative was successfully verified.

[In] Integrate[E^(-a - b\*x)\*x\*(a + b\*x)^3,x]

[Out] (E^(-a - b\*x))\*(-24 - 24\*b\*x - 12\*b^2\*x^2 - 4\*b^3\*x^3 - b^4\*x^4 - a^3\*(1 + b\*x) - 3\*a^2\*(2 + 2\*b\*x + b^2\*x^2) - 3\*a\*(6 + 6\*b\*x + 3\*b^2\*x^2 + b^3\*x^3)) / b^2

**Maple [A]** time = 0.004, size = 102, normalized size = 0.6

$$\frac{(b^4 x^4 + 3 b^3 x^3 a + 3 a^2 b^2 x^2 + 4 b^3 x^3 + a^3 b x + 9 a b^2 x^2 + 6 a^2 b x + 12 b^2 x^2 + a^3 + 18 a b x + 6 a^2 + 24 b x + 18 a + 24) e^{-b x}}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(exp(-b*x-a)*x*(b*x+a)^3,x)`

[Out]  $-(b^4x^4+3ab^3x^3+3a^2b^2x^2+4b^3x^3+a^3bx+9ab^2x^2+6a^2bx+12b^2x^2+a^3+18abx+6a^2+24bx+18a+24)\exp(-bx-a)/b^2$

**Maxima [A]** time = 1.09198, size = 178, normalized size = 0.97

$$\frac{(bx+1)a^3e^{(-bx-a)}}{b^2} - \frac{3(b^2x^2+2bx+2)a^2e^{(-bx-a)}}{b^2} - \frac{3(b^3x^3+3b^2x^2+6bx+6)ae^{(-bx-a)}}{b^2} - \frac{(b^4x^4+4b^3x^3+12b^2x^2+24bx+18a+24)e^{(-bx-a)}}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(-b*x-a)*x*(b*x+a)^3,x, algorithm="maxima")`

[Out]  $-(bx+1)a^3e^{(-bx-a)}/b^2 - 3(b^2x^2+2bx+2)a^2e^{(-bx-a)}/b^2 - 3(b^3x^3+3b^2x^2+6bx+6)ae^{(-bx-a)}/b^2 - (b^4x^4+4b^3x^3+12b^2x^2+24bx+24)e^{(-bx-a)}/b^2$

**Fricas [A]** time = 1.50241, size = 182, normalized size = 0.99

$$\frac{(b^4x^4+(3a+4)b^3x^3+3(a^2+3a+4)b^2x^2+a^3+(a^3+6a^2+18a+24)bx+6a^2+18a+24)e^{(-bx-a)}}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(-b*x-a)*x*(b*x+a)^3,x, algorithm="fricas")`

[Out]  $-(b^4x^4+(3a+4)b^3x^3+3(a^2+3a+4)b^2x^2+a^3+(a^3+6a^2+18a+24)bx+6a^2+18a+24)e^{(-bx-a)}/b^2$

**Sympy [A]** time = 0.163471, size = 148, normalized size = 0.8

$$\begin{cases} \frac{(-a^3bx-a^3-3a^2b^2x^2-6a^2bx-6a^2-3ab^3x^3-9ab^2x^2-18abx-18a-b^4x^4-4b^3x^3-12b^2x^2-24bx-24)e^{-a-bx}}{b^2} & \text{for } b^2 \neq 0 \\ \frac{a^3x^2}{2} + a^2bx^3 + \frac{3ab^2x^4}{4} + \frac{b^3x^5}{5} & \text{otherwise} \end{cases}$$





### 3.59 $\int e^{-a-bx}(a+bx)^3 dx$

**Optimal.** Leaf size=80

$$-\frac{e^{-a-bx}(a+bx)^3}{b} - \frac{3e^{-a-bx}(a+bx)^2}{b} - \frac{6e^{-a-bx}(a+bx)}{b} - \frac{6e^{-a-bx}}{b}$$

[Out]  $(-6E^{-a-bx})/b - (6E^{-a-bx}(a+bx))/b - (3E^{-a-bx}(a+bx)^2)/b - (E^{-a-bx}(a+bx)^3)/b$

**Rubi [A]** time = 0.0683236, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$ , Rules used = {2176, 2194}

$$-\frac{e^{-a-bx}(a+bx)^3}{b} - \frac{3e^{-a-bx}(a+bx)^2}{b} - \frac{6e^{-a-bx}(a+bx)}{b} - \frac{6e^{-a-bx}}{b}$$

Antiderivative was successfully verified.

[In] Int[E<sup>-(a+bx)</sup>(a+bx)<sup>3</sup>, x]

[Out]  $(-6E^{-a-bx})/b - (6E^{-a-bx}(a+bx))/b - (3E^{-a-bx}(a+bx)^2)/b - (E^{-a-bx}(a+bx)^3)/b$

#### Rule 2176

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((c + d*x)^m*(b*F^(g*(e + f*x)))^n)/(f*g*n*Log[F]), x] - Dist[(d*m)/(f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n, x], x]
/; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m] && !$UseGamma === True
```

#### Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol]
:> Simp[(F^(c*(a + b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

#### Rubi steps

$$\begin{aligned}
\int e^{-a-bx}(a+bx)^3 dx &= -\frac{e^{-a-bx}(a+bx)^3}{b} + 3 \int e^{-a-bx}(a+bx)^2 dx \\
&= -\frac{3e^{-a-bx}(a+bx)^2}{b} - \frac{e^{-a-bx}(a+bx)^3}{b} + 6 \int e^{-a-bx}(a+bx) dx \\
&= -\frac{6e^{-a-bx}(a+bx)}{b} - \frac{3e^{-a-bx}(a+bx)^2}{b} - \frac{e^{-a-bx}(a+bx)^3}{b} + 6 \int e^{-a-bx} dx \\
&= -\frac{6e^{-a-bx}}{b} - \frac{6e^{-a-bx}(a+bx)}{b} - \frac{3e^{-a-bx}(a+bx)^2}{b} - \frac{e^{-a-bx}(a+bx)^3}{b}
\end{aligned}$$

**Mathematica [A]** time = 0.046695, size = 41, normalized size = 0.51

$$\frac{e^{-a-bx} \left( -(a+bx)^3 - 3(a+bx)^2 - 6(a+bx) - 6 \right)}{b}$$

Antiderivative was successfully verified.

[In] Integrate[E^(-a - b\*x)\*(a + b\*x)^3,x]

[Out] (E^(-a - b\*x)\*(-6 - 6\*(a + b\*x) - 3\*(a + b\*x)^2 - (a + b\*x)^3))/b

**Maple [A]** time = 0.003, size = 68, normalized size = 0.9

$$\frac{(b^3x^3 + 3b^2x^2a + 3a^2bx + 3b^2x^2 + a^3 + 6abx + 3a^2 + 6bx + 6a + 6)e^{-bx-a}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(-b\*x-a)\*(b\*x+a)^3,x)

[Out] -(b^3\*x^3+3\*a\*b^2\*x^2+3\*a^2\*b\*x+3\*b^2\*x^2+a^3+6\*a\*b\*x+3\*a^2+6\*b\*x+6\*a+6)\*exp(-b\*x-a)/b

**Maxima [A]** time = 1.06245, size = 139, normalized size = 1.74

$$\frac{3(bx+1)a^2e^{(-bx-a)}}{b} - \frac{a^3e^{(-bx-a)}}{b} - \frac{3(b^2x^2+2bx+2)ae^{(-bx-a)}}{b} - \frac{(b^3x^3+3b^2x^2+6bx+6)e^{(-bx-a)}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(-b\*x-a)\*(b\*x+a)^3,x, algorithm="maxima")

[Out]  $-3*(b*x + 1)*a^2*e^{(-b*x - a)}/b - a^3*e^{(-b*x - a)}/b - 3*(b^2*x^2 + 2*b*x + 2)*a*e^{(-b*x - a)}/b - (b^3*x^3 + 3*b^2*x^2 + 6*b*x + 6)*e^{(-b*x - a)}/b$

**Fricas [A]** time = 1.47203, size = 128, normalized size = 1.6

$$\frac{(b^3x^3 + 3(a+1)b^2x^2 + a^3 + 3(a^2 + 2a + 2)bx + 3a^2 + 6a + 6)e^{(-bx-a)}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(-b\*x-a)\*(b\*x+a)^3,x, algorithm="fricas")

[Out]  $-(b^3*x^3 + 3*(a + 1)*b^2*x^2 + a^3 + 3*(a^2 + 2*a + 2)*b*x + 3*a^2 + 6*a + 6)*e^{(-b*x - a)}/b$

**Sympy [A]** time = 0.145624, size = 104, normalized size = 1.3

$$\begin{cases} \frac{((-a^3-3a^2bx-3a^2-3ab^2x^2-6abx-6a-b^3x^3-3b^2x^2-6bx-6)e^{-a-bx}}{a^3x + \frac{3a^2bx^2}{2} + ab^2x^3 + \frac{b^3x^4}{4}} & \text{for } b \neq 0 \\ \text{otherwise} & \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(-b\*x-a)\*(b\*x+a)\*\*3,x)

[Out] Piecewise((( $-a**3 - 3*a**2*b*x - 3*a**2 - 3*a*b**2*x**2 - 6*a*b*x - 6*a - b**3*x**3 - 3*b**2*x**2 - 6*b*x - 6$ ) $*exp(-a - b*x)/b$ , Ne(b, 0)), ( $a**3*x + 3*a**2*b*x**2/2 + a*b**2*x**3 + b**3*x**4/4$ , True))

**Giac [A]** time = 1.31471, size = 117, normalized size = 1.46

$$\frac{(b^6x^3 + 3ab^5x^2 + 3a^2b^4x + 3b^5x^2 + a^3b^3 + 6ab^4x + 3a^2b^3 + 6b^4x + 6ab^3 + 6b^3)e^{(-bx-a)}}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(-b*x-a)*(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -(b^6*x^3 + 3*a*b^5*x^2 + 3*a^2*b^4*x + 3*b^5*x^2 + a^3*b^3 + 6*a*b^4*x + 3*a^2*b^3 + 6*b^4*x + 6*a*b^3 + 6*b^3)*e^(-b*x - a)/b^4
```

$$3.60 \quad \int \frac{e^{-a-bx}(a+bx)^3}{x} dx$$

**Optimal.** Leaf size=102

$$e^{-a}a^3\text{Ei}(-bx) - 3a^2e^{-a-bx} - b^2x^2e^{-a-bx} - 3ae^{-a-bx} - 3abxe^{-a-bx} - 2e^{-a-bx} - 2bx e^{-a-bx}$$

```
[Out] -2*E^(-a - b*x) - 3*a*E^(-a - b*x) - 3*a^2*E^(-a - b*x) - 2*b*E^(-a - b*x)*
x - 3*a*b*E^(-a - b*x)*x - b^2*E^(-a - b*x)*x^2 + (a^3*ExpIntegralEi[-(b*x)
])/E^a
```

**Rubi [A]** time = 0.155647, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 4, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.19$ , Rules used = {2199, 2194, 2178, 2176}

$$e^{-a}a^3\text{Ei}(-bx) - 3a^2e^{-a-bx} - b^2x^2e^{-a-bx} - 3ae^{-a-bx} - 3abxe^{-a-bx} - 2e^{-a-bx} - 2bx e^{-a-bx}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(-a - b*x)*(a + b*x)^3)/x,x]
```

```
[Out] -2*E^(-a - b*x) - 3*a*E^(-a - b*x) - 3*a^2*E^(-a - b*x) - 2*b*E^(-a - b*x)*
x - 3*a*b*E^(-a - b*x)*x - b^2*E^(-a - b*x)*x^2 + (a^3*ExpIntegralEi[-(b*x)
])/E^a
```

#### Rule 2199

```
Int[(F_)^((c_.)*(v_.))*(u_)^(m_.)*(w_), x_Symbol] := Int[ExpandIntegrand[F^(
c*ExpandToSum[v, x]), w*NormalizePowerOfLinear[u, x]^m, x] /; FreeQ[{F,
c}, x] && PolynomialQ[w, x] && LinearQ[v, x] && PowerOfLinearQ[u, x] && In
tegerQ[m] && !$UseGamma === True
```

#### Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] := Simp[(F^(c*(a +
b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

#### Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] := Si
mp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; F
```

```
reeQ[{F, c, d, e, f, g}, x] && !$UseGamma === True
```

### Rule 2176

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[((c + d*x)^m*(b*F^(g*(e + f*x)))^n)/(f*g*n*Log[F]), x] - Dist[(d*m)/(f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m] && !$UseGamma === True
```

### Rubi steps

$$\begin{aligned} \int \frac{e^{-a-bx}(a+bx)^3}{x} dx &= \int \left( 3a^2be^{-a-bx} + \frac{a^3e^{-a-bx}}{x} + 3ab^2e^{-a-bx}x + b^3e^{-a-bx}x^2 \right) dx \\ &= a^3 \int \frac{e^{-a-bx}}{x} dx + (3a^2b) \int e^{-a-bx} dx + (3ab^2) \int e^{-a-bx}x dx + b^3 \int e^{-a-bx}x^2 dx \\ &= -3a^2e^{-a-bx} - 3abe^{-a-bx}x - b^2e^{-a-bx}x^2 + a^3e^{-a}\text{Ei}(-bx) + (3ab) \int e^{-a-bx} dx + (2b^2) \int e^{-a-bx}x dx \\ &= -3ae^{-a-bx} - 3a^2e^{-a-bx} - 2be^{-a-bx}x - 3abe^{-a-bx}x - b^2e^{-a-bx}x^2 + a^3e^{-a}\text{Ei}(-bx) + (2b) \int e^{-a-bx} dx \\ &= -2e^{-a-bx} - 3ae^{-a-bx} - 3a^2e^{-a-bx} - 2be^{-a-bx}x - 3abe^{-a-bx}x - b^2e^{-a-bx}x^2 + a^3e^{-a}\text{Ei}(-bx) \end{aligned}$$

**Mathematica [A]** time = 0.0502071, size = 52, normalized size = 0.51

$$e^{-a-bx} \left( a^3 e^{bx} \text{Ei}(-bx) - 3a^2 - 3a(bx + 1) - b^2x^2 - 2bx - 2 \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(-a - b*x))*(a + b*x)^3)/x,x]
```

```
[Out] E^(-a - b*x)*(-2 - 3*a^2 - 2*b*x - b^2*x^2 - 3*a*(1 + b*x) + a^3*E^(b*x)*ExpIntegralEi[-(b*x)])
```

**Maple [A]** time = 0.008, size = 113, normalized size = 1.1

$$-(-bx - a)^2 e^{-bx-a} + 2(-bx - a)e^{-bx-a} - 2e^{-bx-a} + a \left( (-bx - a)e^{-bx-a} - e^{-bx-a} \right) - a^2 e^{-bx-a} - a^3 e^{-a} \text{Ei}(1, bx)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(exp(-b*x-a)*(b*x+a)^3/x,x)`

[Out]  $-(b*x-a)^2*exp(-b*x-a)+2*(-b*x-a)*exp(-b*x-a)-2*exp(-b*x-a)+a*((-b*x-a)*exp(-b*x-a)-exp(-b*x-a))-a^2*exp(-b*x-a)-a^3*exp(-a)*Ei(1,b*x)$

**Maxima [A]** time = 1.15238, size = 93, normalized size = 0.91

$$a^3 Ei(-bx) e^{(-a)} - 3 (bx + 1) a e^{(-bx-a)} - 3 a^2 e^{(-bx-a)} - (b^2 x^2 + 2 bx + 2) e^{(-bx-a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(-b*x-a)*(b*x+a)^3/x,x, algorithm="maxima")`

[Out]  $a^3 Ei(-b*x)*e^{(-a)} - 3*(b*x + 1)*a*e^{(-b*x - a)} - 3*a^2*e^{(-b*x - a)} - (b^2*x^2 + 2*b*x + 2)*e^{(-b*x - a)}$

**Fricas [A]** time = 1.46542, size = 108, normalized size = 1.06

$$a^3 Ei(-bx) e^{(-a)} - (b^2 x^2 + (3 a + 2) b x + 3 a^2 + 3 a + 2) e^{(-bx-a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(-b*x-a)*(b*x+a)^3/x,x, algorithm="fricas")`

[Out]  $a^3 Ei(-b*x)*e^{(-a)} - (b^2*x^2 + (3*a + 2)*b*x + 3*a^2 + 3*a + 2)*e^{(-b*x - a)}$

**Sympy [A]** time = 15.072, size = 70, normalized size = 0.69

$$(a^3 Ei(-bx) - 3a^2 e^{-bx} - 3a (bx e^{-bx} + e^{-bx}) - b^2 x^2 e^{-bx} - 2bx e^{-bx} - 2e^{-bx}) e^{-a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(-b*x-a)*(b*x+a)**3/x,x)`



[Out]  $(a^3 \text{Ei}(-bx) - 3a^2 \exp(-bx) - 3a(bx \exp(-bx) + \exp(-bx)) - b^2 x^2 \exp(-bx) - 2bx \exp(-bx) - 2 \exp(-bx)) \exp(-a)$

**Giac [A]** time = 1.31972, size = 128, normalized size = 1.25

$$-b^2 x^2 e^{(-bx-a)} + a^3 \text{Ei}(-bx) e^{(-a)} - 3 abx e^{(-bx-a)} - 3 a^2 e^{(-bx-a)} - 2 bx e^{(-bx-a)} - 3 a e^{(-bx-a)} - 2 e^{(-bx-a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(-b*x-a)*(b*x+a)^3/x,x, algorithm="giac")`

[Out]  $-b^2 x^2 e^{(-bx-a)} + a^3 \text{Ei}(-bx) e^{(-a)} - 3a b x e^{(-bx-a)} - 3a^2 e^{(-bx-a)} - 2bx e^{(-bx-a)} - 3a e^{(-bx-a)} - 2e^{(-bx-a)}$

### 3.61 $\int \frac{e^{-a-bx}(a+bx)^3}{x^2} dx$

**Optimal.** Leaf size=94

$$e^{-a}a^3(-b)\text{Ei}(-bx) + 3e^{-a}a^2b\text{Ei}(-bx) - \frac{a^3e^{-a-bx}}{x} - b^2xe^{-a-bx} - 3abe^{-a-bx} - be^{-a-bx}$$

[Out]  $-(bE^{-a-bx}) - 3a*bE^{-a-bx} - (a^3E^{-a-bx})/x - b^2E^{-a-bx}*x + (3a^2*b*ExpIntegralEi[-(b*x))]/E^a - (a^3*b*ExpIntegralEi[-(b*x)])/E^a$

**Rubi [A]** time = 0.160883, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$ , Rules used = {2199, 2194, 2177, 2178, 2176}

$$e^{-a}a^3(-b)\text{Ei}(-bx) + 3e^{-a}a^2b\text{Ei}(-bx) - \frac{a^3e^{-a-bx}}{x} - b^2xe^{-a-bx} - 3abe^{-a-bx} - be^{-a-bx}$$

Antiderivative was successfully verified.

[In] Int[(E^(-a - b\*x)\*(a + b\*x)^3)/x^2,x]

[Out]  $-(bE^{-a-bx}) - 3a*bE^{-a-bx} - (a^3E^{-a-bx})/x - b^2E^{-a-bx}*x + (3a^2*b*ExpIntegralEi[-(b*x))]/E^a - (a^3*b*ExpIntegralEi[-(b*x)])/E^a$

#### Rule 2199

Int[(F\_)^((c\_.)\*(v\_))\*(u\_)^(m\_.)\*(w\_), x\_Symbol] := Int[ExpandIntegrand[F^(c\*ExpandToSum[v, x]), w\*NormalizePowerOfLinear[u, x]^m, x], x] /; FreeQ[{F, c}, x] && PolynomialQ[w, x] && LinearQ[v, x] && PowerOfLinearQ[u, x] && IntegerQ[m] && !\$UseGamma == True

#### Rule 2194

Int[((F\_)^((c\_.)\*((a\_.) + (b\_.)\*(x\_))))^(n\_.), x\_Symbol] := Simp[(F^(c\*(a + b\*x)))^n/(b\*c\*n\*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]

#### Rule 2177

Int[((b\_.)\*(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_), x\_Symbol] := Simp[(c + d\*x)^(m + 1)\*(b\*F^(g\*(e + f\*x)))^n/(d\*(m + 1))

```
, x] - Dist[(f*g*n*Log[F])/(d*(m + 1)), Int[(c + d*x)^(m + 1)*(b*F^(g*(e + f*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && LtQ[m, -1] && IntegerQ[2*m] && !$UseGamma === True
```

### Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; FreeQ[{F, c, d, e, f, g}, x] && !$UseGamma === True
```

### Rule 2176

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[((c + d*x)^m*(b*F^(g*(e + f*x)))^n)/(f*g*n*Log[F]), x] - Dist[(d*m)/(f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m] && !$UseGamma === True
```

### Rubi steps

$$\begin{aligned}
 \int \frac{e^{-a-bx}(a+bx)^3}{x^2} dx &= \int \left( 3ab^2e^{-a-bx} + \frac{a^3e^{-a-bx}}{x^2} + \frac{3a^2be^{-a-bx}}{x} + b^3e^{-a-bx}x \right) dx \\
 &= a^3 \int \frac{e^{-a-bx}}{x^2} dx + (3a^2b) \int \frac{e^{-a-bx}}{x} dx + (3ab^2) \int e^{-a-bx} dx + b^3 \int e^{-a-bx}x dx \\
 &= -3abe^{-a-bx} - \frac{a^3e^{-a-bx}}{x} - b^2e^{-a-bx}x + 3a^2be^{-a} \text{Ei}(-bx) - (a^3b) \int \frac{e^{-a-bx}}{x} dx + b^2 \int e^{-a-bx} dx \\
 &= -be^{-a-bx} - 3abe^{-a-bx} - \frac{a^3e^{-a-bx}}{x} - b^2e^{-a-bx}x + 3a^2be^{-a} \text{Ei}(-bx) - a^3be^{-a} \text{Ei}(-bx)
 \end{aligned}$$

**Mathematica [A]** time = 0.0650001, size = 54, normalized size = 0.57

$$\frac{e^{-a-bx} \left( -(a-3)a^2bx e^{bx} \text{Ei}(-bx) - a^3 - 3abx - bx(bx+1) \right)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(-a - b*x)*(a + b*x)^3)/x^2,x]
```

```
[Out] (E^(-a - b*x)*(-a^3 - 3*a*b*x - b*x*(1 + b*x) - (-3 + a)*a^2*b*E^(b*x)*x*ExpIntegralEi[-(b*x)]))/x
```

---

**Maple [A]** time = 0.008, size = 92, normalized size = 1.

$$b \left( (-bx - a)e^{-bx-a} - e^{-bx-a} - 2ae^{-bx-a} - 3a^2e^{-a}\text{Ei}(1, bx) - a^3 \left( \frac{e^{-bx-a}}{bx} - e^{-a}\text{Ei}(1, bx) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(-b\*x-a)\*(b\*x+a)^3/x^2, x)

[Out] b\*((-b\*x-a)\*exp(-b\*x-a)-exp(-b\*x-a)-2\*a\*exp(-b\*x-a)-3\*a^2\*exp(-a)\*Ei(1, b\*x)-a^3\*(exp(-b\*x-a)/b/x-exp(-a)\*Ei(1, b\*x)))

---

**Maxima [A]** time = 1.1616, size = 82, normalized size = 0.87

$$-a^3be^{(-a)}\Gamma(-1, bx) + 3a^2b\text{Ei}(-bx)e^{(-a)} - (bx + 1)be^{(-bx-a)} - 3abe^{(-bx-a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(-b\*x-a)\*(b\*x+a)^3/x^2, x, algorithm="maxima")

[Out] -a^3\*b\*e^(-a)\*gamma(-1, b\*x) + 3\*a^2\*b\*Ei(-b\*x)\*e^(-a) - (b\*x + 1)\*b\*e^(-b\*x - a) - 3\*a\*b\*e^(-b\*x - a)

---

**Fricas [A]** time = 1.4683, size = 117, normalized size = 1.24

$$\frac{(a^3 - 3a^2)bx\text{Ei}(-bx)e^{(-a)} + (b^2x^2 + a^3 + (3a + 1)bx)e^{(-bx-a)}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(-b\*x-a)\*(b\*x+a)^3/x^2, x, algorithm="fricas")

[Out] -((a^3 - 3\*a^2)\*b\*x\*Ei(-b\*x)\*e^(-a) + (b^2\*x^2 + a^3 + (3\*a + 1)\*b\*x)\*e^(-b\*x - a))/x

---

**Sympy [A]** time = 6.47635, size = 99, normalized size = 1.05

$$-\frac{a^3 e^{-a} E_2(bx)}{x} + 3a^2 b e^{-a} \operatorname{Ei}(-bx) + 3ab^2 \left( \begin{cases} x & \text{for } b = 0 \\ -\frac{e^{-bx}}{b} & \text{otherwise} \end{cases} \right) e^{-a} + b^3 x \left( \begin{cases} x & \text{for } b = 0 \\ -\frac{e^{-bx}}{b} & \text{otherwise} \end{cases} \right) e^{-a} - b^3 \left( \begin{cases} \frac{x^2}{2} \\ -\frac{e^{-bx}}{b} \\ x \end{cases} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(-b\*x-a)\*(b\*x+a)\*\*3/x\*\*2,x)

[Out] -a\*\*3\*exp(-a)\*expint(2, b\*x)/x + 3\*a\*\*2\*b\*exp(-a)\*Ei(-b\*x) + 3\*a\*b\*\*2\*Piecewise((x, Eq(b, 0)), (-exp(-b\*x)/b, True))\*exp(-a) + b\*\*3\*x\*Piecewise((x, Eq(b, 0)), (-exp(-b\*x)/b, True))\*exp(-a) - b\*\*3\*Piecewise((x\*\*2/2, Eq(b, 0)), (-Piecewise((-exp(-b\*x)/b, Ne(b, 0)), (x, True))/b, True))\*exp(-a)

**Giac [A]** time = 1.383, size = 124, normalized size = 1.32

$$\frac{a^3 b x \operatorname{Ei}(-bx) e^{(-a)} - 3 a^2 b x \operatorname{Ei}(-bx) e^{(-a)} + b^2 x^2 e^{(-bx-a)} + a^3 e^{(-bx-a)} + 3 a b x e^{(-bx-a)} + b x e^{(-bx-a)}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(-b\*x-a)\*(b\*x+a)^3/x^2,x, algorithm="giac")

[Out] -(a^3\*b\*x\*Ei(-b\*x)\*e^(-a) - 3\*a^2\*b\*x\*Ei(-b\*x)\*e^(-a) + b^2\*x^2\*e^(-b\*x - a) + a^3\*e^(-b\*x - a) + 3\*a\*b\*x\*e^(-b\*x - a) + b\*x\*e^(-b\*x - a))/x

### 3.62 $\int \frac{e^{-a-bx}(a+bx)^3}{x^3} dx$

**Optimal.** Leaf size=130

$$\frac{1}{2}e^{-a}a^3b^2\text{Ei}(-bx) - 3e^{-a}a^2b^2\text{Ei}(-bx) - \frac{a^3e^{-a-bx}}{2x^2} + \frac{a^3be^{-a-bx}}{2x} - \frac{3a^2be^{-a-bx}}{x} + 3e^{-a}ab^2\text{Ei}(-bx) - b^2e^{-a-bx}$$

[Out]  $-(b^2E^{-a-bx}) - (a^3E^{-a-bx})/(2x^2) - (3a^2bE^{-a-bx})/x + (a^3bE^{-a-bx})/(2x) + (3a^2b^2\text{ExpIntegralEi}[-(bx)])/E^a - (3a^2b^2\text{ExpIntegralEi}[-(bx)])/E^a + (a^3b^2\text{ExpIntegralEi}[-(bx)])/E^a$

**Rubi [A]** time = 0.213394, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 4, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.19$ , Rules used = {2199, 2194, 2177, 2178}

$$\frac{1}{2}e^{-a}a^3b^2\text{Ei}(-bx) - 3e^{-a}a^2b^2\text{Ei}(-bx) - \frac{a^3e^{-a-bx}}{2x^2} + \frac{a^3be^{-a-bx}}{2x} - \frac{3a^2be^{-a-bx}}{x} + 3e^{-a}ab^2\text{Ei}(-bx) - b^2e^{-a-bx}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(E^{-a-bx})(a+bx)^3/x^3, x]$

[Out]  $-(b^2E^{-a-bx}) - (a^3E^{-a-bx})/(2x^2) - (3a^2bE^{-a-bx})/x + (a^3bE^{-a-bx})/(2x) + (3a^2b^2\text{ExpIntegralEi}[-(bx)])/E^a - (3a^2b^2\text{ExpIntegralEi}[-(bx)])/E^a + (a^3b^2\text{ExpIntegralEi}[-(bx)])/E^a$

#### Rule 2199

$\text{Int}[(F_)^((c_.)*(v_))*(u_)^(m_)*(w_), x\_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[F^(\text{c*ExpandToSum}[v, x]), w*\text{NormalizePowerOfLinear}[u, x]^m, x], x] /; \text{FreeQ}\{F, c\}, x] \&\& \text{PolynomialQ}[w, x] \&\& \text{LinearQ}[v, x] \&\& \text{PowerOfLinearQ}[u, x] \&\& \text{IntegerQ}[m] \&\& !\$UseGamma === \text{True}$

#### Rule 2194

$\text{Int}[(F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_), x\_Symbol] \rightarrow \text{Simp}[(F^(c*(a + b*x)))^n/(b*c*n*\text{Log}[F]), x] /; \text{FreeQ}\{F, a, b, c, n\}, x]$

#### Rule 2177

$\text{Int}[(b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_)*((c_.) + (d_.)*(x_))^(m_), x\_Symbol] \rightarrow \text{Simp}[(c + d*x)^(m + 1)*(b*F^(g*(e + f*x)))^n/(d*(m + 1))$

```
, x] - Dist[(f*g*n*Log[F])/(d*(m + 1)), Int[(c + d*x)^(m + 1)*(b*F^(g*(e + f*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && LtQ[m, -1] && IntegerQ[2*m] && !UseGamma == True
```

### Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; FreeQ[{F, c, d, e, f, g}, x] && !UseGamma == True
```

### Rubi steps

$$\begin{aligned} \int \frac{e^{-a-bx}(a+bx)^3}{x^3} dx &= \int \left( b^3 e^{-a-bx} + \frac{a^3 e^{-a-bx}}{x^3} + \frac{3a^2 b e^{-a-bx}}{x^2} + \frac{3ab^2 e^{-a-bx}}{x} \right) dx \\ &= a^3 \int \frac{e^{-a-bx}}{x^3} dx + (3a^2 b) \int \frac{e^{-a-bx}}{x^2} dx + (3ab^2) \int \frac{e^{-a-bx}}{x} dx + b^3 \int e^{-a-bx} dx \\ &= -b^2 e^{-a-bx} - \frac{a^3 e^{-a-bx}}{2x^2} - \frac{3a^2 b e^{-a-bx}}{x} + 3ab^2 e^{-a} \text{Ei}(-bx) - \frac{1}{2} (a^3 b) \int \frac{e^{-a-bx}}{x^2} dx - (3a^2 b^2) \int \frac{e^{-a-bx}}{x} dx \\ &= -b^2 e^{-a-bx} - \frac{a^3 e^{-a-bx}}{2x^2} - \frac{3a^2 b e^{-a-bx}}{x} + \frac{a^3 b e^{-a-bx}}{2x} + 3ab^2 e^{-a} \text{Ei}(-bx) - 3a^2 b^2 e^{-a} \text{Ei}(-bx) + \frac{1}{2} (a^3 b^2) \int \frac{e^{-a-bx}}{x} dx \\ &= -b^2 e^{-a-bx} - \frac{a^3 e^{-a-bx}}{2x^2} - \frac{3a^2 b e^{-a-bx}}{x} + \frac{a^3 b e^{-a-bx}}{2x} + 3ab^2 e^{-a} \text{Ei}(-bx) - 3a^2 b^2 e^{-a} \text{Ei}(-bx) + \frac{1}{2} a^3 b^2 e^{-a} \int \frac{e^{-a-bx}}{x} dx \end{aligned}$$

**Mathematica [A]** time = 0.0724097, size = 68, normalized size = 0.52

$$\frac{e^{-a-bx} \left( (a^2 - 6a + 6) ab^2 x^2 e^{bx} \text{Ei}(-bx) + a^3 (bx - 1) - 6a^2 bx - 2b^2 x^2 \right)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(-a - b*x)*(a + b*x)^3)/x^3,x]
```

```
[Out] (E^(-a - b*x)*(-6*a^2*b*x - 2*b^2*x^2 + a^3*(-1 + b*x) + a*(6 - 6*a + a^2)*b^2*E^(b*x)*x^2*ExpIntegralEi[-(b*x)]))/(2*x^2)
```

**Maple [A]** time = 0.009, size = 112, normalized size = 0.9

$$-b^2 \left( e^{-bx-a} + 3ae^{-a} \text{Ei}(1, bx) - a^3 \left( -\frac{e^{-bx-a}}{2b^2 x^2} + \frac{e^{-bx-a}}{2bx} - \frac{e^{-a} \text{Ei}(1, bx)}{2} \right) + 3a^2 \left( \frac{e^{-bx-a}}{bx} - e^{-a} \text{Ei}(1, bx) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(exp(-b*x-a)*(b*x+a)^3/x^3,x)`

[Out]  $-b^2*(\exp(-b*x-a)+3*a*\exp(-a)*\text{Ei}(1,b*x)-a^3*(-1/2*\exp(-b*x-a)/b^2/x^2+1/2*\exp(-b*x-a)/b/x-1/2*\exp(-a)*\text{Ei}(1,b*x))+3*a^2*(\exp(-b*x-a)/b/x-\exp(-a)*\text{Ei}(1,b*x)))$

**Maxima [A]** time = 1.2718, size = 86, normalized size = 0.66

$$-a^3 b^2 e^{(-a)} \Gamma(-2, bx) - 3 a^2 b^2 e^{(-a)} \Gamma(-1, bx) + 3 a b^2 \text{Ei}(-bx) e^{(-a)} - b^2 e^{(-bx-a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(-b*x-a)*(b*x+a)^3/x^3,x, algorithm="maxima")`

[Out]  $-a^3 b^2 e^{(-a)} \gamma(-2, bx) - 3 a^2 b^2 e^{(-a)} \gamma(-1, bx) + 3 a b^2 \text{Ei}(-bx) e^{(-a)} - b^2 e^{(-bx-a)}$

**Fricas [A]** time = 1.47975, size = 146, normalized size = 1.12

$$\frac{(a^3 - 6 a^2 + 6 a) b^2 x^2 \text{Ei}(-bx) e^{(-a)} - (2 b^2 x^2 + a^3 - (a^3 - 6 a^2) bx) e^{(-bx-a)}}{2 x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(-b*x-a)*(b*x+a)^3/x^3,x, algorithm="fricas")`

[Out]  $1/2*((a^3 - 6*a^2 + 6*a)*b^2*x^2*\text{Ei}(-b*x)*e^{(-a)} - (2*b^2*x^2 + a^3 - (a^3 - 6*a^2)*b*x)*e^{(-b*x - a)})/x^2$

**Sympy [A]** time = 6.58302, size = 56, normalized size = 0.43

$$\left( -\frac{a^3 \text{E}_3(bx)}{x^2} - \frac{3a^2 b \text{E}_2(bx)}{x} + 3ab^2 \text{Ei}(-bx) + b^3 \left( \begin{array}{ll} x & \text{for } b = 0 \\ -\frac{e^{-bx}}{b} & \text{otherwise} \end{array} \right) \right) e^{-a}$$

Verification of antiderivative is not currently implemented for this CAS.



[In] integrate(exp(-b\*x-a)\*(b\*x+a)\*\*3/x\*\*3,x)

[Out] (-a\*\*3\*expint(3, b\*x)/x\*\*2 - 3\*a\*\*2\*b\*expint(2, b\*x)/x + 3\*a\*b\*\*2\*Ei(-b\*x) + b\*\*3\*Piecewise((x, Eq(b, 0)), (-exp(-b\*x)/b, True)))\*exp(-a)

**Giac [A]** time = 1.33416, size = 169, normalized size = 1.3

$$\frac{a^3 b^2 x^2 \operatorname{Ei}(-bx) e^{(-a)} - 6 a^2 b^2 x^2 \operatorname{Ei}(-bx) e^{(-a)} + 6 a b^2 x^2 \operatorname{Ei}(-bx) e^{(-a)} + a^3 b x e^{(-bx-a)} - 6 a^2 b x e^{(-bx-a)} - 2 b^2 x^2 e^{(-bx-a)} - a^3 e^{(-a)}}{2 x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(-b\*x-a)\*(b\*x+a)^3/x^3,x, algorithm="giac")

[Out] 1/2\*(a^3\*b^2\*x^2\*Ei(-b\*x)\*e^(-a) - 6\*a^2\*b^2\*x^2\*Ei(-b\*x)\*e^(-a) + 6\*a\*b^2\*x^2\*Ei(-b\*x)\*e^(-a) + a^3\*b\*x\*e^(-b\*x - a) - 6\*a^2\*b\*x\*e^(-b\*x - a) - 2\*b^2\*x^2\*e^(-b\*x - a) - a^3\*e^(-b\*x - a))/x^2

### 3.63 $\int \frac{e^{-a-bx}(a+bx)^3}{x^4} dx$

**Optimal.** Leaf size=198

$$-\frac{1}{6}e^{-a}a^3b^3\text{Ei}(-bx) + \frac{3}{2}e^{-a}a^2b^3\text{Ei}(-bx) - \frac{a^3b^2e^{-a-bx}}{6x} + \frac{3a^2b^2e^{-a-bx}}{2x} + \frac{a^3be^{-a-bx}}{6x^2} - \frac{a^3e^{-a-bx}}{3x^3} - \frac{3a^2be^{-a-bx}}{2x^2} - 3e^{-a}ab^3\text{Ei}(-bx)$$

[Out]  $-(a^3E^{-a-bx})/(3x^3) - (3a^2bE^{-a-bx})/(2x^2) + (a^3bE^{-a-bx})/(6x^2) - (3a^2b^2E^{-a-bx})/x + (3a^2b^2E^{-a-bx})/(2x) - (a^3b^2E^{-a-bx})/(6x) + (b^3\text{ExpIntegralEi}[-(bx)])/E^a - (3a^3b^3\text{ExpIntegralEi}[-(bx)])/E^a + (3a^2b^3\text{ExpIntegralEi}[-(bx)])/(2E^a) - (a^3b^3\text{ExpIntegralEi}[-(bx)])/(6E^a)$

**Rubi [A]** time = 0.290449, antiderivative size = 198, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 3, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {2199, 2177, 2178}

$$-\frac{1}{6}e^{-a}a^3b^3\text{Ei}(-bx) + \frac{3}{2}e^{-a}a^2b^3\text{Ei}(-bx) - \frac{a^3b^2e^{-a-bx}}{6x} + \frac{3a^2b^2e^{-a-bx}}{2x} + \frac{a^3be^{-a-bx}}{6x^2} - \frac{a^3e^{-a-bx}}{3x^3} - \frac{3a^2be^{-a-bx}}{2x^2} - 3e^{-a}ab^3\text{Ei}(-bx)$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(E^{-a-bx}*(a+bx)^3)/x^4, x]$

[Out]  $-(a^3E^{-a-bx})/(3x^3) - (3a^2bE^{-a-bx})/(2x^2) + (a^3bE^{-a-bx})/(6x^2) - (3a^2b^2E^{-a-bx})/x + (3a^2b^2E^{-a-bx})/(2x) - (a^3b^2E^{-a-bx})/(6x) + (b^3\text{ExpIntegralEi}[-(bx)])/E^a - (3a^3b^3\text{ExpIntegralEi}[-(bx)])/E^a + (3a^2b^3\text{ExpIntegralEi}[-(bx)])/(2E^a) - (a^3b^3\text{ExpIntegralEi}[-(bx)])/(6E^a)$

#### Rule 2199

$\text{Int}[(F_)^((c_.)*(v_))*(u_)^(m_)*(w_), x\_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[F^(\text{c*ExpandToSum}[v, x]), w*\text{NormalizePowerOfLinear}[u, x]^m, x], x] /; \text{FreeQ}\{F, c\}, x] \&\& \text{PolynomialQ}[w, x] \&\& \text{LinearQ}[v, x] \&\& \text{PowerOfLinearQ}[u, x] \&\& \text{IntegerQ}[m] \&\& !\$UseGamma === \text{True}$

#### Rule 2177

$\text{Int}[(b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_), x\_Symbol] \rightarrow \text{Simp}[(c + dx)^(m + 1)*(b*F^(g*(e + fx)))^n]/(d*(m + 1)), x] - \text{Dist}[(f*g*n*\text{Log}[F])/d*(m + 1), \text{Int}[(c + dx)^(m + 1)*(b*F^(g*(e +$

f\*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && LtQ[m, -1] && IntegerQ[2\*m] && !UseGamma == True

### Rule 2178

Int[(F\_)^((g\_)\*((e\_) + (f\_)\*(x\_)))/((c\_) + (d\_)\*(x\_)), x\_Symbol] :> Simp[(F^(g\*(e - (c\*f)/d))\*ExpIntegralEi[(f\*g\*(c + d\*x)\*Log[F])/d])/d, x] /; FreeQ[{F, c, d, e, f, g}, x] && !UseGamma == True

### Rubi steps

$$\begin{aligned}
 \int \frac{e^{-a-bx}(a+bx)^3}{x^4} dx &= \int \left( \frac{a^3 e^{-a-bx}}{x^4} + \frac{3a^2 b e^{-a-bx}}{x^3} + \frac{3ab^2 e^{-a-bx}}{x^2} + \frac{b^3 e^{-a-bx}}{x} \right) dx \\
 &= a^3 \int \frac{e^{-a-bx}}{x^4} dx + (3a^2 b) \int \frac{e^{-a-bx}}{x^3} dx + (3ab^2) \int \frac{e^{-a-bx}}{x^2} dx + b^3 \int \frac{e^{-a-bx}}{x} dx \\
 &= -\frac{a^3 e^{-a-bx}}{3x^3} - \frac{3a^2 b e^{-a-bx}}{2x^2} - \frac{3ab^2 e^{-a-bx}}{x} + b^3 e^{-a} \text{Ei}(-bx) - \frac{1}{3} (a^3 b) \int \frac{e^{-a-bx}}{x^3} dx - \frac{1}{2} (3a^2 b^2) \int \frac{e^{-a-bx}}{x^2} dx \\
 &= -\frac{a^3 e^{-a-bx}}{3x^3} - \frac{3a^2 b e^{-a-bx}}{2x^2} + \frac{a^3 b e^{-a-bx}}{6x^2} - \frac{3ab^2 e^{-a-bx}}{x} + \frac{3a^2 b^2 e^{-a-bx}}{2x} + b^3 e^{-a} \text{Ei}(-bx) - 3ab^3 e^{-a} \text{Ei}(-bx) \\
 &= -\frac{a^3 e^{-a-bx}}{3x^3} - \frac{3a^2 b e^{-a-bx}}{2x^2} + \frac{a^3 b e^{-a-bx}}{6x^2} - \frac{3ab^2 e^{-a-bx}}{x} + \frac{3a^2 b^2 e^{-a-bx}}{2x} - \frac{a^3 b^2 e^{-a-bx}}{6x} + b^3 e^{-a} \text{Ei}(-bx) \\
 &= -\frac{a^3 e^{-a-bx}}{3x^3} - \frac{3a^2 b e^{-a-bx}}{2x^2} + \frac{a^3 b e^{-a-bx}}{6x^2} - \frac{3ab^2 e^{-a-bx}}{x} + \frac{3a^2 b^2 e^{-a-bx}}{2x} - \frac{a^3 b^2 e^{-a-bx}}{6x} + b^3 e^{-a} \text{Ei}(-bx)
 \end{aligned}$$

**Mathematica [A]** time = 0.105909, size = 81, normalized size = 0.41

$$\frac{1}{6} e^{-a} \left( - (a^3 - 9a^2 + 18a - 6) b^3 \text{Ei}(-bx) - \frac{ae^{-bx} (a^2 (b^2 x^2 - bx + 2) - 9abx(bx - 1) + 18b^2 x^2)}{x^3} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(E^(-a - b\*x)\*(a + b\*x)^3)/x^4, x]

[Out] (-((a\*(18\*b^2\*x^2 - 9\*a\*b\*x\*(-1 + b\*x) + a^2\*(2 - b\*x + b^2\*x^2)))/(E^(b\*x)\*x^3)) - (-6 + 18\*a - 9\*a^2 + a^3)\*b^3\*ExpIntegralEi[-(b\*x)])/(6\*E^a)

**Maple [A]** time = 0.008, size = 167, normalized size = 0.8

$$b^3 \left( -e^{-a} \text{Ei}(1, bx) - a^3 \left( \frac{e^{-bx-a}}{3b^3 x^3} - \frac{e^{-bx-a}}{6b^2 x^2} + \frac{e^{-bx-a}}{6bx} - \frac{e^{-a} \text{Ei}(1, bx)}{6} \right) + 3a^2 \left( -1/2 \frac{e^{-bx-a}}{b^2 x^2} + 1/2 \frac{e^{-bx-a}}{bx} - 1/2 e^{-a} \text{Ei}(1, bx) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(exp(-b*x-a)*(b*x+a)^3/x^4,x)`

[Out]  $b^3*(-\exp(-a)*\text{Ei}(1,b*x)-a^3*(1/3*\exp(-b*x-a)/b^3/x^3-1/6*\exp(-b*x-a)/b^2/x^2+1/6*\exp(-b*x-a)/b/x-1/6*\exp(-a)*\text{Ei}(1,b*x))+3*a^2*(-1/2*\exp(-b*x-a)/b^2/x^2+1/2*\exp(-b*x-a)/b/x-1/2*\exp(-a)*\text{Ei}(1,b*x))-3*a*(\exp(-b*x-a)/b/x-\exp(-a)*\text{Ei}(1,b*x))$

**Maxima [A]** time = 1.23261, size = 85, normalized size = 0.43

$$-a^3 b^3 e^{(-a)} \Gamma(-3, bx) - 3 a^2 b^3 e^{(-a)} \Gamma(-2, bx) - 3 a b^3 e^{(-a)} \Gamma(-1, bx) + b^3 \text{Ei}(-bx) e^{(-a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(-b*x-a)*(b*x+a)^3/x^4,x, algorithm="maxima")`

[Out]  $-a^3 b^3 e^{(-a)} \text{gamma}(-3, b*x) - 3 a^2 b^3 e^{(-a)} \text{gamma}(-2, b*x) - 3 a b^3 e^{(-a)} \text{gamma}(-1, b*x) + b^3 \text{Ei}(-b*x) e^{(-a)}$

**Fricas [A]** time = 1.44072, size = 182, normalized size = 0.92

$$\frac{(a^3 - 9a^2 + 18a - 6)b^3 x^3 \text{Ei}(-bx) e^{(-a)} + ((a^3 - 9a^2 + 18a)b^2 x^2 + 2a^3 - (a^3 - 9a^2)bx) e^{(-bx-a)}}{6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(-b*x-a)*(b*x+a)^3/x^4,x, algorithm="fricas")`

[Out]  $-1/6*((a^3 - 9a^2 + 18a - 6)*b^3*x^3*\text{Ei}(-b*x)*e^{(-a)} + ((a^3 - 9a^2 + 18a)*b^2*x^2 + 2*a^3 - (a^3 - 9a^2)*b*x)*e^{(-b*x - a)})/x^3$

**Sympy [A]** time = 6.89928, size = 53, normalized size = 0.27

$$\left( -\frac{a^3 E_4(bx)}{x^3} - \frac{3a^2 b E_3(bx)}{x^2} - \frac{3ab^2 E_2(bx)}{x} + b^3 \text{Ei}(-bx) \right) e^{-a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(-b\*x-a)\*(b\*x+a)\*\*3/x\*\*4,x)

[Out] (-a\*\*3\*expint(4, b\*x)/x\*\*3 - 3\*a\*\*2\*b\*expint(3, b\*x)/x\*\*2 - 3\*a\*b\*\*2\*expint(2, b\*x)/x + b\*\*3\*Ei(-b\*x))\*exp(-a)

**Giac [A]** time = 1.36748, size = 247, normalized size = 1.25

$$\frac{a^3 b^3 x^3 \operatorname{Ei}(-bx) e^{(-a)} - 9 a^2 b^3 x^3 \operatorname{Ei}(-bx) e^{(-a)} + 18 a b^3 x^3 \operatorname{Ei}(-bx) e^{(-a)} + a^3 b^2 x^2 e^{(-bx-a)} - 6 b^3 x^3 \operatorname{Ei}(-bx) e^{(-a)} - 9 a^2 b^2 x^2}{6 x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(-b\*x-a)\*(b\*x+a)^3/x^4,x, algorithm="giac")

[Out] 
$$\frac{-1/6*(a^3*b^3*x^3*\operatorname{Ei}(-b*x)*e^{(-a)} - 9*a^2*b^3*x^3*\operatorname{Ei}(-b*x)*e^{(-a)} + 18*a*b^3*x^3*\operatorname{Ei}(-b*x)*e^{(-a)} + a^3*b^2*x^2*e^{(-b*x - a)} - 6*b^3*x^3*\operatorname{Ei}(-b*x)*e^{(-a)}) - 9*a^2*b^2*x^2*e^{(-b*x - a)} - a^3*b*x*e^{(-b*x - a)} + 18*a*b^2*x^2*e^{(-b*x - a)} + 9*a^2*b*x*e^{(-b*x - a)} + 2*a^3*e^{(-b*x - a)})}{x^3}$$

### 3.64 $\int F^{a+b(c+dx)} x^m (e + fx)^2 dx$

**Optimal.** Leaf size=139

$$\frac{2efx^m F^{a+bc} (-bdx \log(F))^{-m} \Gamma(m+2, -bdx \log(F))}{b^2 d^2 \log^2(F)} + \frac{f^2 x^m F^{a+bc} (-bdx \log(F))^{-m} \Gamma(m+3, -bdx \log(F))}{b^3 d^3 \log^3(F)}$$

[Out] (f^2 \* F^(a + b\*c) \* x^m \* Gamma[3 + m, -(b\*d\*x\*Log[F])]) / (b^3 \* d^3 \* Log[F]^3 \* (-(b\*d\*x\*Log[F]))^m) - (2 \* e \* f \* F^(a + b\*c) \* x^m \* Gamma[2 + m, -(b\*d\*x\*Log[F])]) / (b^2 \* d^2 \* Log[F]^2 \* (-(b\*d\*x\*Log[F]))^m) + (e^2 \* F^(a + b\*c) \* x^m \* Gamma[1 + m, -(b\*d\*x\*Log[F])]) / (b\*d \* Log[F] \* (-(b\*d\*x\*Log[F]))^m)

**Rubi [A]** time = 0.309197, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$ , Rules used = {2199, 2181}

$$\frac{2efx^m F^{a+bc} (-bdx \log(F))^{-m} \Gamma(m+2, -bdx \log(F))}{b^2 d^2 \log^2(F)} + \frac{f^2 x^m F^{a+bc} (-bdx \log(F))^{-m} \Gamma(m+3, -bdx \log(F))}{b^3 d^3 \log^3(F)}$$

Antiderivative was successfully verified.

[In] Int[F^(a + b\*(c + d\*x))\*x^m\*(e + f\*x)^2,x]

[Out] (f^2 \* F^(a + b\*c) \* x^m \* Gamma[3 + m, -(b\*d\*x\*Log[F])]) / (b^3 \* d^3 \* Log[F]^3 \* (-(b\*d\*x\*Log[F]))^m) - (2 \* e \* f \* F^(a + b\*c) \* x^m \* Gamma[2 + m, -(b\*d\*x\*Log[F])]) / (b^2 \* d^2 \* Log[F]^2 \* (-(b\*d\*x\*Log[F]))^m) + (e^2 \* F^(a + b\*c) \* x^m \* Gamma[1 + m, -(b\*d\*x\*Log[F])]) / (b\*d \* Log[F] \* (-(b\*d\*x\*Log[F]))^m)

#### Rule 2199

```
Int[(F_)^((c_.)*(v_.))*(u_)^(m_.)*(w_), x_Symbol] := Int[ExpandIntegrand[F^(c*ExpandToSum[v, x]), w*NormalizePowerOfLinear[u, x]^m, x], x] /; FreeQ[{F, c}, x] && PolynomialQ[w, x] && LinearQ[v, x] && PowerOfLinearQ[u, x] && IntegerQ[m] && !$UseGamma === True
```

#### Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_.)))*((c_.) + (d_.)*(x_.))^(m_), x_Symbol] := -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-(f*g*Log[F])/d)*(c + d*x])]/(d*(-(f*g*Log[F])/d))^(IntPart[m] + 1)*(-(f*g*Log[F])*(c + d*x)/d)^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !I
```

ntegerQ[m]

### Rubi steps

$$\begin{aligned} \int F^{a+b(c+dx)} x^m (e+fx)^2 dx &= \int \left( e^2 F^{a+bc+bdx} x^m + 2ef F^{a+bc+bdx} x^{1+m} + f^2 F^{a+bc+bdx} x^{2+m} \right) dx \\ &= e^2 \int F^{a+bc+bdx} x^m dx + (2ef) \int F^{a+bc+bdx} x^{1+m} dx + f^2 \int F^{a+bc+bdx} x^{2+m} dx \\ &= \frac{f^2 F^{a+bc} x^m \Gamma(3+m, -bdx \log(F)) (-bdx \log(F))^{-m}}{b^3 d^3 \log^3(F)} - \frac{2ef F^{a+bc} x^m \Gamma(2+m, -bdx \log(F)) (-bdx \log(F))^{-m}}{b^2 d^2 \log^2(F)} \end{aligned}$$

**Mathematica [A]** time = 0.124451, size = 86, normalized size = 0.62

$$\frac{x^m F^{a+bc} (-bdx \log(F))^{-m} \left( bde \log(F) (bde \log(F) \Gamma(m+1, -bdx \log(F)) - 2f \Gamma(m+2, -bdx \log(F))) + f^2 \Gamma(m+3, -bdx \log(F)) \right)}{b^3 d^3 \log^3(F)}$$

Antiderivative was successfully verified.

[In] Integrate[F^(a + b\*(c + d\*x))\*x^m\*(e + f\*x)^2,x]

[Out] (F^(a + b\*c)\*x^m\*(f^2\*Gamma[3 + m, -(b\*d\*x\*Log[F])]) + b\*d\*e\*Log[F]\*(-2\*f\*Gamma[2 + m, -(b\*d\*x\*Log[F])]) + b\*d\*e\*Gamma[1 + m, -(b\*d\*x\*Log[F])]\*Log[F]))/(b^3\*d^3\*Log[F]^3\*(-(b\*d\*x\*Log[F]))^m)

**Maple [B]** time = 0.097, size = 433, normalized size = 3.1

$$\frac{(\ln(F))^{-3-m} (-bd)^{-m} F^{bc+a} f^2 \left( x^m (-bd)^m (\ln(F))^m m (m^2 + 3m + 2) \Gamma(m) (-bdx \ln(F))^{-m} - x^m (-bd)^m (\ln(F))^m (b^2 d^2) \right)}{b^3 d^3 \log^3(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(a+b\*(d\*x+c))\*x^m\*(f\*x+e)^2,x)

[Out] -1/b^3/d^3\*ln(F)^(-3-m)\*(-b\*d)^(-m)\*F^(b\*c+a)\*f^2\*(x^m\*(-b\*d)^m\*ln(F)^m\*(m^2+3\*m+2)\*GAMMA(m)\*(-b\*d\*x\*ln(F))^(-m)-x^m\*(-b\*d)^m\*ln(F)^m\*(b^2\*d^2\*x^2\*ln(F)^2-m\*b\*d\*x\*ln(F)+m^2-2\*b\*d\*x\*ln(F)+3\*m+2)\*exp(b\*d\*x\*ln(F))-x^m\*(-b\*d)^m\*ln(F)^m\*(m^2+3\*m+2)\*(-b\*d\*x\*ln(F))^(-m)\*GAMMA(m,-b\*d\*x\*ln(F)))+2/b^2/d^2\*ln(F)^(-2-m)\*(-b\*d)^(-m)\*F^(b\*c+a)\*f\*e\*(x^m\*(-b\*d)^m\*ln(F)^m\*(1+m)\*GAMMA

$$\begin{aligned} & (m)*(-b*d*x*\ln(F))^{(-m)+x^m*(-b*d)^m*\ln(F)^m*(b*d*x*\ln(F)-1-m)*\exp(b*d*x*\ln(F)) \\ & (F))-x^m*(-b*d)^m*\ln(F)^m*(1+m)*m*(-b*d*x*\ln(F))^{(-m)*\text{GAMMA}(m,-b*d*x*\ln(F))} \\ & )-F^{(b*c+a)*(-b*d)^{(-m)*\ln(F)^{(-m-1)*e^{2/b/d*(x^m*(-b*d)^m*\ln(F)^m*m*\text{GAMMA}(m)*(-b*d*x*\ln(F))^{(-m)-x^m*(-b*d)^m*\ln(F)^m*\exp(b*d*x*\ln(F))-x^m*(-b*d)^m*\ln(F)^m*m*(-b*d*x*\ln(F))^{(-m)*\text{GAMMA}(m,-b*d*x*\ln(F))} \end{aligned}$$

**Maxima [A]** time = 1.27436, size = 166, normalized size = 1.19

$$-(-b dx \log(F))^{-m-3} F^{bc+a} f^2 x^{m+3} \Gamma(m+3, -b dx \log(F)) - 2 (-b dx \log(F))^{-m-2} F^{bc+a} e f x^{m+2} \Gamma(m+2, -b dx \log(F)) - (-b dx \log(F))^{-m-1} F^{bc+a} e^2 f x^{m+1} \Gamma(m+1, -b dx \log(F))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*(d\*x+c))\*x^m\*(f\*x+e)^2,x, algorithm="maxima")

[Out] 
$$-(-b*d*x*\log(F))^{(-m-3)*F^{(b*c+a)*f^2*x^{(m+3)*\text{gamma}(m+3,-b*d*x*\log(F))} - 2*(-b*d*x*\log(F))^{(-m-2)*F^{(b*c+a)*e*f*x^{(m+2)*\text{gamma}(m+2,-b*d*x*\log(F))} - (-b*d*x*\log(F))^{(-m-1)*F^{(b*c+a)*e^2*x^{(m+1)*\text{gamma}(m+1,-b*d*x*\log(F))}}$$

**Fricas [A]** time = 1.60851, size = 386, normalized size = 2.78

$$\frac{\left( (b d f^2 m + 2 b d f^2) x \log(F) - (b^2 d^2 f^2 x^2 + 2 b^2 d^2 e f x) \log(F)^2 \right) F^{b d x + b c + a} x^m - (b^2 d^2 e^2 \log(F)^2 + f^2 m^2 + 3 f^2 m + 2 f^2 - b^3 d^3 \log(F)^3}{b^3 d^3 \log(F)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*(d\*x+c))\*x^m\*(f\*x+e)^2,x, algorithm="fricas")

[Out] 
$$-(((b*d*f^2*m + 2*b*d*f^2)*x*\log(F) - (b^2*d^2*f^2*x^2 + 2*b^2*d^2*e*f*x)*\log(F)^2)*F^{(b*d*x + b*c + a)*x^m} - (b^2*d^2*e^2*\log(F)^2 + f^2*m^2 + 3*f^2*m + 2*f^2 - 2*(b*d*e*f*m + b*d*e*f)*\log(F))*e^{(-m*\log(-b*d*\log(F)) + (b*c + a)*\log(F))*\text{gamma}(m + 1, -b*d*x*\log(F))})/(b^3*d^3*\log(F)^3)$$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int F^{a+b(c+dx)} x^m (e + fx)^2 dx$$



Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F**(a+b*(d*x+c))*x**m*(f*x+e)**2,x)`

[Out] `Integral(F**(a + b*(c + d*x))*x**m*(e + f*x)**2, x)`

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (fx + e)^2 F^{(dx+c)b+a} x^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(a+b*(d*x+c))*x^m*(f*x+e)^2,x, algorithm="giac")`

[Out] `integrate((f*x + e)^2*F^((d*x + c)*b + a)*x^m, x)`

### 3.65 $\int F^{a+b(c+dx)} x^3 (e + fx)^2 dx$

**Optimal.** Leaf size=414

$$-\frac{3e^2 x^2 F^{a+bc+bdx}}{b^2 d^2 \log^2(F)} + \frac{6e^2 x F^{a+bc+bdx}}{b^3 d^3 \log^3(F)} - \frac{6e^2 F^{a+bc+bdx}}{b^4 d^4 \log^4(F)} - \frac{8efx^3 F^{a+bc+bdx}}{b^2 d^2 \log^2(F)} + \frac{24efx^2 F^{a+bc+bdx}}{b^3 d^3 \log^3(F)} - \frac{48efx F^{a+bc+bdx}}{b^4 d^4 \log^4(F)} + \frac{48ef F^{a+bc+bdx}}{b^5 d^5 \log^5(F)}$$

[Out]  $(-120*f^2*F^{(a + b*c + b*d*x)})/(b^6*d^6*\text{Log}[F]^6) + (48*e*f*F^{(a + b*c + b*d*x)})/(b^5*d^5*\text{Log}[F]^5) + (120*f^2*F^{(a + b*c + b*d*x)*x})/(b^5*d^5*\text{Log}[F]^5) - (6*e^2*F^{(a + b*c + b*d*x)})/(b^4*d^4*\text{Log}[F]^4) - (48*e*f*F^{(a + b*c + b*d*x)*x})/(b^4*d^4*\text{Log}[F]^4) - (60*f^2*F^{(a + b*c + b*d*x)*x^2})/(b^4*d^4*\text{Log}[F]^4) + (6*e^2*F^{(a + b*c + b*d*x)*x})/(b^3*d^3*\text{Log}[F]^3) + (24*e*f*F^{(a + b*c + b*d*x)*x^2})/(b^3*d^3*\text{Log}[F]^3) + (20*f^2*F^{(a + b*c + b*d*x)*x^3})/(b^3*d^3*\text{Log}[F]^3) - (3*e^2*F^{(a + b*c + b*d*x)*x^2})/(b^2*d^2*\text{Log}[F]^2) - (8*e*f*F^{(a + b*c + b*d*x)*x^3})/(b^2*d^2*\text{Log}[F]^2) - (5*f^2*F^{(a + b*c + b*d*x)*x^4})/(b^2*d^2*\text{Log}[F]^2) + (e^2*F^{(a + b*c + b*d*x)*x^3})/(b*d*\text{Log}[F]) + (2*e*f*F^{(a + b*c + b*d*x)*x^4})/(b*d*\text{Log}[F]) + (f^2*F^{(a + b*c + b*d*x)*x^5})/(b*d*\text{Log}[F])$

---

**Rubi [A]** time = 0.671541, antiderivative size = 414, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 3, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$ , Rules used = {2196, 2176, 2194}

$$-\frac{3e^2 x^2 F^{a+bc+bdx}}{b^2 d^2 \log^2(F)} + \frac{6e^2 x F^{a+bc+bdx}}{b^3 d^3 \log^3(F)} - \frac{6e^2 F^{a+bc+bdx}}{b^4 d^4 \log^4(F)} - \frac{8efx^3 F^{a+bc+bdx}}{b^2 d^2 \log^2(F)} + \frac{24efx^2 F^{a+bc+bdx}}{b^3 d^3 \log^3(F)} - \frac{48efx F^{a+bc+bdx}}{b^4 d^4 \log^4(F)} + \frac{48ef F^{a+bc+bdx}}{b^5 d^5 \log^5(F)}$$

Antiderivative was successfully verified.

[In] Int[F^(a + b\*(c + d\*x))\*x^3\*(e + f\*x)^2,x]

[Out]  $(-120*f^2*F^{(a + b*c + b*d*x)})/(b^6*d^6*\text{Log}[F]^6) + (48*e*f*F^{(a + b*c + b*d*x)})/(b^5*d^5*\text{Log}[F]^5) + (120*f^2*F^{(a + b*c + b*d*x)*x})/(b^5*d^5*\text{Log}[F]^5) - (6*e^2*F^{(a + b*c + b*d*x)})/(b^4*d^4*\text{Log}[F]^4) - (48*e*f*F^{(a + b*c + b*d*x)*x})/(b^4*d^4*\text{Log}[F]^4) - (60*f^2*F^{(a + b*c + b*d*x)*x^2})/(b^4*d^4*\text{Log}[F]^4) + (6*e^2*F^{(a + b*c + b*d*x)*x})/(b^3*d^3*\text{Log}[F]^3) + (24*e*f*F^{(a + b*c + b*d*x)*x^2})/(b^3*d^3*\text{Log}[F]^3) + (20*f^2*F^{(a + b*c + b*d*x)*x^3})/(b^3*d^3*\text{Log}[F]^3) - (3*e^2*F^{(a + b*c + b*d*x)*x^2})/(b^2*d^2*\text{Log}[F]^2) - (8*e*f*F^{(a + b*c + b*d*x)*x^3})/(b^2*d^2*\text{Log}[F]^2) - (5*f^2*F^{(a + b*c + b*d*x)*x^4})/(b^2*d^2*\text{Log}[F]^2) + (e^2*F^{(a + b*c + b*d*x)*x^3})/(b*d*\text{Log}[F]) + (2*e*f*F^{(a + b*c + b*d*x)*x^4})/(b*d*\text{Log}[F]) + (f^2*F^{(a + b*c + b*d*x)*x^5})/(b*d*\text{Log}[F])$

Rule 2196

```
Int[(F_)^((c_)*(v_))*(u_), x_Symbol] := Int[ExpandIntegrand[F^(c*ExpandToSum[v, x]), u, x], x] /; FreeQ[{F, c}, x] && PolynomialQ[u, x] && LinearQ[v, x] && !$UseGamma === True
```

Rule 2176

```
Int[((b_)*(F_)^((g_)*((e_) + (f_)*(x_))))^(n_)*((c_) + (d_)*(x_))^(m_), x_Symbol] := Simp[((c + d*x)^m*(b*F^(g*(e + f*x)))^n)/(f*g*n*Log[F]), x] - Dist[(d*m)/(f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m] && !$UseGamma === True
```

Rule 2194

```
Int[((F_)^((c_)*((a_) + (b_)*(x_))))^(n_), x_Symbol] := Simp[(F^(c*(a + b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned}
\int F^{a+b(c+dx)} x^3 (e+fx)^2 dx &= \int (e^2 F^{a+bc+bdx} x^3 + 2ef F^{a+bc+bdx} x^4 + f^2 F^{a+bc+bdx} x^5) dx \\
&= e^2 \int F^{a+bc+bdx} x^3 dx + (2ef) \int F^{a+bc+bdx} x^4 dx + f^2 \int F^{a+bc+bdx} x^5 dx \\
&= \frac{e^2 F^{a+bc+bdx} x^3}{bd \log(F)} + \frac{2ef F^{a+bc+bdx} x^4}{bd \log(F)} + \frac{f^2 F^{a+bc+bdx} x^5}{bd \log(F)} - \frac{(3e^2) \int F^{a+bc+bdx} x^2 dx}{bd \log(F)} - \frac{(8ef) \int F^{a+bc+bdx} x dx}{bd \log(F)} \\
&= -\frac{3e^2 F^{a+bc+bdx} x^2}{b^2 d^2 \log^2(F)} - \frac{8ef F^{a+bc+bdx} x^3}{b^2 d^2 \log^2(F)} - \frac{5f^2 F^{a+bc+bdx} x^4}{b^2 d^2 \log^2(F)} + \frac{e^2 F^{a+bc+bdx} x^3}{bd \log(F)} + \frac{2ef F^{a+bc+bdx} x^4}{bd \log(F)} \\
&= \frac{6e^2 F^{a+bc+bdx} x}{b^3 d^3 \log^3(F)} + \frac{24ef F^{a+bc+bdx} x^2}{b^3 d^3 \log^3(F)} + \frac{20f^2 F^{a+bc+bdx} x^3}{b^3 d^3 \log^3(F)} - \frac{3e^2 F^{a+bc+bdx} x^2}{b^2 d^2 \log^2(F)} - \frac{8ef F^{a+bc+bdx} x^3}{b^2 d^2 \log^2(F)} \\
&= -\frac{6e^2 F^{a+bc+bdx}}{b^4 d^4 \log^4(F)} - \frac{48ef F^{a+bc+bdx} x}{b^4 d^4 \log^4(F)} - \frac{60f^2 F^{a+bc+bdx} x^2}{b^4 d^4 \log^4(F)} + \frac{6e^2 F^{a+bc+bdx} x}{b^3 d^3 \log^3(F)} + \frac{24ef F^{a+bc+bdx} x^2}{b^3 d^3 \log^3(F)} \\
&= \frac{48ef F^{a+bc+bdx}}{b^5 d^5 \log^5(F)} + \frac{120f^2 F^{a+bc+bdx} x}{b^5 d^5 \log^5(F)} - \frac{6e^2 F^{a+bc+bdx}}{b^4 d^4 \log^4(F)} - \frac{48ef F^{a+bc+bdx} x}{b^4 d^4 \log^4(F)} - \frac{60f^2 F^{a+bc+bdx} x^2}{b^4 d^4 \log^4(F)} \\
&= -\frac{120f^2 F^{a+bc+bdx}}{b^6 d^6 \log^6(F)} + \frac{48ef F^{a+bc+bdx}}{b^5 d^5 \log^5(F)} + \frac{120f^2 F^{a+bc+bdx} x}{b^5 d^5 \log^5(F)} - \frac{6e^2 F^{a+bc+bdx}}{b^4 d^4 \log^4(F)} - \frac{48ef F^{a+bc+bdx} x}{b^4 d^4 \log^4(F)}
\end{aligned}$$

**Mathematica [A]** time = 0.119706, size = 159, normalized size = 0.38

$$\frac{F^{a+b(c+dx)} \left( -b^4 d^4 x^2 \log^4(F) (3e^2 + 8efx + 5f^2 x^2) + 2b^3 d^3 x \log^3(F) (3e^2 + 12efx + 10f^2 x^2) - 6b^2 d^2 \log^2(F) (e^2 + 8efx + 5f^2 x^2) \right)}{b^6 d^6 \log^6(F)}$$

Antiderivative was successfully verified.

[In] Integrate[F^(a + b\*(c + d\*x))\*x^3\*(e + f\*x)^2,x]

[Out] (F^(a + b\*(c + d\*x))\*(-120\*f^2 + 24\*b\*d\*f\*(2\*e + 5\*f\*x)\*Log[F] - 6\*b^2\*d^2\*(e^2 + 8\*e\*f\*x + 10\*f^2\*x^2)\*Log[F]^2 + 2\*b^3\*d^3\*x\*(3\*e^2 + 12\*e\*f\*x + 10\*f^2\*x^2)\*Log[F]^3 - b^4\*d^4\*x^2\*(3\*e^2 + 8\*e\*f\*x + 5\*f^2\*x^2)\*Log[F]^4 + b^5\*d^5\*x^3\*(e + f\*x)^2\*Log[F]^5))/(b^6\*d^6\*Log[F]^6)

**Maple [A]** time = 0.009, size = 250, normalized size = 0.6

$$\frac{(f^2 x^5 (\ln(F))^5 b^5 d^5 + 2 (\ln(F))^5 b^5 d^5 e f x^4 + (\ln(F))^5 b^5 d^5 e^2 x^3 - 5 (\ln(F))^4 b^4 d^4 f^2 x^4 - 8 (\ln(F))^4 b^4 d^4 e f x^3 - 3 (\ln(F))^4 b^4 d^4 e^2 x^2 + 20 (\ln(F))^3 b^3 d^3 f^2 x^3 + 24 (\ln(F))^3 b^3 d^3 e f x^2 + 6 (\ln(F))^3 b^3 d^3 e^2 x - 60 (\ln(F))^2 b^2 d^2 f^2 x^2 - 48 (\ln(F))^2 b^2 d^2 e f x - 6 (\ln(F))^2 b^2 d^2 e^2 + 120 (\ln(F)) b d f^2 x + 48 f e \ln(F) b d - 120 f^2) F^{(b d x + b c + a)}}{b^6 d^6 \ln(F)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(a+b\*(d\*x+c))\*x^3\*(f\*x+e)^2,x)

[Out] (f^2\*x^5\*ln(F)^5\*b^5\*d^5+2\*ln(F)^5\*b^5\*d^5\*e\*f\*x^4+ln(F)^5\*b^5\*d^5\*e^2\*x^3-5\*ln(F)^4\*b^4\*d^4\*f^2\*x^4-8\*ln(F)^4\*b^4\*d^4\*e\*f\*x^3-3\*ln(F)^4\*b^4\*d^4\*e^2\*x^2+20\*ln(F)^3\*b^3\*d^3\*f^2\*x^3+24\*ln(F)^3\*b^3\*d^3\*e\*f\*x^2+6\*ln(F)^3\*b^3\*d^3\*e^2\*x-60\*ln(F)^2\*b^2\*d^2\*f^2\*x^2-48\*ln(F)^2\*b^2\*d^2\*e\*f\*x-6\*ln(F)^2\*b^2\*d^2\*e^2+120\*ln(F)\*b\*d\*f^2\*x+48\*f\*e\*ln(F)\*b\*d-120\*f^2)\*F^(b\*d\*x+b\*c+a)/ln(F)^6/b^6/d^6

**Maxima [A]** time = 1.06091, size = 443, normalized size = 1.07

$$\frac{(F^{bc+a} b^3 d^3 x^3 \log(F)^3 - 3 F^{bc+a} b^2 d^2 x^2 \log(F)^2 + 6 F^{bc+a} b d x \log(F) - 6 F^{bc+a}) F^{bdx} e^2}{b^4 d^4 \log(F)^4} + \frac{2 (F^{bc+a} b^4 d^4 x^4 \log(F)^4 - 4 F^{bc+a} b^3 d^3 x^3 \log(F)^3 + 6 F^{bc+a} b^2 d^2 x^2 \log(F)^2 - 4 F^{bc+a} b d x \log(F) + 2 F^{bc+a}) F^{bdx} e^2}{b^6 d^6 \log(F)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*(d\*x+c))\*x^3\*(f\*x+e)^2,x, algorithm="maxima")

```
[Out] (F^(b*c + a)*b^3*d^3*x^3*log(F)^3 - 3*F^(b*c + a)*b^2*d^2*x^2*log(F)^2 + 6*
F^(b*c + a)*b*d*x*log(F) - 6*F^(b*c + a))*F^(b*d*x)*e^2/(b^4*d^4*log(F)^4)
+ 2*(F^(b*c + a)*b^4*d^4*x^4*log(F)^4 - 4*F^(b*c + a)*b^3*d^3*x^3*log(F)^3
+ 12*F^(b*c + a)*b^2*d^2*x^2*log(F)^2 - 24*F^(b*c + a)*b*d*x*log(F) + 24*F^
(b*c + a))*F^(b*d*x)*e*f/(b^5*d^5*log(F)^5) + (F^(b*c + a)*b^5*d^5*x^5*log(
F)^5 - 5*F^(b*c + a)*b^4*d^4*x^4*log(F)^4 + 20*F^(b*c + a)*b^3*d^3*x^3*log(
F)^3 - 60*F^(b*c + a)*b^2*d^2*x^2*log(F)^2 + 120*F^(b*c + a)*b*d*x*log(F) -
120*F^(b*c + a))*F^(b*d*x)*f^2/(b^6*d^6*log(F)^6)
```

**Fricas [A]** time = 1.55277, size = 493, normalized size = 1.19

$$\frac{\left( (b^5 d^5 f^2 x^5 + 2 b^5 d^5 e f x^4 + b^5 d^5 e^2 x^3) \log(F)^5 - (5 b^4 d^4 f^2 x^4 + 8 b^4 d^4 e f x^3 + 3 b^4 d^4 e^2 x^2) \log(F)^4 + 2 (10 b^3 d^3 f^2 x^3 + 12 b^3 d^3 e f x^2 + 6 b^3 d^3 e^2 x) \log(F)^3 - 6 (10 b^2 d^2 f^2 x^2 + 8 b^2 d^2 e f x + b^2 d^2 e^2) \log(F)^2 - 120 f^2 + 24 (5 b d f^2 x + 2 b d e f) \log(F) \right) F^{b d x + b c + a}}{b^6 d^6 \log(F)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(a+b*(d*x+c))*x^3*(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] ((b^5*d^5*f^2*x^5 + 2*b^5*d^5*e*f*x^4 + b^5*d^5*e^2*x^3)*log(F)^5 - (5*b^4*d^4*
d^4*f^2*x^4 + 8*b^4*d^4*e*f*x^3 + 3*b^4*d^4*e^2*x^2)*log(F)^4 + 2*(10*b^3*d^3*
d^3*f^2*x^3 + 12*b^3*d^3*e*f*x^2 + 3*b^3*d^3*e^2*x)*log(F)^3 - 6*(10*b^2*d^2*
d^2*f^2*x^2 + 8*b^2*d^2*e*f*x + b^2*d^2*e^2)*log(F)^2 - 120*f^2 + 24*(5*b*d*f^
2*x + 2*b*d*e*f)*log(F))*F^(b*d*x + b*c + a)/(b^6*d^6*log(F)^6)
```

**Sympy [A]** time = 0.209837, size = 323, normalized size = 0.78

$$\frac{\left( F^{a+b(c+dx)} (b^5 d^5 e^2 x^3 \log(F)^5 + 2 b^5 d^5 e f x^4 \log(F)^5 + b^5 d^5 f^2 x^5 \log(F)^5 - 3 b^4 d^4 e^2 x^2 \log(F)^4 - 8 b^4 d^4 e f x^3 \log(F)^4 - 5 b^4 d^4 f^2 x^4 \log(F)^4 + 6 b^3 d^3 e^2 x \log(F)^3 + 24 b^3 d^3 e f x^2 \log(F)^3 - 6 (10 b^2 d^2 f^2 x^2 + 8 b^2 d^2 e f x + b^2 d^2 e^2) \log(F)^2 - 120 f^2 + 24 (5 b d f^2 x + 2 b d e f) \log(F) \right) F^{b d x + b c + a}}{b^6 d^6 \log(F)^6}$$

$$\frac{e^2 x^4}{4} + \frac{2 e f x^5}{5} + \frac{f^2 x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(a+b*(d*x+c))*x**3*(f*x+e)**2,x)
```

```
[Out] Piecewise((F**(a + b*(c + d*x))*(b**5*d**5*e**2*x**3*log(F)**5 + 2*b**5*d**
5*e*f*x**4*log(F)**5 + b**5*d**5*f**2*x**5*log(F)**5 - 3*b**4*d**4*e**2*x**
2*log(F)**4 - 8*b**4*d**4*e*f*x**3*log(F)**4 - 5*b**4*d**4*f**2*x**4*log(F)
**4 + 6*b**3*d**3*e**2*x*log(F)**3 + 24*b**3*d**3*e*f*x**2*log(F)**3 + 20*b
**3*d**3*f**2*x**3*log(F)**3 - 6*b**2*d**2*e**2*log(F)**2 - 48*b**2*d**2*e
```

```
f*x*log(F)**2 - 60*b**2*d**2*f**2*x**2*log(F)**2 + 48*b*d*e*f*log(F) + 120*
b*d*f**2*x*log(F) - 120*f**2)/(b**6*d**6*log(F)**6), Ne(b**6*d**6*log(F)**6
, 0)), (e**2*x**4/4 + 2*e*f*x**5/5 + f**2*x**6/6, True))
```

---

**Giac [C]** time = 1.85998, size = 13437, normalized size = 32.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(a+b*(d*x+c))*x^3*(f*x+e)^2,x, algorithm="giac")
```

```
[Out] ((4*(pi^3*b^3*d^3*x^3*sgn(F) - 3*pi*b^3*d^3*x^3*log(abs(F))^2*sgn(F) - pi^3
*b^3*d^3*x^3 + 3*pi*b^3*d^3*x^3*log(abs(F))^2 + 6*pi*b^2*d^2*x^2*log(abs(F)
)*sgn(F) - 6*pi*b^2*d^2*x^2*log(abs(F)) - 6*pi*b*d*x*sgn(F) + 6*pi*b*d*x)*(
pi^3*b^4*d^4*log(abs(F))*sgn(F) - pi*b^4*d^4*log(abs(F))^3*sgn(F) - pi^3*b^
4*d^4*log(abs(F)) + pi*b^4*d^4*log(abs(F))^3)/((pi^4*b^4*d^4*sgn(F) - 6*pi^
2*b^4*d^4*log(abs(F))^2*sgn(F) - pi^4*b^4*d^4 + 6*pi^2*b^4*d^4*log(abs(F))^
2 - 2*b^4*d^4*log(abs(F))^4)^2 + 16*(pi^3*b^4*d^4*log(abs(F))*sgn(F) - pi*b
^4*d^4*log(abs(F))^3*sgn(F) - pi^3*b^4*d^4*log(abs(F)) + pi*b^4*d^4*log(abs
(F))^3)^2) - (pi^4*b^4*d^4*sgn(F) - 6*pi^2*b^4*d^4*log(abs(F))^2*sgn(F) - p
i^4*b^4*d^4 + 6*pi^2*b^4*d^4*log(abs(F))^2 - 2*b^4*d^4*log(abs(F))^4)*(3*pi
^2*b^3*d^3*x^3*log(abs(F))*sgn(F) - 3*pi^2*b^3*d^3*x^3*log(abs(F)) + 2*b^3*
d^3*x^3*log(abs(F))^3 - 3*pi^2*b^2*d^2*x^2*sgn(F) + 3*pi^2*b^2*d^2*x^2 - 6*
b^2*d^2*x^2*log(abs(F))^2 + 12*b*d*x*log(abs(F)) - 12)/((pi^4*b^4*d^4*sgn(F)
) - 6*pi^2*b^4*d^4*log(abs(F))^2*sgn(F) - pi^4*b^4*d^4 + 6*pi^2*b^4*d^4*log
(abs(F))^2 - 2*b^4*d^4*log(abs(F))^4)^2 + 16*(pi^3*b^4*d^4*log(abs(F))*sgn(
F) - pi*b^4*d^4*log(abs(F))^3*sgn(F) - pi^3*b^4*d^4*log(abs(F)) + pi*b^4*d^
4*log(abs(F))^3)^2))*cos(-1/2*pi*b*d*x*sgn(F) + 1/2*pi*b*d*x - 1/2*pi*b*c*s
gn(F) + 1/2*pi*b*c - 1/2*pi*a*sgn(F) + 1/2*pi*a) + ((pi^4*b^4*d^4*sgn(F) -
6*pi^2*b^4*d^4*log(abs(F))^2*sgn(F) - pi^4*b^4*d^4 + 6*pi^2*b^4*d^4*log(abs
(F))^2 - 2*b^4*d^4*log(abs(F))^4)*(pi^3*b^3*d^3*x^3*sgn(F) - 3*pi*b^3*d^3*x
^3*log(abs(F))^2*sgn(F) - pi^3*b^3*d^3*x^3 + 3*pi*b^3*d^3*x^3*log(abs(F))^2
+ 6*pi*b^2*d^2*x^2*log(abs(F))*sgn(F) - 6*pi*b^2*d^2*x^2*log(abs(F)) - 6*p
i*b*d*x*sgn(F) + 6*pi*b*d*x)/((pi^4*b^4*d^4*sgn(F) - 6*pi^2*b^4*d^4*log(abs
(F))^2*sgn(F) - pi^4*b^4*d^4 + 6*pi^2*b^4*d^4*log(abs(F))^2 - 2*b^4*d^4*log
(abs(F))^4)^2 + 16*(pi^3*b^4*d^4*log(abs(F))*sgn(F) - pi*b^4*d^4*log(abs(F)
))^3*sgn(F) - pi^3*b^4*d^4*log(abs(F)) + pi*b^4*d^4*log(abs(F))^3)^2) + 4*(p
i^3*b^4*d^4*log(abs(F))*sgn(F) - pi*b^4*d^4*log(abs(F))^3*sgn(F) - pi^3*b^4
*d^4*log(abs(F)) + pi*b^4*d^4*log(abs(F))^3)*(3*pi^2*b^3*d^3*x^3*log(abs(F)
))*sgn(F) - 3*pi^2*b^3*d^3*x^3*log(abs(F)) + 2*b^3*d^3*x^3*log(abs(F))^3 - 3
*pi^2*b^2*d^2*x^2*sgn(F) + 3*pi^2*b^2*d^2*x^2 - 6*b^2*d^2*x^2*log(abs(F))^2
```

$$\begin{aligned}
& + 12*b*d*x*\log(\text{abs}(F)) - 12)/((\pi^4*b^4*d^4*\text{sgn}(F) - 6*\pi^2*b^4*d^4*\log(\text{abs}(F))^2*\text{sgn}(F) - \pi^4*b^4*d^4 + 6*\pi^2*b^4*d^4*\log(\text{abs}(F))^2 - 2*b^4*d^4*\log(\text{abs}(F))^4)^2 + 16*(\pi^3*b^4*d^4*\log(\text{abs}(F))*\text{sgn}(F) - \pi*b^4*d^4*\log(\text{abs}(F))^3*\text{sgn}(F) - \pi^3*b^4*d^4*\log(\text{abs}(F)) + \pi*b^4*d^4*\log(\text{abs}(F))^3)^2)*\sin(-1/2*\pi*b*d*x*\text{sgn}(F) + 1/2*\pi*b*d*x - 1/2*\pi*b*c*\text{sgn}(F) + 1/2*\pi*b*c - 1/2*\pi*a*\text{sgn}(F) + 1/2*\pi*a))*e^{(b*d*x*\log(\text{abs}(F)) + b*c*\log(\text{abs}(F)) + a*\log(\text{abs}(F)) + 2) - 1/2*I*((8*\pi^3*b^3*d^3*x^3*\text{sgn}(F) + 24*I*\pi^2*b^3*d^3*x^3*\log(\text{abs}(F))*\text{sgn}(F) - 24*\pi*b^3*d^3*x^3*\log(\text{abs}(F))^2*\text{sgn}(F) - 8*\pi^3*b^3*d^3*x^3 - 24*I*\pi^2*b^3*d^3*x^3*\log(\text{abs}(F)) + 24*\pi*b^3*d^3*x^3*\log(\text{abs}(F))^2 + 16*I*b^3*d^3*x^3*\log(\text{abs}(F))^3 - 24*I*\pi^2*b^2*d^2*x^2*\text{sgn}(F) + 48*\pi*b^2*d^2*x^2*\log(\text{abs}(F))*\text{sgn}(F) + 24*I*\pi^2*b^2*d^2*x^2 - 48*\pi*b^2*d^2*x^2*\log(\text{abs}(F)) - 48*I*b^2*d^2*x^2*\log(\text{abs}(F))^2 - 48*\pi*b*d*x*\text{sgn}(F) + 48*\pi*b*d*x + 96*I*b*d*x*\log(\text{abs}(F)) - 96*I)*e^{(1/2*I*\pi*b*d*x*\text{sgn}(F) - 1/2*I*\pi*b*d*x + 1/2*I*\pi*b*c*\text{sgn}(F) - 1/2*I*\pi*b*c + 1/2*I*\pi*a*\text{sgn}(F) - 1/2*I*\pi*a)/(8*\pi^4*b^4*d^4*\text{sgn}(F) + 32*I*\pi^3*b^4*d^4*\log(\text{abs}(F))*\text{sgn}(F) - 48*\pi^2*b^4*d^4*\log(\text{abs}(F))^2*\text{sgn}(F) - 32*I*\pi*b^4*d^4*\log(\text{abs}(F))^3*\text{sgn}(F) - 8*\pi^4*b^4*d^4 - 32*I*\pi^3*b^4*d^4*\log(\text{abs}(F)) + 48*\pi^2*b^4*d^4*\log(\text{abs}(F))^2 + 32*I*\pi*b^4*d^4*\log(\text{abs}(F))^3 - 16*b^4*d^4*\log(\text{abs}(F))^4) + (8*\pi^3*b^3*d^3*x^3*\text{sgn}(F) - 24*I*\pi^2*b^3*d^3*x^3*\log(\text{abs}(F))*\text{sgn}(F) - 24*\pi*b^3*d^3*x^3*\log(\text{abs}(F))^2*\text{sgn}(F) - 8*\pi^3*b^3*d^3*x^3 + 24*I*\pi^2*b^3*d^3*x^3*\log(\text{abs}(F)) + 24*\pi*b^3*d^3*x^3*\log(\text{abs}(F))^2 - 16*I*b^3*d^3*x^3*\log(\text{abs}(F))^3 + 24*I*\pi^2*b^2*d^2*x^2*\text{sgn}(F) + 48*\pi*b^2*d^2*x^2*\log(\text{abs}(F))*\text{sgn}(F) - 24*I*\pi^2*b^2*d^2*x^2 - 48*\pi*b^2*d^2*x^2*\log(\text{abs}(F)) + 48*I*b^2*d^2*x^2*\log(\text{abs}(F))^2 - 48*\pi*b*d*x*\text{sgn}(F) + 48*\pi*b*d*x - 96*I*b*d*x*\log(\text{abs}(F)) + 96*I)*e^{(-1/2*I*\pi*b*d*x*\text{sgn}(F) + 1/2*I*\pi*b*d*x - 1/2*I*\pi*b*c*\text{sgn}(F) + 1/2*I*\pi*b*c - 1/2*I*\pi*a*\text{sgn}(F) + 1/2*I*\pi*a)/(8*\pi^4*b^4*d^4*\text{sgn}(F) - 32*I*\pi^3*b^4*d^4*\log(\text{abs}(F))*\text{sgn}(F) - 48*\pi^2*b^4*d^4*\log(\text{abs}(F))^2*\text{sgn}(F) + 32*I*\pi*b^4*d^4*\log(\text{abs}(F))^3*\text{sgn}(F) - 8*\pi^4*b^4*d^4 + 32*I*\pi^3*b^4*d^4*\log(\text{abs}(F)) + 48*\pi^2*b^4*d^4*\log(\text{abs}(F))^2 - 32*I*\pi*b^4*d^4*\log(\text{abs}(F))^3 - 16*b^4*d^4*\log(\text{abs}(F))^4)}*e^{(b*d*x*\log(\text{abs}(F)) + b*c*\log(\text{abs}(F)) + a*\log(\text{abs}(F)) + 2) - 2*((4*(\pi^3*b^4*d^4*f*x^4*\log(\text{abs}(F))*\text{sgn}(F) - \pi*b^4*d^4*f*x^4*\log(\text{abs}(F))^3*\text{sgn}(F) - \pi^3*b^4*d^4*f*x^4*\log(\text{abs}(F)) + \pi*b^4*d^4*f*x^4*\log(\text{abs}(F))^3 - \pi^3*b^3*d^3*f*x^3*\text{sgn}(F) + 3*\pi*b^3*d^3*f*x^3*\log(\text{abs}(F))^2*\text{sgn}(F) + \pi^3*b^3*d^3*f*x^3 - 3*\pi*b^3*d^3*f*x^3*\log(\text{abs}(F))^2 - 6*\pi*b^2*d^2*f*x^2*\log(\text{abs}(F))*\text{sgn}(F) + 6*\pi*b^2*d^2*f*x^2*\log(\text{abs}(F)) + 6*\pi*b*d*f*x*\text{sgn}(F) - 6*\pi*b*d*f*x*(\pi^5*b^5*d^5*\text{sgn}(F) - 10*\pi^3*b^5*d^5*\log(\text{abs}(F))^2*\text{sgn}(F) + 5*\pi*b^5*d^5*\log(\text{abs}(F))^4*\text{sgn}(F) - \pi^5*b^5*d^5 + 10*\pi^3*b^5*d^5*\log(\text{abs}(F))^2 - 5*\pi*b^5*d^5*\log(\text{abs}(F))^4)/((\pi^5*b^5*d^5*\text{sgn}(F) - 10*\pi^3*b^5*d^5*\log(\text{abs}(F))^2*\text{sgn}(F) + 5*\pi*b^5*d^5*\log(\text{abs}(F))^4*\text{sgn}(F) - \pi^5*b^5*d^5 + 10*\pi^3*b^5*d^5*\log(\text{abs}(F))^2 - 5*\pi*b^5*d^5*\log(\text{abs}(F))^4)^2 + (5*\pi^4*b^5*d^5*\log(\text{abs}(F))*\text{sgn}(F) - 10*\pi^2*b^5*d^5*\log(\text{abs}(F))^3*\text{sgn}(F) - 5*\pi^4*b^5*d^5*\log(\text{abs}(F)) + 10*\pi^2*b^5*d^5*\log(\text{abs}(F))^3 - 2*b^5*d^5*\log(\text{abs}(F))^5)^2) - (\pi^4*b^4*d^4*f*x^4*\text{sgn}(F) - 6*\pi^2*b^4*d^4*f*x^4*\log(\text{abs}(F))^2*\text{sgn}(F) - \pi^4*b^4*d^4*f*x^4 + 6*\pi^2*b^4*d^4*f*x^4*\log(\text{abs}(F))^2 - 2*b^4*d^4*f*x^4*\log(\text{abs}(F))^4 + 12*\pi^2*b^3*d^3*f*x^3*\log(\text{abs}(F))*\text{sgn}(F) - 12*\pi^2*b^3*d^3*
\end{aligned}$$

$$\begin{aligned}
& f*x^3*\log(\text{abs}(F)) + 8*b^3*d^3*f*x^3*\log(\text{abs}(F))^3 - 12*\pi^2*b^2*d^2*f*x^2*s \\
& \text{gn}(F) + 12*\pi^2*b^2*d^2*f*x^2 - 24*b^2*d^2*f*x^2*\log(\text{abs}(F))^2 + 48*b*d*f*x \\
& *\log(\text{abs}(F)) - 48*f)*(5*\pi^4*b^5*d^5*\log(\text{abs}(F))*\text{sgn}(F) - 10*\pi^2*b^5*d^5*l \\
& \text{og}(\text{abs}(F))^3*\text{sgn}(F) - 5*\pi^4*b^5*d^5*\log(\text{abs}(F)) + 10*\pi^2*b^5*d^5*\log(\text{abs}( \\
& F))^3 - 2*b^5*d^5*\log(\text{abs}(F))^5)/((\pi^5*b^5*d^5*\text{sgn}(F) - 10*\pi^3*b^5*d^5*lo \\
& \text{g}(\text{abs}(F))^2*\text{sgn}(F) + 5*\pi*b^5*d^5*\log(\text{abs}(F))^4*\text{sgn}(F) - \pi^5*b^5*d^5 + 10* \\
& \pi^3*b^5*d^5*\log(\text{abs}(F))^2 - 5*\pi*b^5*d^5*\log(\text{abs}(F))^4)^2 + (5*\pi^4*b^5*d^ \\
& 5*\log(\text{abs}(F))*\text{sgn}(F) - 10*\pi^2*b^5*d^5*\log(\text{abs}(F))^3*\text{sgn}(F) - 5*\pi^4*b^5*d^ \\
& 5*\log(\text{abs}(F)) + 10*\pi^2*b^5*d^5*\log(\text{abs}(F))^3 - 2*b^5*d^5*\log(\text{abs}(F))^5)^2) \\
& )*\cos(-1/2*\pi*b*d*x*\text{sgn}(F) + 1/2*\pi*b*d*x - 1/2*\pi*b*c*\text{sgn}(F) + 1/2*\pi*b*c \\
& - 1/2*\pi*a*\text{sgn}(F) + 1/2*\pi*a) - ((\pi^4*b^4*d^4*f*x^4*\text{sgn}(F) - 6*\pi^2*b^4*d^ \\
& 4*f*x^4*\log(\text{abs}(F))^2*\text{sgn}(F) - \pi^4*b^4*d^4*f*x^4 + 6*\pi^2*b^4*d^4*f*x^4*lo \\
& \text{g}(\text{abs}(F))^2 - 2*b^4*d^4*f*x^4*\log(\text{abs}(F))^4 + 12*\pi^2*b^3*d^3*f*x^3*\log(\text{abs} \\
& (F))*\text{sgn}(F) - 12*\pi^2*b^3*d^3*f*x^3*\log(\text{abs}(F)) + 8*b^3*d^3*f*x^3*\log(\text{abs}(F \\
& ))^3 - 12*\pi^2*b^2*d^2*f*x^2*\text{sgn}(F) + 12*\pi^2*b^2*d^2*f*x^2 - 24*b^2*d^2*f* \\
& x^2*\log(\text{abs}(F))^2 + 48*b*d*f*x*\log(\text{abs}(F)) - 48*f)*(\pi^5*b^5*d^5*\text{sgn}(F) - 1 \\
& 0*\pi^3*b^5*d^5*\log(\text{abs}(F))^2*\text{sgn}(F) + 5*\pi*b^5*d^5*\log(\text{abs}(F))^4*\text{sgn}(F) - \pi \\
& i^5*b^5*d^5 + 10*\pi^3*b^5*d^5*\log(\text{abs}(F))^2 - 5*\pi*b^5*d^5*\log(\text{abs}(F))^4)/ \\
& ((\pi^5*b^5*d^5*\text{sgn}(F) - 10*\pi^3*b^5*d^5*\log(\text{abs}(F))^2*\text{sgn}(F) + 5*\pi*b^5*d^5* \\
& \log(\text{abs}(F))^4*\text{sgn}(F) - \pi^5*b^5*d^5 + 10*\pi^3*b^5*d^5*\log(\text{abs}(F))^2 - 5*\pi* \\
& b^5*d^5*\log(\text{abs}(F))^4)^2 + (5*\pi^4*b^5*d^5*\log(\text{abs}(F))*\text{sgn}(F) - 10*\pi^2*b^5 \\
& *d^5*\log(\text{abs}(F))^3*\text{sgn}(F) - 5*\pi^4*b^5*d^5*\log(\text{abs}(F)) + 10*\pi^2*b^5*d^5*lo \\
& \text{g}(\text{abs}(F))^3 - 2*b^5*d^5*\log(\text{abs}(F))^5)^2) + 4*(\pi^3*b^4*d^4*f*x^4*\log(\text{abs}(F \\
& ))*\text{sgn}(F) - \pi*b^4*d^4*f*x^4*\log(\text{abs}(F))^3*\text{sgn}(F) - \pi^3*b^4*d^4*f*x^4*\log( \\
& \text{abs}(F)) + \pi*b^4*d^4*f*x^4*\log(\text{abs}(F))^3 - \pi^3*b^3*d^3*f*x^3*\text{sgn}(F) + 3*\pi \\
& *b^3*d^3*f*x^3*\log(\text{abs}(F))^2*\text{sgn}(F) + \pi^3*b^3*d^3*f*x^3 - 3*\pi*b^3*d^3*f*x \\
& ^3*\log(\text{abs}(F))^2 - 6*\pi*b^2*d^2*f*x^2*\log(\text{abs}(F))*\text{sgn}(F) + 6*\pi*b^2*d^2*f*x \\
& ^2*\log(\text{abs}(F)) + 6*\pi*b*d*f*x*\text{sgn}(F) - 6*\pi*b*d*f*x)*(5*\pi^4*b^5*d^5*\log(ab \\
& s(F))*\text{sgn}(F) - 10*\pi^2*b^5*d^5*\log(\text{abs}(F))^3*\text{sgn}(F) - 5*\pi^4*b^5*d^5*\log(ab \\
& s(F)) + 10*\pi^2*b^5*d^5*\log(\text{abs}(F))^3 - 2*b^5*d^5*\log(\text{abs}(F))^5)/((\pi^5*b^5 \\
& *d^5*\text{sgn}(F) - 10*\pi^3*b^5*d^5*\log(\text{abs}(F))^2*\text{sgn}(F) + 5*\pi*b^5*d^5*\log(\text{abs}(F \\
& ))^4*\text{sgn}(F) - \pi^5*b^5*d^5 + 10*\pi^3*b^5*d^5*\log(\text{abs}(F))^2 - 5*\pi*b^5*d^5*l \\
& \text{og}(\text{abs}(F))^4)^2 + (5*\pi^4*b^5*d^5*\log(\text{abs}(F))*\text{sgn}(F) - 10*\pi^2*b^5*d^5*\log( \\
& \text{abs}(F))^3*\text{sgn}(F) - 5*\pi^4*b^5*d^5*\log(\text{abs}(F)) + 10*\pi^2*b^5*d^5*\log(\text{abs}(F)) \\
& ^3 - 2*b^5*d^5*\log(\text{abs}(F))^5)^2))*\sin(-1/2*\pi*b*d*x*\text{sgn}(F) + 1/2*\pi*b*d*x - \\
& 1/2*\pi*b*c*\text{sgn}(F) + 1/2*\pi*b*c - 1/2*\pi*a*\text{sgn}(F) + 1/2*\pi*a)*e^{(b*d*x*\log \\
& (\text{abs}(F)) + b*c*\log(\text{abs}(F)) + a*\log(\text{abs}(F)) + 1) + 1/2*I*((-32*I*\pi^4*b^4*d^ \\
& 4*f*x^4*\text{sgn}(F) + 128*\pi^3*b^4*d^4*f*x^4*\log(\text{abs}(F))*\text{sgn}(F) + 192*I*\pi^2*b^4 \\
& *d^4*f*x^4*\log(\text{abs}(F))^2*\text{sgn}(F) - 128*\pi*b^4*d^4*f*x^4*\log(\text{abs}(F))^3*\text{sgn}(F) \\
& + 32*I*\pi^4*b^4*d^4*f*x^4 - 128*\pi^3*b^4*d^4*f*x^4*\log(\text{abs}(F)) - 192*I*\pi^ \\
& 2*b^4*d^4*f*x^4*\log(\text{abs}(F))^2 + 128*\pi*b^4*d^4*f*x^4*\log(\text{abs}(F))^3 + 64*I*b \\
& ^4*d^4*f*x^4*\log(\text{abs}(F))^4 - 128*\pi^3*b^3*d^3*f*x^3*\text{sgn}(F) - 384*I*\pi^2*b^3 \\
& *d^3*f*x^3*\log(\text{abs}(F))*\text{sgn}(F) + 384*\pi*b^3*d^3*f*x^3*\log(\text{abs}(F))^2*\text{sgn}(F) + \\
& 128*\pi^3*b^3*d^3*f*x^3 + 384*I*\pi^2*b^3*d^3*f*x^3*\log(\text{abs}(F)) - 384*\pi*b^3 \\
& *d^3*f*x^3*\log(\text{abs}(F))^2 - 256*I*b^3*d^3*f*x^3*\log(\text{abs}(F))^3 + 384*I*\pi^2*b
\end{aligned}$$



$$\begin{aligned}
& ^2d^2f*x^2*sgn(F) - 768*pi*b^2*d^2*f*x^2*log(abs(F))*sgn(F) - 384*I*pi^2* \\
& b^2*d^2*f*x^2 + 768*pi*b^2*d^2*f*x^2*log(abs(F)) + 768*I*b^2*d^2*f*x^2*log( \\
& abs(F))^2 + 768*pi*b*d*f*x*sgn(F) - 768*pi*b*d*f*x - 1536*I*b*d*f*x*log(abs \\
& (F)) + 1536*I*f)*e^(1/2*I*pi*b*d*x*sgn(F) - 1/2*I*pi*b*d*x + 1/2*I*pi*b*c*s \\
& gn(F) - 1/2*I*pi*b*c + 1/2*I*pi*a*sgn(F) - 1/2*I*pi*a)/(16*I*pi^5*b^5*d^5*s \\
& gn(F) - 80*pi^4*b^5*d^5*log(abs(F))*sgn(F) - 160*I*pi^3*b^5*d^5*log(abs(F)) \\
& ^2*sgn(F) + 160*pi^2*b^5*d^5*log(abs(F))^3*sgn(F) + 80*I*pi*b^5*d^5*log(abs \\
& (F))^4*sgn(F) - 16*I*pi^5*b^5*d^5 + 80*pi^4*b^5*d^5*log(abs(F)) + 160*I*pi^ \\
& 3*b^5*d^5*log(abs(F))^2 - 160*pi^2*b^5*d^5*log(abs(F))^3 - 80*I*pi*b^5*d^5* \\
& log(abs(F))^4 + 32*b^5*d^5*log(abs(F))^5) - (-32*I*pi^4*b^4*d^4*f*x^4*sgn(F \\
& ) - 128*pi^3*b^4*d^4*f*x^4*log(abs(F))*sgn(F) + 192*I*pi^2*b^4*d^4*f*x^4*lo \\
& g(abs(F))^2*sgn(F) + 128*pi*b^4*d^4*f*x^4*log(abs(F))^3*sgn(F) + 32*I*pi^4* \\
& b^4*d^4*f*x^4 + 128*pi^3*b^4*d^4*f*x^4*log(abs(F)) - 192*I*pi^2*b^4*d^4*f*x \\
& ^4*log(abs(F))^2 - 128*pi*b^4*d^4*f*x^4*log(abs(F))^3 + 64*I*b^4*d^4*f*x^4* \\
& log(abs(F))^4 + 128*pi^3*b^3*d^3*f*x^3*sgn(F) - 384*I*pi^2*b^3*d^3*f*x^3*lo \\
& g(abs(F))*sgn(F) - 384*pi*b^3*d^3*f*x^3*log(abs(F))^2*sgn(F) - 128*pi^3*b^3 \\
& *d^3*f*x^3 + 384*I*pi^2*b^3*d^3*f*x^3*log(abs(F)) + 384*pi*b^3*d^3*f*x^3*lo \\
& g(abs(F))^2 - 256*I*b^3*d^3*f*x^3*log(abs(F))^3 + 384*I*pi^2*b^2*d^2*f*x^2* \\
& sgn(F) + 768*pi*b^2*d^2*f*x^2*log(abs(F))*sgn(F) - 384*I*pi^2*b^2*d^2*f*x^2 \\
& - 768*pi*b^2*d^2*f*x^2*log(abs(F)) + 768*I*b^2*d^2*f*x^2*log(abs(F))^2 - 7 \\
& 68*pi*b*d*f*x*sgn(F) + 768*pi*b*d*f*x - 1536*I*b*d*f*x*log(abs(F)) + 1536*I \\
& *f)*e^(-1/2*I*pi*b*d*x*sgn(F) + 1/2*I*pi*b*d*x - 1/2*I*pi*b*c*sgn(F) + 1/2* \\
& I*pi*b*c - 1/2*I*pi*a*sgn(F) + 1/2*I*pi*a)/(-16*I*pi^5*b^5*d^5*sgn(F) - 80* \\
& pi^4*b^5*d^5*log(abs(F))*sgn(F) + 160*I*pi^3*b^5*d^5*log(abs(F))^2*sgn(F) + \\
& 160*pi^2*b^5*d^5*log(abs(F))^3*sgn(F) - 80*I*pi*b^5*d^5*log(abs(F))^4*sgn(F) \\
& + 16*I*pi^5*b^5*d^5 + 80*pi^4*b^5*d^5*log(abs(F)) - 160*I*pi^3*b^5*d^5* \\
& log(abs(F))^2 - 160*pi^2*b^5*d^5*log(abs(F))^3 + 80*I*pi*b^5*d^5*log(abs(F)) \\
& ^4 + 32*b^5*d^5*log(abs(F))^5)*e^(b*d*x*log(abs(F)) + b*c*log(abs(F)) + a* \\
& log(abs(F)) + 1) - (((5*pi^4*b^5*d^5*f^2*x^5*log(abs(F))*sgn(F) - 10*pi^2*b \\
& ^5*d^5*f^2*x^5*log(abs(F))^3*sgn(F) - 5*pi^4*b^5*d^5*f^2*x^5*log(abs(F)) + \\
& 10*pi^2*b^5*d^5*f^2*x^5*log(abs(F))^3 - 2*b^5*d^5*f^2*x^5*log(abs(F))^5 - 5 \\
& *pi^4*b^4*d^4*f^2*x^4*sgn(F) + 30*pi^2*b^4*d^4*f^2*x^4*log(abs(F))^2*sgn(F) \\
& + 5*pi^4*b^4*d^4*f^2*x^4 - 30*pi^2*b^4*d^4*f^2*x^4*log(abs(F))^2 + 10*b^4* \\
& d^4*f^2*x^4*log(abs(F))^4 - 60*pi^2*b^3*d^3*f^2*x^3*log(abs(F))*sgn(F) + 60 \\
& *pi^2*b^3*d^3*f^2*x^3*log(abs(F)) - 40*b^3*d^3*f^2*x^3*log(abs(F))^3 + 60*pi \\
& i^2*b^2*d^2*f^2*x^2*sgn(F) - 60*pi^2*b^2*d^2*f^2*x^2 + 120*b^2*d^2*f^2*x^2* \\
& log(abs(F))^2 - 240*b*d*f^2*x*log(abs(F)) + 240*f^2)*(pi^6*b^6*d^6*sgn(F) - \\
& 15*pi^4*b^6*d^6*log(abs(F))^2*sgn(F) + 15*pi^2*b^6*d^6*log(abs(F))^4*sgn(F) \\
& ) - pi^6*b^6*d^6 + 15*pi^4*b^6*d^6*log(abs(F))^2 - 15*pi^2*b^6*d^6*log(abs( \\
& F))^4 + 2*b^6*d^6*log(abs(F))^6)/((pi^6*b^6*d^6*sgn(F) - 15*pi^4*b^6*d^6*lo \\
& g(abs(F))^2*sgn(F) + 15*pi^2*b^6*d^6*log(abs(F))^4*sgn(F) - pi^6*b^6*d^6 + \\
& 15*pi^4*b^6*d^6*log(abs(F))^2 - 15*pi^2*b^6*d^6*log(abs(F))^4 + 2*b^6*d^6* \\
& log(abs(F))^6)^2 + 4*(3*pi^5*b^6*d^6*log(abs(F))*sgn(F) - 10*pi^3*b^6*d^6*lo \\
& g(abs(F))^3*sgn(F) + 3*pi*b^6*d^6*log(abs(F))^5*sgn(F) - 3*pi^5*b^6*d^6*log \\
& (abs(F)) + 10*pi^3*b^6*d^6*log(abs(F))^3 - 3*pi*b^6*d^6*log(abs(F))^5)^2) -
\end{aligned}$$

$$\begin{aligned}
& 2*(\pi^5*b^5*d^5*f^2*x^5*\operatorname{sgn}(F) - 10*\pi^3*b^5*d^5*f^2*x^5*\log(\operatorname{abs}(F))^2*\operatorname{sgn}(F) + 5*\pi*b^5*d^5*f^2*x^5*\log(\operatorname{abs}(F))^4*\operatorname{sgn}(F) - \pi^5*b^5*d^5*f^2*x^5 + 10 \\
& * \pi^3*b^5*d^5*f^2*x^5*\log(\operatorname{abs}(F))^2 - 5*\pi*b^5*d^5*f^2*x^5*\log(\operatorname{abs}(F))^4 + 20*\pi^3*b^4*d^4*f^2*x^4*\log(\operatorname{abs}(F))*\operatorname{sgn}(F) - 20*\pi*b^4*d^4*f^2*x^4*\log(\operatorname{abs}(F))^3*\operatorname{sgn}(F) - 20*\pi^3*b^4*d^4*f^2*x^4*\log(\operatorname{abs}(F)) + 20*\pi*b^4*d^4*f^2*x^4* \\
& \log(\operatorname{abs}(F))^3 - 20*\pi^3*b^3*d^3*f^2*x^3*\operatorname{sgn}(F) + 60*\pi*b^3*d^3*f^2*x^3*\log(\operatorname{abs}(F))^2*\operatorname{sgn}(F) + 20*\pi^3*b^3*d^3*f^2*x^3 - 60*\pi*b^3*d^3*f^2*x^3*\log(\operatorname{abs}(F))^2 - 120*\pi*b^2*d^2*f^2*x^2*\log(\operatorname{abs}(F))*\operatorname{sgn}(F) + 120*\pi*b^2*d^2*f^2*x^2* \\
& \log(\operatorname{abs}(F)) + 120*\pi*b*d*f^2*x*\operatorname{sgn}(F) - 120*\pi*b*d*f^2*x)*(3*\pi^5*b^6*d^6*\log(\operatorname{abs}(F))*\operatorname{sgn}(F) - 10*\pi^3*b^6*d^6*\log(\operatorname{abs}(F))^3*\operatorname{sgn}(F) + 3*\pi*b^6*d^6*\log(\operatorname{abs}(F))^5*\operatorname{sgn}(F) - 3*\pi^5*b^6*d^6*\log(\operatorname{abs}(F)) + 10*\pi^3*b^6*d^6*\log(\operatorname{abs}(F))^3 - 3*\pi*b^6*d^6*\log(\operatorname{abs}(F))^5)/((\pi^6*b^6*d^6*\operatorname{sgn}(F) - 15*\pi^4*b^6*d^6*\log(\operatorname{abs}(F))^2*\operatorname{sgn}(F) + 15*\pi^2*b^6*d^6*\log(\operatorname{abs}(F))^4*\operatorname{sgn}(F) - \pi^6*b^6*d^6 + 15*\pi^4*b^6*d^6*\log(\operatorname{abs}(F))^2 - 15*\pi^2*b^6*d^6*\log(\operatorname{abs}(F))^4 + 2*b^6*d^6*\log(\operatorname{abs}(F))^6)^2 + 4*(3*\pi^5*b^6*d^6*\log(\operatorname{abs}(F))*\operatorname{sgn}(F) - 10*\pi^3*b^6*d^6*\log(\operatorname{abs}(F))^3*\operatorname{sgn}(F) + 3*\pi*b^6*d^6*\log(\operatorname{abs}(F))^5*\operatorname{sgn}(F) - 3*\pi^5*b^6*d^6*\log(\operatorname{abs}(F)) + 10*\pi^3*b^6*d^6*\log(\operatorname{abs}(F))^3 - 3*\pi*b^6*d^6*\log(\operatorname{abs}(F))^5)^2)) \\
& *\cos(-1/2*\pi*b*d*x*\operatorname{sgn}(F) + 1/2*\pi*b*d*x - 1/2*\pi*b*c*\operatorname{sgn}(F) + 1/2*\pi*b*c - 1/2*\pi*a*\operatorname{sgn}(F) + 1/2*\pi*a) - ((\pi^5*b^5*d^5*f^2*x^5*\operatorname{sgn}(F) - 10*\pi^3*b^5*d^5*f^2*x^5*\log(\operatorname{abs}(F))^2*\operatorname{sgn}(F) + 5*\pi*b^5*d^5*f^2*x^5*\log(\operatorname{abs}(F))^4*\operatorname{sgn}(F) - \pi^5*b^5*d^5*f^2*x^5 + 10*\pi^3*b^5*d^5*f^2*x^5*\log(\operatorname{abs}(F))^2 - 5*\pi*b^5*d^5*f^2*x^5*\log(\operatorname{abs}(F))^4 + 20*\pi^3*b^4*d^4*f^2*x^4*\log(\operatorname{abs}(F))*\operatorname{sgn}(F) - 20*\pi*b^4*d^4*f^2*x^4*\log(\operatorname{abs}(F))^3*\operatorname{sgn}(F) - 20*\pi^3*b^4*d^4*f^2*x^4*\log(\operatorname{abs}(F)) + 20*\pi*b^4*d^4*f^2*x^4*\log(\operatorname{abs}(F))^3 - 20*\pi^3*b^3*d^3*f^2*x^3*\operatorname{sgn}(F) + 60*\pi*b^3*d^3*f^2*x^3*\log(\operatorname{abs}(F))^2*\operatorname{sgn}(F) + 20*\pi^3*b^3*d^3*f^2*x^3 - 60*\pi*b^3*d^3*f^2*x^3*\log(\operatorname{abs}(F))^2 - 120*\pi*b^2*d^2*f^2*x^2*\log(\operatorname{abs}(F))*\operatorname{sgn}(F) + 120*\pi*b^2*d^2*f^2*x^2*\log(\operatorname{abs}(F)) + 120*\pi*b*d*f^2*x*\operatorname{sgn}(F) - 120*\pi*b*d*f^2*x)*(\pi^6*b^6*d^6*\operatorname{sgn}(F) - 15*\pi^4*b^6*d^6*\log(\operatorname{abs}(F))^2*\operatorname{sgn}(F) + 15*\pi^2*b^6*d^6*\log(\operatorname{abs}(F))^4*\operatorname{sgn}(F) - \pi^6*b^6*d^6 + 15*\pi^4*b^6*d^6*\log(\operatorname{abs}(F))^2 - 15*\pi^2*b^6*d^6*\log(\operatorname{abs}(F))^4 + 2*b^6*d^6*\log(\operatorname{abs}(F))^6)/((\pi^6*b^6*d^6*\operatorname{sgn}(F) - 15*\pi^4*b^6*d^6*\log(\operatorname{abs}(F))^2*\operatorname{sgn}(F) + 15*\pi^2*b^6*d^6*\log(\operatorname{abs}(F))^4*\operatorname{sgn}(F) - \pi^6*b^6*d^6 + 15*\pi^4*b^6*d^6*\log(\operatorname{abs}(F))^2 - 15*\pi^2*b^6*d^6*\log(\operatorname{abs}(F))^4 + 2*b^6*d^6*\log(\operatorname{abs}(F))^6)^2 + 4*(3*\pi^5*b^6*d^6*\log(\operatorname{abs}(F))*\operatorname{sgn}(F) - 10*\pi^3*b^6*d^6*\log(\operatorname{abs}(F))^3*\operatorname{sgn}(F) + 3*\pi*b^6*d^6*\log(\operatorname{abs}(F))^5*\operatorname{sgn}(F) - 3*\pi^5*b^6*d^6*\log(\operatorname{abs}(F)) + 10*\pi^3*b^6*d^6*\log(\operatorname{abs}(F))^3 - 3*\pi*b^6*d^6*\log(\operatorname{abs}(F))^5)^2) + 2*(5*\pi^4*b^5*d^5*f^2*x^5*\log(\operatorname{abs}(F))*\operatorname{sgn}(F) - 10*\pi^2*b^5*d^5*f^2*x^5*\log(\operatorname{abs}(F))^3*\operatorname{sgn}(F) - 5*\pi^4*b^5*d^5*f^2*x^5*\log(\operatorname{abs}(F)) + 10*\pi^2*b^5*d^5*f^2*x^5*\log(\operatorname{abs}(F))^3 - 2*b^5*d^5*f^2*x^5*\log(\operatorname{abs}(F))^5 - 5*\pi^4*b^4*d^4*f^2*x^4*\operatorname{sgn}(F) + 30*\pi^2*b^4*d^4*f^2*x^4*\log(\operatorname{abs}(F))^2*\operatorname{sgn}(F) + 5*\pi^4*b^4*d^4*f^2*x^4 - 30*\pi^2*b^4*d^4*f^2*x^4*\log(\operatorname{abs}(F))^2 + 10*b^4*d^4*f^2*x^4*\log(\operatorname{abs}(F))^4 - 60*\pi^2*b^3*d^3*f^2*x^3*\log(\operatorname{abs}(F))*\operatorname{sgn}(F) + 60*\pi^2*b^3*d^3*f^2*x^3*\log(\operatorname{abs}(F)) - 40*b^3*d^3*f^2*x^3*\log(\operatorname{abs}(F))^3 + 60*\pi^2*b^2*d^2*f^2*x^2*\operatorname{sgn}(F) - 60*\pi^2*b^2*d^2*f^2*x^2 + 120*b^2*d^2*f^2*x^2*\log(\operatorname{abs}(F))^2 - 240*b*d*f^2*x*\log(\operatorname{abs}(F)) + 240*f^2)*(3*\pi^5*b^6*d^6*\log(\operatorname{abs}(F))*\operatorname{sgn}(F) - 10*\pi^3*b^6*d^6*\log(\operatorname{abs}(F))^3*\operatorname{sgn}(F) + 3*\pi*
\end{aligned}$$

$$\begin{aligned}
& b^6 d^6 \log(\operatorname{abs}(F))^5 \operatorname{sgn}(F) - 3 \pi^5 b^6 d^6 \log(\operatorname{abs}(F)) + 10 \pi^3 b^6 d^6 \\
& * \log(\operatorname{abs}(F))^3 - 3 \pi b^6 d^6 \log(\operatorname{abs}(F))^5 / ((\pi^6 b^6 d^6 \operatorname{sgn}(F) - 15 \pi^4 \\
& b^6 d^6 \log(\operatorname{abs}(F))^2 \operatorname{sgn}(F) + 15 \pi^2 b^6 d^6 \log(\operatorname{abs}(F))^4 \operatorname{sgn}(F) - \pi^6 \\
& b^6 d^6 + 15 \pi^4 b^6 d^6 \log(\operatorname{abs}(F))^2 - 15 \pi^2 b^6 d^6 \log(\operatorname{abs}(F))^4 + \\
& 2 b^6 d^6 \log(\operatorname{abs}(F))^6)^2 + 4 * (3 \pi^5 b^6 d^6 \log(\operatorname{abs}(F)) * \operatorname{sgn}(F) - 10 \pi^3 \\
& b^6 d^6 \log(\operatorname{abs}(F))^3 \operatorname{sgn}(F) + 3 \pi b^6 d^6 \log(\operatorname{abs}(F))^5 \operatorname{sgn}(F) - 3 \pi^5 \\
& b^6 d^6 \log(\operatorname{abs}(F)) + 10 \pi^3 b^6 d^6 \log(\operatorname{abs}(F))^3 - 3 \pi b^6 d^6 \log(\operatorname{abs}(F))^5)^2) * \sin(-1/2 \pi b^6 d^6 \operatorname{sgn}(F) + 1/2 \pi b^6 d^6 \\
& - 1/2 \pi b^6 c - 1/2 \pi a \operatorname{sgn}(F) + 1/2 \pi a)) * e^{(b^6 d^6 \log(\operatorname{abs}(F)) + b^6 c \log(\operatorname{abs}(F)) + a \log(\operatorname{abs}(F)))} - 1/2 I * ((32 \pi^5 b^5 d^5 f^2 x^5 \operatorname{sgn}(F) + 160 I \pi^4 \\
& b^5 d^5 f^2 x^5 \log(\operatorname{abs}(F)) * \operatorname{sgn}(F) - 320 \pi^3 b^5 d^5 f^2 x^5 \log(\operatorname{abs}(F))^2 \\
& * \operatorname{sgn}(F) - 320 I \pi^2 b^5 d^5 f^2 x^5 \log(\operatorname{abs}(F))^3 * \operatorname{sgn}(F) + 160 \pi b^5 d^5 \\
& f^2 x^5 \log(\operatorname{abs}(F))^4 * \operatorname{sgn}(F) - 32 \pi^5 b^5 d^5 f^2 x^5 - 160 I \pi^4 b^5 d^5 \\
& f^2 x^5 \log(\operatorname{abs}(F)) + 320 \pi^3 b^5 d^5 f^2 x^5 \log(\operatorname{abs}(F))^2 + 320 I \pi^2 \\
& b^5 d^5 f^2 x^5 \log(\operatorname{abs}(F))^3 - 160 \pi b^5 d^5 f^2 x^5 \log(\operatorname{abs}(F))^4 - 64 I \\
& b^5 d^5 f^2 x^5 \log(\operatorname{abs}(F))^5 - 160 I \pi^4 b^4 d^4 f^2 x^4 * \operatorname{sgn}(F) + 640 \pi \\
& i^3 b^4 d^4 f^2 x^4 \log(\operatorname{abs}(F)) * \operatorname{sgn}(F) + 960 I \pi^2 b^4 d^4 f^2 x^4 \log(\operatorname{abs}(F))^2 \\
& * \operatorname{sgn}(F) - 640 \pi b^4 d^4 f^2 x^4 \log(\operatorname{abs}(F))^3 * \operatorname{sgn}(F) + 160 I \pi^4 b^4 \\
& d^4 f^2 x^4 - 640 \pi^3 b^4 d^4 f^2 x^4 \log(\operatorname{abs}(F)) - 960 I \pi^2 b^4 d^4 f^2 \\
& x^4 \log(\operatorname{abs}(F))^2 + 640 \pi b^4 d^4 f^2 x^4 \log(\operatorname{abs}(F))^3 + 320 I b^4 d^4 \\
& f^2 x^4 \log(\operatorname{abs}(F))^4 - 640 \pi^3 b^3 d^3 f^2 x^3 * \operatorname{sgn}(F) - 1920 I \pi^2 b^3 d^3 \\
& f^2 x^3 \log(\operatorname{abs}(F)) * \operatorname{sgn}(F) + 1920 \pi b^3 d^3 f^2 x^3 \log(\operatorname{abs}(F))^2 * \operatorname{sgn}(F) \\
& + 640 \pi^3 b^3 d^3 f^2 x^3 + 1920 I \pi^2 b^3 d^3 f^2 x^3 \log(\operatorname{abs}(F)) - 1 \\
& 920 \pi b^3 d^3 f^2 x^3 \log(\operatorname{abs}(F))^2 - 1280 I b^3 d^3 f^2 x^3 \log(\operatorname{abs}(F))^3 \\
& + 1920 I \pi^2 b^2 d^2 f^2 x^2 * \operatorname{sgn}(F) - 3840 \pi b^2 d^2 f^2 x^2 \log(\operatorname{abs}(F)) \\
& * \operatorname{sgn}(F) - 1920 I \pi^2 b^2 d^2 f^2 x^2 + 3840 \pi b^2 d^2 f^2 x^2 \log(\operatorname{abs}(F)) \\
& + 3840 I b^2 d^2 f^2 x^2 \log(\operatorname{abs}(F))^2 + 3840 \pi b^2 d^2 f^2 x^2 * \operatorname{sgn}(F) - 3840 \pi \\
& i b^2 d^2 f^2 x^2 - 7680 I b^2 d^2 f^2 x^2 \log(\operatorname{abs}(F)) + 7680 I f^2) * e^{(1/2 I \pi b^2 d^2 x^2 \\
& \operatorname{sgn}(F) - 1/2 I \pi b^2 d^2 x + 1/2 I \pi b^2 c * \operatorname{sgn}(F) - 1/2 I \pi b^2 c + 1/2 I \pi a * \operatorname{sgn}(F) \\
& - 1/2 I \pi a) / (32 \pi^6 b^6 d^6 \operatorname{sgn}(F) + 192 I \pi^5 b^6 d^6 \log(\operatorname{abs}(F)) * \operatorname{sgn}(F) - 480 \pi^4 \\
& b^6 d^6 \log(\operatorname{abs}(F))^2 * \operatorname{sgn}(F) - 640 I \pi^3 b^6 d^6 \log(\operatorname{abs}(F))^3 * \operatorname{sgn}(F) + 480 \pi^2 \\
& b^6 d^6 \log(\operatorname{abs}(F))^4 * \operatorname{sgn}(F) + 192 I \pi b^6 d^6 \log(\operatorname{abs}(F))^5 * \operatorname{sgn}(F) - 32 \pi^6 b^6 d^6 \\
& - 192 I \pi^5 b^6 d^6 \log(\operatorname{abs}(F)) + 480 \pi^4 b^6 d^6 \log(\operatorname{abs}(F))^2 + 640 I \pi^3 b^6 d^6 \log(\operatorname{abs}(F))^3 \\
& - 480 \pi^2 b^6 d^6 \log(\operatorname{abs}(F))^4 - 192 I \pi b^6 d^6 \log(\operatorname{abs}(F))^5 + 64 b^6 d^6 \log(\operatorname{abs}(F))^6) + \\
& (32 \pi^5 b^5 d^5 f^2 x^5 \operatorname{sgn}(F) - 160 I \pi^4 b^5 d^5 f^2 x^5 \log(\operatorname{abs}(F)) * \operatorname{sgn}(F) - 320 \pi^3 \\
& b^5 d^5 f^2 x^5 \log(\operatorname{abs}(F))^2 * \operatorname{sgn}(F) + 320 I \pi^2 b^5 d^5 f^2 x^5 \log(\operatorname{abs}(F))^3 * \operatorname{sgn}(F) + \\
& 160 \pi b^5 d^5 f^2 x^5 \log(\operatorname{abs}(F))^4 * \operatorname{sgn}(F) - 32 \pi^5 b^5 d^5 f^2 x^5 + 160 I \pi^4 b^5 d^5 f^2 x^5 \\
& \log(\operatorname{abs}(F)) + 320 \pi^3 b^5 d^5 f^2 x^5 \log(\operatorname{abs}(F))^2 - 320 I \pi^2 b^5 d^5 f^2 x^5 \log(\operatorname{abs}(F))^3 \\
& - 160 \pi b^5 d^5 f^2 x^5 \log(\operatorname{abs}(F))^4 + 64 I b^5 d^5 f^2 x^5 \log(\operatorname{abs}(F))^5 + 160 I \pi^4 b^4 d^4 \\
& f^2 x^4 * \operatorname{sgn}(F) + 640 \pi^3 b^4 d^4 f^2 x^4 \log(\operatorname{abs}(F)) * \operatorname{sgn}(F) - 960 I \pi^2 b^4 d^4 f^2 x^4 \\
& \log(\operatorname{abs}(F))^2 * \operatorname{sgn}(F) - 640 \pi b^4 d^4 f^2 x^4 \log(\operatorname{abs}(F))^3 * \operatorname{sgn}(F) - 160 I \pi^4 b^4 d^4 f^2 x^4 \\
& - 640 \pi^3 b^4 d^4 f^2 x^4 \log(\operatorname{abs}(F)) + 960 I \pi^2 b^4 d^4 f^2 x^4 \log(\operatorname{abs}(F))^2
\end{aligned}$$

$$\begin{aligned}
& + 640\pi b^4 d^4 f^2 x^4 \log(\operatorname{abs}(F))^3 - 320I\pi b^4 d^4 f^2 x^4 \log(\operatorname{abs}(F)) \\
& ^4 - 640\pi^3 b^3 d^3 f^2 x^3 \operatorname{sgn}(F) + 1920I\pi^2 b^3 d^3 f^2 x^3 \log(\operatorname{abs}(F)) \operatorname{sgn}(F) + 1920\pi^3 b^3 d^3 f^2 x^3 \log(\operatorname{abs}(F))^2 \operatorname{sgn}(F) + 640\pi^3 b^3 d^3 f^2 x^3 \\
& - 1920I\pi^2 b^3 d^3 f^2 x^3 \log(\operatorname{abs}(F)) - 1920\pi b^3 d^3 f^2 x^3 \log(\operatorname{abs}(F))^2 + 1280I\pi b^3 d^3 f^2 x^3 \log(\operatorname{abs}(F))^3 - 1920I\pi^2 b^2 d^2 f^2 x^2 \operatorname{sgn}(F) - 3840\pi b^2 d^2 f^2 x^2 \log(\operatorname{abs}(F)) \operatorname{sgn}(F) + 1920I\pi^2 b^2 d^2 f^2 x^2 \\
& + 3840\pi b^2 d^2 f^2 x^2 \log(\operatorname{abs}(F)) - 3840I\pi b^2 d^2 f^2 x^2 \log(\operatorname{abs}(F))^2 + 3840\pi b d f^2 x \operatorname{sgn}(F) - 3840\pi b d f^2 x + 7680I \\
& * b d f^2 x \log(\operatorname{abs}(F)) - 7680I f^2 * e^{(-1/2I\pi b d x \operatorname{sgn}(F) + 1/2I\pi b d x - 1/2I\pi b c \operatorname{sgn}(F) + 1/2I\pi b c - 1/2I\pi a \operatorname{sgn}(F) + 1/2I\pi a)} \\
& / (32\pi^6 b^6 d^6 \operatorname{sgn}(F) - 192I\pi^5 b^6 d^6 \log(\operatorname{abs}(F)) \operatorname{sgn}(F) - 480\pi^4 b^6 d^6 \log(\operatorname{abs}(F))^2 \operatorname{sgn}(F) + 640I\pi^3 b^6 d^6 \log(\operatorname{abs}(F))^3 \operatorname{sgn}(F) + 4 \\
& 80\pi^2 b^6 d^6 \log(\operatorname{abs}(F))^4 \operatorname{sgn}(F) - 192I\pi b^6 d^6 \log(\operatorname{abs}(F))^5 \operatorname{sgn}(F) - 32\pi^6 b^6 d^6 + 192I\pi^5 b^6 d^6 \log(\operatorname{abs}(F)) + 480\pi^4 b^6 d^6 \log(\operatorname{abs}(F))^2 \\
& - 640I\pi^3 b^6 d^6 \log(\operatorname{abs}(F))^3 - 480\pi^2 b^6 d^6 \log(\operatorname{abs}(F))^4 + 192I\pi b^6 d^6 \log(\operatorname{abs}(F))^5 + 64 b^6 d^6 \log(\operatorname{abs}(F))^6) * e^{(b d x \log(\operatorname{abs}(F)) + b c \log(\operatorname{abs}(F)) + a \log(\operatorname{abs}(F)))}
\end{aligned}$$

### 3.66 $\int F^{a+b(c+dx)} x^2 (e + fx)^2 dx$

**Optimal.** Leaf size=328

$$-\frac{2e^2 x F^{a+bc+bdx}}{b^2 d^2 \log^2(F)} + \frac{2e^2 F^{a+bc+bdx}}{b^3 d^3 \log^3(F)} - \frac{6efx^2 F^{a+bc+bdx}}{b^2 d^2 \log^2(F)} + \frac{12efx F^{a+bc+bdx}}{b^3 d^3 \log^3(F)} - \frac{12ef F^{a+bc+bdx}}{b^4 d^4 \log^4(F)} - \frac{4f^2 x^3 F^{a+bc+bdx}}{b^2 d^2 \log^2(F)} + \frac{12f^2 x^2 F^{a+bc+bdx}}{b^3 d^3 \log^3(F)}$$

[Out]  $(24*f^2*F^{(a + b*c + b*d*x)})/(b^5*d^5*Log[F]^5) - (12*e*f*F^{(a + b*c + b*d*x)})/(b^4*d^4*Log[F]^4) - (24*f^2*F^{(a + b*c + b*d*x)*x})/(b^4*d^4*Log[F]^4) + (2*e^2*F^{(a + b*c + b*d*x)})/(b^3*d^3*Log[F]^3) + (12*e*f*F^{(a + b*c + b*d*x)*x})/(b^3*d^3*Log[F]^3) + (12*f^2*F^{(a + b*c + b*d*x)*x^2})/(b^3*d^3*Log[F]^3) - (2*e^2*F^{(a + b*c + b*d*x)*x})/(b^2*d^2*Log[F]^2) - (6*e*f*F^{(a + b*c + b*d*x)*x^2})/(b^2*d^2*Log[F]^2) - (4*f^2*F^{(a + b*c + b*d*x)*x^3})/(b^2*d^2*Log[F]^2) + (e^2*F^{(a + b*c + b*d*x)*x^2})/(b*d*Log[F]) + (2*e*f*F^{(a + b*c + b*d*x)*x^3})/(b*d*Log[F]) + (f^2*F^{(a + b*c + b*d*x)*x^4})/(b*d*Log[F])$

**Rubi [A]** time = 0.53245, antiderivative size = 328, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 3, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$ , Rules used = {2196, 2176, 2194}

$$-\frac{2e^2 x F^{a+bc+bdx}}{b^2 d^2 \log^2(F)} + \frac{2e^2 F^{a+bc+bdx}}{b^3 d^3 \log^3(F)} - \frac{6efx^2 F^{a+bc+bdx}}{b^2 d^2 \log^2(F)} + \frac{12efx F^{a+bc+bdx}}{b^3 d^3 \log^3(F)} - \frac{12ef F^{a+bc+bdx}}{b^4 d^4 \log^4(F)} - \frac{4f^2 x^3 F^{a+bc+bdx}}{b^2 d^2 \log^2(F)} + \frac{12f^2 x^2 F^{a+bc+bdx}}{b^3 d^3 \log^3(F)}$$

Antiderivative was successfully verified.

[In] Int[F^(a + b\*(c + d\*x))\*x^2\*(e + f\*x)^2,x]

[Out]  $(24*f^2*F^{(a + b*c + b*d*x)})/(b^5*d^5*Log[F]^5) - (12*e*f*F^{(a + b*c + b*d*x)})/(b^4*d^4*Log[F]^4) - (24*f^2*F^{(a + b*c + b*d*x)*x})/(b^4*d^4*Log[F]^4) + (2*e^2*F^{(a + b*c + b*d*x)})/(b^3*d^3*Log[F]^3) + (12*e*f*F^{(a + b*c + b*d*x)*x})/(b^3*d^3*Log[F]^3) + (12*f^2*F^{(a + b*c + b*d*x)*x^2})/(b^3*d^3*Log[F]^3) - (2*e^2*F^{(a + b*c + b*d*x)*x})/(b^2*d^2*Log[F]^2) - (6*e*f*F^{(a + b*c + b*d*x)*x^2})/(b^2*d^2*Log[F]^2) - (4*f^2*F^{(a + b*c + b*d*x)*x^3})/(b^2*d^2*Log[F]^2) + (e^2*F^{(a + b*c + b*d*x)*x^2})/(b*d*Log[F]) + (2*e*f*F^{(a + b*c + b*d*x)*x^3})/(b*d*Log[F]) + (f^2*F^{(a + b*c + b*d*x)*x^4})/(b*d*Log[F])$

#### Rule 2196

Int[(F\_)^((c\_.)\*(v\_))\*(u\_), x\_Symbol] := Int[ExpandIntegrand[F^(c\*ExpandToSum[v, x]), u, x], x] /; FreeQ[{F, c}, x] && PolynomialQ[u, x] && LinearQ[v, x] && !\$UseGamma == True

Rule 2176

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] := Simp[((c + d*x)^m*(b*F^(g*(e + f*x)))^n)/(f*g*n*Log[F]),
x] - Dist[(d*m)/(f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] && !$UseGamma === True
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] := Simp[(F^(c*(a +
b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned}
\int F^{a+b(c+dx)} x^2 (e+fx)^2 dx &= \int (e^2 F^{a+bc+bdx} x^2 + 2ef F^{a+bc+bdx} x^3 + f^2 F^{a+bc+bdx} x^4) dx \\
&= e^2 \int F^{a+bc+bdx} x^2 dx + (2ef) \int F^{a+bc+bdx} x^3 dx + f^2 \int F^{a+bc+bdx} x^4 dx \\
&= \frac{e^2 F^{a+bc+bdx} x^2}{bd \log(F)} + \frac{2ef F^{a+bc+bdx} x^3}{bd \log(F)} + \frac{f^2 F^{a+bc+bdx} x^4}{bd \log(F)} - \frac{(2e^2) \int F^{a+bc+bdx} x dx}{bd \log(F)} - \frac{(6ef) \int F^{a+bc+bdx} x^2 dx}{bd \log(F)} \\
&= -\frac{2e^2 F^{a+bc+bdx} x}{b^2 d^2 \log^2(F)} - \frac{6ef F^{a+bc+bdx} x^2}{b^2 d^2 \log^2(F)} - \frac{4f^2 F^{a+bc+bdx} x^3}{b^2 d^2 \log^2(F)} + \frac{e^2 F^{a+bc+bdx} x^2}{bd \log(F)} + \frac{2ef F^{a+bc+bdx} x^3}{bd \log(F)} + \frac{f^2 F^{a+bc+bdx} x^4}{bd \log(F)} \\
&= \frac{2e^2 F^{a+bc+bdx}}{b^3 d^3 \log^3(F)} + \frac{12ef F^{a+bc+bdx} x}{b^3 d^3 \log^3(F)} + \frac{12f^2 F^{a+bc+bdx} x^2}{b^3 d^3 \log^3(F)} - \frac{2e^2 F^{a+bc+bdx} x}{b^2 d^2 \log^2(F)} - \frac{6ef F^{a+bc+bdx} x^2}{b^2 d^2 \log^2(F)} - \frac{4f^2 F^{a+bc+bdx} x^3}{b^2 d^2 \log^2(F)} \\
&= -\frac{12ef F^{a+bc+bdx}}{b^4 d^4 \log^4(F)} - \frac{24f^2 F^{a+bc+bdx} x}{b^4 d^4 \log^4(F)} + \frac{2e^2 F^{a+bc+bdx}}{b^3 d^3 \log^3(F)} + \frac{12ef F^{a+bc+bdx} x}{b^3 d^3 \log^3(F)} + \frac{12f^2 F^{a+bc+bdx} x^2}{b^3 d^3 \log^3(F)} - \frac{2e^2 F^{a+bc+bdx} x^2}{b^3 d^3 \log^3(F)} \\
&= \frac{24f^2 F^{a+bc+bdx}}{b^5 d^5 \log^5(F)} - \frac{12ef F^{a+bc+bdx}}{b^4 d^4 \log^4(F)} - \frac{24f^2 F^{a+bc+bdx} x}{b^4 d^4 \log^4(F)} + \frac{2e^2 F^{a+bc+bdx}}{b^3 d^3 \log^3(F)} + \frac{12ef F^{a+bc+bdx} x}{b^3 d^3 \log^3(F)} + \frac{12f^2 F^{a+bc+bdx} x^2}{b^3 d^3 \log^3(F)} - \frac{2e^2 F^{a+bc+bdx} x^2}{b^3 d^3 \log^3(F)}
\end{aligned}$$

**Mathematica [A]** time = 0.237664, size = 121, normalized size = 0.37

$$\frac{F^{a+b(c+dx)} \left( -2b^3 d^3 x \log^3(F) (e^2 + 3efx + 2f^2 x^2) + 2b^2 d^2 \log^2(F) (e^2 + 6efx + 6f^2 x^2) + b^4 d^4 x^2 \log^4(F) (e + fx)^2 - 12bdf \right)}{b^5 d^5 \log^5(F)}$$

Antiderivative was successfully verified.

```
[In] Integrate[F^(a + b*(c + d*x))*x^2*(e + f*x)^2,x]
```

[Out]  $(F^{(a + b*(c + d*x))} * (24*f^2 - 12*b*d*f*(e + 2*f*x) * \text{Log}[F] + 2*b^2*d^2*(e^2 + 6*e*f*x + 6*f^2*x^2) * \text{Log}[F]^2 - 2*b^3*d^3*x*(e^2 + 3*e*f*x + 2*f^2*x^2) * \text{Log}[F]^3 + b^4*d^4*x^2*(e + f*x)^2 * \text{Log}[F]^4)) / (b^5*d^5 * \text{Log}[F]^5)$

**Maple [A]** time = 0.007, size = 197, normalized size = 0.6

$$\frac{((\ln(F))^4 b^4 d^4 f^2 x^4 + 2 (\ln(F))^4 b^4 d^4 e f x^3 + (\ln(F))^4 b^4 d^4 e^2 x^2 - 4 (\ln(F))^3 b^3 d^3 f^2 x^3 - 6 (\ln(F))^3 b^3 d^3 e f x^2 - 2 (\ln(F))^3 b^3 d^3 e^2 x - 2 (\ln(F))^2 b^2 d^2 f^2 x^4 + 2 (\ln(F))^2 b^2 d^2 e f x^3 + (\ln(F))^2 b^2 d^2 e^2 x^2 - 4 (\ln(F))^2 b^2 d^2 f^2 x^3 - 6 (\ln(F))^2 b^2 d^2 e f x^2 - 2 (\ln(F))^2 b^2 d^2 e^2 x - 24 (\ln(F)) b^2 d^2 f^2 x^2 - 12 (\ln(F)) b^2 d^2 e f x - 12 (\ln(F)) b^2 d^2 e^2 x - 24 (\ln(F)) b^2 d^2 f^2 x - 12 (\ln(F)) b^2 d^2 e f x - 12 (\ln(F)) b^2 d^2 e^2 x) F^{(b*d*x+b*c+a)} / \ln(F)^5 / b^5 / d^5$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}(F^{(a+b*(d*x+c))} * x^2 * (f*x+e)^2, x)$

[Out]  $(\ln(F)^4 * b^4 * d^4 * f^2 * x^4 + 2 * \ln(F)^4 * b^4 * d^4 * e * f * x^3 + \ln(F)^4 * b^4 * d^4 * e^2 * x^2 - 4 * \ln(F)^3 * b^3 * d^3 * f^2 * x^3 - 6 * \ln(F)^3 * b^3 * d^3 * e * f * x^2 - 2 * \ln(F)^3 * b^3 * d^3 * e^2 * x - 12 * \ln(F)^2 * b^2 * d^2 * f^2 * x^4 + 2 * \ln(F)^2 * b^2 * d^2 * e * f * x^3 + (\ln(F))^2 * b^2 * d^2 * e^2 * x^2 - 4 * (\ln(F))^2 * b^2 * d^2 * f^2 * x^3 - 6 * (\ln(F))^2 * b^2 * d^2 * e * f * x^2 - 2 * (\ln(F))^2 * b^2 * d^2 * e^2 * x - 24 * (\ln(F)) * b^2 * d^2 * f^2 * x^2 - 12 * (\ln(F)) * b^2 * d^2 * e * f * x - 12 * (\ln(F)) * b^2 * d^2 * e^2 * x - 24 * (\ln(F)) * b^2 * d^2 * f^2 * x - 12 * (\ln(F)) * b^2 * d^2 * e * f * x - 12 * (\ln(F)) * b^2 * d^2 * e^2 * x) * F^{(b*d*x+b*c+a)} / \ln(F)^5 / b^5 / d^5$

**Maxima [A]** time = 1.04444, size = 354, normalized size = 1.08

$$\frac{(F^{bc+a} b^2 d^2 x^2 \log(F)^2 - 2 F^{bc+a} b d x \log(F) + 2 F^{bc+a}) F^{bdx} e^2}{b^3 d^3 \log(F)^3} + \frac{2 (F^{bc+a} b^3 d^3 x^3 \log(F)^3 - 3 F^{bc+a} b^2 d^2 x^2 \log(F)^2 + 6 F^{bc+a} b d x \log(F) - 6 F^{bc+a}) F^{bdx} e^2}{b^4 d^4 \log(F)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}(F^{(a+b*(d*x+c))} * x^2 * (f*x+e)^2, x, \text{algorithm}="maxima")$

[Out]  $(F^{(b*c + a)} * b^2 * d^2 * x^2 * \log(F)^2 - 2 * F^{(b*c + a)} * b * d * x * \log(F) + 2 * F^{(b*c + a)}) * F^{(b*d*x)} * e^2 / (b^3 * d^3 * \log(F)^3) + 2 * (F^{(b*c + a)} * b^3 * d^3 * x^3 * \log(F)^3 - 3 * F^{(b*c + a)} * b^2 * d^2 * x^2 * \log(F)^2 + 6 * F^{(b*c + a)} * b * d * x * \log(F) - 6 * F^{(b*c + a)}) * F^{(b*d*x)} * e^2 / (b^4 * d^4 * \log(F)^4) + (F^{(b*c + a)} * b^4 * d^4 * x^4 * \log(F)^4 - 4 * F^{(b*c + a)} * b^3 * d^3 * x^3 * \log(F)^3 + 12 * F^{(b*c + a)} * b^2 * d^2 * x^2 * \log(F)^2 - 24 * F^{(b*c + a)} * b * d * x * \log(F) + 24 * F^{(b*c + a)}) * F^{(b*d*x)} * e^2 / (b^5 * d^5 * \log(F)^5)$

**Fricas [A]** time = 1.48157, size = 386, normalized size = 1.18

$$\frac{((b^4 d^4 f^2 x^4 + 2 b^4 d^4 e f x^3 + b^4 d^4 e^2 x^2) \log(F)^4 - 2 (2 b^3 d^3 f^2 x^3 + 3 b^3 d^3 e f x^2 + b^3 d^3 e^2 x) \log(F)^3 + 2 (6 b^2 d^2 f^2 x^2 + 6 b^2 d^2 e f x + 6 b^2 d^2 e^2 x) \log(F)^2 - 24 b^2 d^2 f^2 x^2 - 24 b^2 d^2 e f x - 24 b^2 d^2 e^2 x) F^{(b*d*x+b*c+a)} / \ln(F)^5 / b^5 / d^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(a+b*(d*x+c))*x^2*(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] ((b^4*d^4*f^2*x^4 + 2*b^4*d^4*e*f*x^3 + b^4*d^4*e^2*x^2)*log(F)^4 - 2*(2*b^3*d^3*f^2*x^3 + 3*b^3*d^3*e*f*x^2 + b^3*d^3*e^2*x)*log(F)^3 + 2*(6*b^2*d^2*f^2*x^2 + 6*b^2*d^2*e*f*x + b^2*d^2*e^2)*log(F)^2 + 24*f^2 - 12*(2*b*d*f^2*x + b*d*e*f)*log(F))*F^(b*d*x + b*c + a)/(b^5*d^5*log(F)^5)
```

**Sympy [A]** time = 0.194154, size = 260, normalized size = 0.79

$$\left\{ \frac{F^{a+b(c+dx)}(b^4 d^4 e^2 x^2 \log(F)^4 + 2b^4 d^4 e f x^3 \log(F)^4 + b^4 d^4 f^2 x^4 \log(F)^4 - 2b^3 d^3 e^2 x \log(F)^3 - 6b^3 d^3 e f x^2 \log(F)^3 - 4b^3 d^3 f^2 x^3 \log(F)^3 + 2b^2 d^2 e^2 \log(F)^2 + 12b^2 d^2 e f x \log(F)^2 + 24f^2 \log(F)^2 - 12(2bd f^2 x + bde f) \log(F)) F^{b d x + b c + a}}{b^5 d^5 \log(F)^5} \right\}$$

$$\left\{ \frac{e^2 x^3}{3} + \frac{e f x^4}{2} + \frac{f^2 x^5}{5} \right\}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(a+b*(d*x+c))*x**2*(f*x+e)**2,x)
```

```
[Out] Piecewise((F**(a + b*(c + d*x))*(b**4*d**4*e**2*x**2*log(F)**4 + 2*b**4*d**4*e*f*x**3*log(F)**4 + b**4*d**4*f**2*x**4*log(F)**4 - 2*b**3*d**3*e**2*x*log(F)**3 - 6*b**3*d**3*e*f*x**2*log(F)**3 - 4*b**3*d**3*f**2*x**3*log(F)**3 + 2*b**2*d**2*e**2*log(F)**2 + 12*b**2*d**2*e*f*x*log(F)**2 + 12*b**2*d**2*f**2*x**2*log(F)**2 - 12*b*d*e*f*log(F) - 24*b*d*f**2*x*log(F) + 24*f**2)/(b**5*d**5*log(F)**5), Ne(b**5*d**5*log(F)**5, 0)), (e**2*x**3/3 + e*f*x**4/2 + f**2*x**5/5, True))
```

**Giac [C]** time = 2.00664, size = 9532, normalized size = 29.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(a+b*(d*x+c))*x^2*(f*x+e)^2,x, algorithm="giac")
```

```
[Out] (((3*pi^2*b^3*d^3*log(abs(F))*sgn(F) - 3*pi^2*b^3*d^3*log(abs(F)) + 2*b^3*d^3*log(abs(F))^3)*(pi^2*b^2*d^2*x^2*sgn(F) - pi^2*b^2*d^2*x^2 + 2*b^2*d^2*x^2*log(abs(F))^2 - 4*b*d*x*log(abs(F)) + 4)/((pi^3*b^3*d^3*sgn(F) - 3*pi*b^3*d^3*log(abs(F))^2*sgn(F) - pi^3*b^3*d^3 + 3*pi*b^3*d^3*log(abs(F))^2)^2 +
```



$$\begin{aligned}
& (3\pi^2 b^3 d^3 \log(\text{abs}(F)) \text{sgn}(F) - 3\pi^2 b^3 d^3 \log(\text{abs}(F)) + 2b^3 d^3 \log(\text{abs}(F))^3)^2 - 2(\pi^3 b^3 d^3 \text{sgn}(F) - 3\pi b^3 d^3 \log(\text{abs}(F))^2 \text{sgn}(F) - \pi^3 b^3 d^3 + 3\pi b^3 d^3 \log(\text{abs}(F))^2) (\pi b^2 d^2 x^2 \log(\text{abs}(F)) \text{sgn}(F) - \pi b^2 d^2 x^2 \log(\text{abs}(F)) - \pi b d x \text{sgn}(F) + \pi b d x) / ((\pi^3 b^3 d^3 \text{sgn}(F) - 3\pi b^3 d^3 \log(\text{abs}(F))^2 \text{sgn}(F) - \pi^3 b^3 d^3 + 3\pi b^3 d^3 \log(\text{abs}(F))^2)^2 + (3\pi^2 b^3 d^3 \log(\text{abs}(F)) \text{sgn}(F) - 3\pi^2 b^3 d^3 \log(\text{abs}(F)) + 2b^3 d^3 \log(\text{abs}(F))^3)^2) \cos(-1/2 \pi b d x \text{sgn}(F) + 1/2 \pi b d x - 1/2 \pi b c \text{sgn}(F) + 1/2 \pi b c - 1/2 \pi a \text{sgn}(F) + 1/2 \pi a) \\
& + ((\pi^3 b^3 d^3 \text{sgn}(F) - 3\pi b^3 d^3 \log(\text{abs}(F))^2 \text{sgn}(F) - \pi^3 b^3 d^3 + 3\pi b^3 d^3 \log(\text{abs}(F))^2) (\pi^2 b^2 d^2 x^2 \text{sgn}(F) - \pi^2 b^2 d^2 x^2 + 2b^2 d^2 x^2 \log(\text{abs}(F))^2 - 4b d x \log(\text{abs}(F)) + 4) / ((\pi^3 b^3 d^3 \text{sgn}(F) - 3\pi b^3 d^3 \log(\text{abs}(F))^2 \text{sgn}(F) - \pi^3 b^3 d^3 + 3\pi b^3 d^3 \log(\text{abs}(F))^2)^2 + (3\pi^2 b^3 d^3 \log(\text{abs}(F)) \text{sgn}(F) - 3\pi^2 b^3 d^3 \log(\text{abs}(F)) + 2b^3 d^3 \log(\text{abs}(F))^3)^2) + 2(3\pi^2 b^3 d^3 \log(\text{abs}(F)) \text{sgn}(F) - 3\pi^2 b^3 d^3 \log(\text{abs}(F)) + 2b^3 d^3 \log(\text{abs}(F))^3) (\pi b^2 d^2 x^2 \log(\text{abs}(F)) \text{sgn}(F) - \pi b^2 d^2 x^2 \log(\text{abs}(F)) - \pi b d x \text{sgn}(F) + \pi b d x) / ((\pi^3 b^3 d^3 \text{sgn}(F) - 3\pi b^3 d^3 \log(\text{abs}(F))^2 \text{sgn}(F) - \pi^3 b^3 d^3 + 3\pi b^3 d^3 \log(\text{abs}(F))^2)^2 + (3\pi^2 b^3 d^3 \log(\text{abs}(F)) \text{sgn}(F) - 3\pi^2 b^3 d^3 \log(\text{abs}(F)) + 2b^3 d^3 \log(\text{abs}(F))^3)^2) \sin(-1/2 \pi b d x \text{sgn}(F) + 1/2 \pi b d x - 1/2 \pi b c \text{sgn}(F) + 1/2 \pi b c - 1/2 \pi a \text{sgn}(F) + 1/2 \pi a) \\
& ) e^{(b d x \log(\text{abs}(F)) + b c \log(\text{abs}(F)) + a \log(\text{abs}(F)) + 2) + 1/2 I ((4 I \pi^2 b^2 d^2 x^2 \text{sgn}(F) - 8\pi b^2 d^2 x^2 \log(\text{abs}(F)) \text{sgn}(F) - 4 I \pi^2 b^2 d^2 x^2 + 8\pi b^2 d^2 x^2 \log(\text{abs}(F)) + 8 I b^2 d^2 x^2 \log(\text{abs}(F))^2 + 8\pi b d x \text{sgn}(F) - 8\pi b d x - 16 I b d x \log(\text{abs}(F)) + 16 I) e^{(1/2 I \pi b d x \text{sgn}(F) - 1/2 I \pi b d x + 1/2 I \pi b c \text{sgn}(F) - 1/2 I \pi b c + 1/2 I \pi a \text{sgn}(F) - 1/2 I \pi a) / (-4 I \pi^3 b^3 d^3 \text{sgn}(F) + 12 \pi^2 b^3 d^3 \log(\text{abs}(F)) \text{sgn}(F) + 12 I \pi b^3 d^3 \log(\text{abs}(F))^2 \text{sgn}(F) + 4 I \pi^3 b^3 d^3 - 12 \pi^2 b^3 d^3 \log(\text{abs}(F)) - 12 I \pi b^3 d^3 \log(\text{abs}(F))^2 + 8 b^3 d^3 \log(\text{abs}(F))^3 - (4 I \pi^2 b^2 d^2 x^2 \text{sgn}(F) + 8\pi b^2 d^2 x^2 \log(\text{abs}(F)) \text{sgn}(F) - 4 I \pi^2 b^2 d^2 x^2 - 8\pi b^2 d^2 x^2 \log(\text{abs}(F)) + 8 I b^2 d^2 x^2 \log(\text{abs}(F))^2 - 8\pi b d x \text{sgn}(F) + 8\pi b d x - 16 I b d x \log(\text{abs}(F)) + 16 I) e^{(-1/2 I \pi b d x \text{sgn}(F) + 1/2 I \pi b d x - 1/2 I \pi b c \text{sgn}(F) + 1/2 I \pi b c - 1/2 I \pi a \text{sgn}(F) + 1/2 I \pi a) / (4 I \pi^3 b^3 d^3 \text{sgn}(F) + 12 \pi^2 b^3 d^3 \log(\text{abs}(F)) \text{sgn}(F) - 12 I \pi b^3 d^3 \log(\text{abs}(F))^2 \text{sgn}(F) - 4 I \pi^3 b^3 d^3 - 12 \pi^2 b^3 d^3 \log(\text{abs}(F)) + 12 I \pi b^3 d^3 \log(\text{abs}(F))^2 + 8 b^3 d^3 \log(\text{abs}(F))^3)} e^{(b d x \log(\text{abs}(F)) + b c \log(\text{abs}(F)) + a \log(\text{abs}(F)) + 2) - 2(((3\pi^2 b^3 d^3 f x^3 \log(\text{abs}(F)) \text{sgn}(F) - 3\pi^2 b^3 d^3 f x^3 \log(\text{abs}(F)) + 2b^3 d^3 f x^3 \log(\text{abs}(F))^3 - 3\pi^2 b^2 d^2 f x^2 \text{sgn}(F) + 3\pi^2 b^2 d^2 f x^2 - 6b^2 d^2 f x^2 \log(\text{abs}(F))^2 + 12 b d f x \log(\text{abs}(F)) - 12 f) (\pi^4 b^4 d^4 \text{sgn}(F) - 6\pi^2 b^4 d^4 \log(\text{abs}(F))^2 \text{sgn}(F) - \pi^4 b^4 d^4 + 6\pi^2 b^4 d^4 \log(\text{abs}(F))^2 - 2b^4 d^4 \log(\text{abs}(F))^4) / ((\pi^4 b^4 d^4 \text{sgn}(F) - 6\pi^2 b^4 d^4 \log(\text{abs}(F))^2 \text{sgn}(F) - \pi^4 b^4 d^4 + 6\pi^2 b^4 d^4 \log(\text{abs}(F))^2 - 2b^4 d^4 \log(\text{abs}(F))^4)^2 + 16(\pi^3 b^4 d^4 \log(\text{abs}(F)) \text{sgn}(F) - \pi b^4 d^4 \log(\text{abs}(F))^3 \text{sgn}(F) - \pi^3 b^4 d^4 \log(\text{abs}(F)) + \pi b^4 d^4 \log(\text{abs}(F))^3)^2) - 4(\pi^3 b^3 d^3 f x^3 \text{sgn}(
\end{aligned}$$

$$\begin{aligned}
& F) - 3\pi b^3 d^3 f x^3 \log(\text{abs}(F))^2 \text{sgn}(F) - \pi^3 b^3 d^3 f x^3 + 3\pi b^3 d^3 f x^3 \log(\text{abs}(F))^2 + 6\pi b^2 d^2 f x^2 \log(\text{abs}(F)) \text{sgn}(F) - 6\pi b^2 d^2 f x^2 \log(\text{abs}(F)) - 6\pi b^2 d^2 f x^2 \log(\text{abs}(F)) - 6\pi b^2 d^2 f x^2 \log(\text{abs}(F)) \text{sgn}(F) + 6\pi b^2 d^2 f x^2 \log(\text{abs}(F)) \text{sgn}(F) - \pi b^4 d^4 \log(\text{abs}(F))^3 \text{sgn}(F) - \pi^3 b^4 d^4 \log(\text{abs}(F)) + \pi b^4 d^4 \log(\text{abs}(F))^3 / ((\pi^4 b^4 d^4 \text{sgn}(F) - 6\pi^2 b^4 d^4 \log(\text{abs}(F))^2 \text{sgn}(F) - \pi^4 b^4 d^4 + 6\pi^2 b^4 d^4 \log(\text{abs}(F))^2 - 2b^4 d^4 \log(\text{abs}(F))^4)^2 + 16(\pi^3 b^4 d^4 \log(\text{abs}(F)) \text{sgn}(F) - \pi b^4 d^4 \log(\text{abs}(F))^3 \text{sgn}(F) - \pi^3 b^4 d^4 \log(\text{abs}(F)) + \pi b^4 d^4 \log(\text{abs}(F))^3)^2)) * \\
& \cos(-1/2\pi b^2 d^2 f x^2 \text{sgn}(F) + 1/2\pi b^2 d^2 f x^2 - 1/2\pi b^2 c \text{sgn}(F) + 1/2\pi b^2 c - 1/2\pi a \text{sgn}(F) + 1/2\pi a) - ((\pi^3 b^3 d^3 f x^3 \text{sgn}(F) - 3\pi b^3 d^3 f x^3 \log(\text{abs}(F))^2 \text{sgn}(F) - \pi^3 b^3 d^3 f x^3 + 3\pi b^3 d^3 f x^3 \log(\text{abs}(F))^2 + 6\pi b^2 d^2 f x^2 \log(\text{abs}(F)) \text{sgn}(F) - 6\pi b^2 d^2 f x^2 \log(\text{abs}(F)) - 6\pi b^2 d^2 f x^2 \log(\text{abs}(F)) \text{sgn}(F) + 6\pi b^2 d^2 f x^2 \log(\text{abs}(F)) \text{sgn}(F) * (\pi^4 b^4 d^4 \text{sgn}(F) - 6\pi^2 b^4 d^4 \log(\text{abs}(F))^2 \text{sgn}(F) - \pi^4 b^4 d^4 + 6\pi^2 b^4 d^4 \log(\text{abs}(F))^2 - 2b^4 d^4 \log(\text{abs}(F))^4)^2 + 16(\pi^3 b^4 d^4 \log(\text{abs}(F)) \text{sgn}(F) - \pi b^4 d^4 \log(\text{abs}(F))^3 \text{sgn}(F) - \pi^3 b^4 d^4 \log(\text{abs}(F)) + \pi b^4 d^4 \log(\text{abs}(F))^3)^2) + 4*(3\pi^2 b^3 d^3 f x^3 \log(\text{abs}(F)) \text{sgn}(F) - 3\pi^2 b^3 d^3 f x^3 \log(\text{abs}(F)) + 2b^3 d^3 f x^3 \log(\text{abs}(F))^3 - 3\pi^2 b^2 d^2 f x^2 \text{sgn}(F) + 3\pi^2 b^2 d^2 f x^2 - 6b^2 d^2 f x^2 \log(\text{abs}(F))^2 + 12b^2 d^2 f x^2 \log(\text{abs}(F)) - 12f)(\pi^3 b^4 d^4 \log(\text{abs}(F)) \text{sgn}(F) - \pi b^4 d^4 \log(\text{abs}(F))^3 \text{sgn}(F) - \pi^3 b^4 d^4 \log(\text{abs}(F)) + \pi b^4 d^4 \log(\text{abs}(F))^3) / ((\pi^4 b^4 d^4 \text{sgn}(F) - 6\pi^2 b^4 d^4 \log(\text{abs}(F))^2 \text{sgn}(F) - \pi^4 b^4 d^4 + 6\pi^2 b^4 d^4 \log(\text{abs}(F))^2 - 2b^4 d^4 \log(\text{abs}(F))^4)^2 + 16(\pi^3 b^4 d^4 \log(\text{abs}(F)) \text{sgn}(F) - \pi b^4 d^4 \log(\text{abs}(F))^3 \text{sgn}(F) - \pi^3 b^4 d^4 \log(\text{abs}(F)) + \pi b^4 d^4 \log(\text{abs}(F))^3)^2)) * \sin(-1/2\pi b^2 d^2 f x^2 \text{sgn}(F) + 1/2\pi b^2 d^2 f x^2 - 1/2\pi b^2 c \text{sgn}(F) + 1/2\pi b^2 c - 1/2\pi a \text{sgn}(F) + 1/2\pi a) * e^{(b^2 d^2 f x^2 \log(\text{abs}(F)) + b^2 c \log(\text{abs}(F)) + a \log(\text{abs}(F)) + 1) - 1/2 I * ((16\pi^3 b^3 d^3 f x^3 \text{sgn}(F) + 48 I \pi^2 b^3 d^3 f x^3 \log(\text{abs}(F)) \text{sgn}(F) - 48\pi b^3 d^3 f x^3 \log(\text{abs}(F))^2 \text{sgn}(F) - 16\pi^3 b^3 d^3 f x^3 - 48 I \pi^2 b^3 d^3 f x^3 \log(\text{abs}(F)) + 48\pi b^3 d^3 f x^3 \log(\text{abs}(F))^2 + 32 I b^3 d^3 f x^3 \log(\text{abs}(F))^3 - 48 I \pi^2 b^2 d^2 f x^2 \text{sgn}(F) + 96\pi b^2 d^2 f x^2 \log(\text{abs}(F)) \text{sgn}(F) + 48 I \pi^2 b^2 d^2 f x^2 - 96\pi b^2 d^2 f x^2 \log(\text{abs}(F)) - 96 I b^2 d^2 f x^2 \log(\text{abs}(F))^2 - 96\pi b^2 d^2 f x^2 \text{sgn}(F) + 96\pi b^2 d^2 f x^2 + 192 I b^2 d^2 f x^2 \log(\text{abs}(F)) - 192 I f) * e^{(1/2 I \pi b^2 d^2 f x^2 \text{sgn}(F) - 1/2 I \pi b^2 d^2 f x^2 + 1/2 I \pi b^2 c \text{sgn}(F) - 1/2 I \pi b^2 c + 1/2 I \pi a \text{sgn}(F) - 1/2 I \pi a) / (8\pi^4 b^4 d^4 \text{sgn}(F) + 32 I \pi^3 b^4 d^4 \log(\text{abs}(F)) \text{sgn}(F) - 48\pi^2 b^4 d^4 \log(\text{abs}(F))^2 \text{sgn}(F) - 32 I \pi b^4 d^4 \log(\text{abs}(F))^3 \text{sgn}(F) - 8\pi^4 b^4 d^4 - 32 I \pi^3 b^4 d^4 \log(\text{abs}(F)) + 48\pi^2 b^4 d^4 \log(\text{abs}(F))^2 + 32 I \pi b^4 d^4 \log(\text{abs}(F))^3 - 16b^4 d^4 \log(\text{abs}(F))^4) + (16\pi^3 b^3 d^3 f x^3 \text{sgn}(F) - 48 I \pi^2 b^3 d^3 f x^3 \log(\text{abs}(F)) \text{sgn}(F) - 48\pi b^3 d^3 f x^3 \log(\text{abs}(F))^2 \text{sgn}(F) - 16\pi^3 b^3 d^3 f x^3 + 48 I \pi^2 b^3 d^3 f x^3 \log(\text{abs}(F)) + 48\pi b^3 d^3 f x^3 \log(\text{abs}(F))^2 - 32 I b^3 d^3 f x^3 \log(\text{abs}(F))^3 + 48 I \pi^2 b^2 d^2 f x^2 \text{sgn}(F) + 96\pi b^2 d^2 f x^2 \log(\text{abs}(F)) \text{sgn}(F) - 48 I \pi^2 b^2 d^2 f x^2}
\end{aligned}$$



$$\begin{aligned}
& i^2 b^5 d^5 \log(\operatorname{abs}(F))^3 \operatorname{sgn}(F) - 5 \pi^4 b^5 d^5 \log(\operatorname{abs}(F)) + 10 \pi^2 b^5 \\
& d^5 \log(\operatorname{abs}(F))^3 - 2 b^5 d^5 \log(\operatorname{abs}(F))^5)^2 + 4 (\pi^3 b^4 d^4 f^2 x^4 * \\
& \log(\operatorname{abs}(F)) * \operatorname{sgn}(F) - \pi b^4 d^4 f^2 x^4 * \log(\operatorname{abs}(F))^3 * \operatorname{sgn}(F) - \pi^3 b^4 d^4 \\
& f^2 x^4 * \log(\operatorname{abs}(F)) + \pi b^4 d^4 f^2 x^4 * \log(\operatorname{abs}(F))^3 - \pi^3 b^3 d^3 f^2 * \\
& x^3 * \operatorname{sgn}(F) + 3 \pi b^3 d^3 f^2 x^3 * \log(\operatorname{abs}(F))^2 * \operatorname{sgn}(F) + \pi^3 b^3 d^3 f^2 x \\
& ^3 - 3 \pi b^3 d^3 f^2 x^3 * \log(\operatorname{abs}(F))^2 - 6 \pi b^2 d^2 f^2 x^2 * \log(\operatorname{abs}(F)) * \\
& \operatorname{sgn}(F) + 6 \pi b^2 d^2 f^2 x^2 * \log(\operatorname{abs}(F)) + 6 \pi b d f^2 x * \operatorname{sgn}(F) - 6 \pi b * \\
& d f^2 x) * (5 \pi^4 b^5 d^5 \log(\operatorname{abs}(F)) * \operatorname{sgn}(F) - 10 \pi^2 b^5 d^5 \log(\operatorname{abs}(F))^3 \\
& * \operatorname{sgn}(F) - 5 \pi^4 b^5 d^5 \log(\operatorname{abs}(F)) + 10 \pi^2 b^5 d^5 \log(\operatorname{abs}(F))^3 - 2 b^5 \\
& d^5 \log(\operatorname{abs}(F))^5) / ((\pi^5 b^5 d^5 * \operatorname{sgn}(F) - 10 \pi^3 b^5 d^5 \log(\operatorname{abs}(F))^2 * \\
& \operatorname{sgn}(F) + 5 \pi b^5 d^5 \log(\operatorname{abs}(F))^4 * \operatorname{sgn}(F) - \pi^5 b^5 d^5 + 10 \pi^3 b^5 d^5 \\
& * \log(\operatorname{abs}(F))^2 - 5 \pi b^5 d^5 \log(\operatorname{abs}(F))^4)^2 + (5 \pi^4 b^5 d^5 \log(\operatorname{abs}(F)) \\
& ) * \operatorname{sgn}(F) - 10 \pi^2 b^5 d^5 \log(\operatorname{abs}(F))^3 * \operatorname{sgn}(F) - 5 \pi^4 b^5 d^5 \log(\operatorname{abs}(F)) \\
& ) + 10 \pi^2 b^5 d^5 \log(\operatorname{abs}(F))^3 - 2 b^5 d^5 \log(\operatorname{abs}(F))^5)^2) * \sin(-1/2 \pi \\
& i b d x * \operatorname{sgn}(F) + 1/2 \pi i b d x - 1/2 \pi i b c * \operatorname{sgn}(F) + 1/2 \pi i b c - 1/2 \pi i a * \operatorname{sgn}(F) \\
& + 1/2 \pi i a) * e^{(b d x * \log(\operatorname{abs}(F)) + b c * \log(\operatorname{abs}(F)) + a * \log(\operatorname{abs}(F)))} \\
& + 1/2 I * ((-16 I \pi^4 b^4 d^4 f^2 x^4 * \operatorname{sgn}(F) + 64 \pi^3 b^4 d^4 f^2 x^4 * \log(\operatorname{abs}(F)) * \operatorname{sgn}(F) \\
& + 96 I \pi^2 b^4 d^4 f^2 x^4 * \log(\operatorname{abs}(F))^2 * \operatorname{sgn}(F) - 64 \pi b^4 d^4 f^2 x^4 * \log(\operatorname{abs}(F))^3 * \operatorname{sgn}(F) \\
& + 16 I \pi^4 b^4 d^4 f^2 x^4 - 64 \pi^3 b^4 d^4 f^2 x^4 * \log(\operatorname{abs}(F)) - 96 I \pi^2 b^4 d^4 f^2 x^4 * \log(\operatorname{abs}(F))^2 \\
& + 64 \pi b^4 d^4 f^2 x^4 * \log(\operatorname{abs}(F))^3 + 32 I b^4 d^4 f^2 x^4 * \log(\operatorname{abs}(F))^4 - 64 \pi^3 \\
& b^3 d^3 f^2 x^3 * \operatorname{sgn}(F) - 192 I \pi^2 b^3 d^3 f^2 x^3 * \log(\operatorname{abs}(F)) * \operatorname{sgn}(F) + 1 \\
& 92 \pi b^3 d^3 f^2 x^3 * \log(\operatorname{abs}(F))^2 * \operatorname{sgn}(F) + 64 \pi^3 b^3 d^3 f^2 x^3 + 192 * \\
& I \pi^2 b^3 d^3 f^2 x^3 * \log(\operatorname{abs}(F)) - 192 \pi b^3 d^3 f^2 x^3 * \log(\operatorname{abs}(F))^2 - \\
& 128 I b^3 d^3 f^2 x^3 * \log(\operatorname{abs}(F))^3 + 192 I \pi^2 b^2 d^2 f^2 x^2 * \operatorname{sgn}(F) - \\
& 384 \pi b^2 d^2 f^2 x^2 * \log(\operatorname{abs}(F)) * \operatorname{sgn}(F) - 192 I \pi^2 b^2 d^2 f^2 x^2 + 38 \\
& 4 \pi b^2 d^2 f^2 x^2 * \log(\operatorname{abs}(F)) + 384 I b^2 d^2 f^2 x^2 * \log(\operatorname{abs}(F))^2 + 38 \\
& 4 \pi b d f^2 x * \operatorname{sgn}(F) - 384 \pi b d f^2 x - 768 I b d f^2 x * \log(\operatorname{abs}(F)) + 76 \\
& 8 I f^2) * e^{(1/2 I \pi i b d x * \operatorname{sgn}(F) - 1/2 I \pi i b d x + 1/2 I \pi i b c * \operatorname{sgn}(F) - \\
& 1/2 I \pi i b c + 1/2 I \pi i a * \operatorname{sgn}(F) - 1/2 I \pi i a) / (16 I \pi^5 b^5 d^5 * \operatorname{sgn}(F) - \\
& 80 \pi^4 b^5 d^5 \log(\operatorname{abs}(F)) * \operatorname{sgn}(F) - 160 I \pi^3 b^5 d^5 \log(\operatorname{abs}(F))^2 * \operatorname{sgn}(F) \\
& ) + 160 \pi^2 b^5 d^5 \log(\operatorname{abs}(F))^3 * \operatorname{sgn}(F) + 80 I \pi b^5 d^5 \log(\operatorname{abs}(F))^4 * \operatorname{sgn}(F) \\
& - 16 I \pi^5 b^5 d^5 + 80 \pi^4 b^5 d^5 \log(\operatorname{abs}(F)) + 160 I \pi^3 b^5 d^5 \log(\operatorname{abs}(F))^2 \\
& - 160 \pi^2 b^5 d^5 \log(\operatorname{abs}(F))^3 - 80 I \pi b^5 d^5 \log(\operatorname{abs}(F))^4 + 32 b^5 d^5 \log(\operatorname{abs}(F))^5 - \\
& (-16 I \pi^4 b^4 d^4 f^2 x^4 * \operatorname{sgn}(F) - 64 \\
& \pi^3 b^4 d^4 f^2 x^4 * \log(\operatorname{abs}(F)) * \operatorname{sgn}(F) + 96 I \pi^2 b^4 d^4 f^2 x^4 * \log(\operatorname{abs}(F))^2 * \operatorname{sgn}(F) \\
& + 64 \pi b^4 d^4 f^2 x^4 * \log(\operatorname{abs}(F))^3 * \operatorname{sgn}(F) + 16 I \pi^4 b^4 \\
& d^4 f^2 x^4 + 64 \pi^3 b^4 d^4 f^2 x^4 * \log(\operatorname{abs}(F)) - 96 I \pi^2 b^4 d^4 f^2 x \\
& ^4 * \log(\operatorname{abs}(F))^2 - 64 \pi b^4 d^4 f^2 x^4 * \log(\operatorname{abs}(F))^3 + 32 I b^4 d^4 f^2 x \\
& ^4 * \log(\operatorname{abs}(F))^4 + 64 \pi^3 b^3 d^3 f^2 x^3 * \operatorname{sgn}(F) - 192 I \pi^2 b^3 d^3 f^2 \\
& x^3 * \log(\operatorname{abs}(F)) * \operatorname{sgn}(F) - 192 \pi b^3 d^3 f^2 x^3 * \log(\operatorname{abs}(F))^2 * \operatorname{sgn}(F) - 64 \\
& \pi^3 b^3 d^3 f^2 x^3 + 192 I \pi^2 b^3 d^3 f^2 x^3 * \log(\operatorname{abs}(F)) + 192 \pi b^3 * \\
& d^3 f^2 x^3 * \log(\operatorname{abs}(F))^2 - 128 I b^3 d^3 f^2 x^3 * \log(\operatorname{abs}(F))^3 + 192 I \pi^2 \\
& b^2 d^2 f^2 x^2 * \operatorname{sgn}(F) + 384 \pi b^2 d^2 f^2 x^2 * \log(\operatorname{abs}(F)) * \operatorname{sgn}(F) - 192 * \\
& I \pi^2 b^2 d^2 f^2 x^2 - 384 \pi b^2 d^2 f^2 x^2 * \log(\operatorname{abs}(F)) + 384 I b^2 d^2
\end{aligned}$$

$$\begin{aligned}
& *f^2*x^2*\log(\text{abs}(F))^2 - 384*\pi*b*d*f^2*x*\text{sgn}(F) + 384*\pi*b*d*f^2*x - 768*I \\
& *b*d*f^2*x*\log(\text{abs}(F)) + 768*I*f^2)*e^{(-1/2*I*\pi*b*d*x*\text{sgn}(F) + 1/2*I*\pi*b* \\
& d*x - 1/2*I*\pi*b*c*\text{sgn}(F) + 1/2*I*\pi*b*c - 1/2*I*\pi*a*\text{sgn}(F) + 1/2*I*\pi*a)/} \\
& (-16*I*\pi^5*b^5*d^5*\text{sgn}(F) - 80*\pi^4*b^5*d^5*\log(\text{abs}(F))*\text{sgn}(F) + 160*I*\pi^ \\
& 3*b^5*d^5*\log(\text{abs}(F))^2*\text{sgn}(F) + 160*\pi^2*b^5*d^5*\log(\text{abs}(F))^3*\text{sgn}(F) - 80 \\
& *I*\pi*b^5*d^5*\log(\text{abs}(F))^4*\text{sgn}(F) + 16*I*\pi^5*b^5*d^5 + 80*\pi^4*b^5*d^5*\log \\
& (\text{abs}(F)) - 160*I*\pi^3*b^5*d^5*\log(\text{abs}(F))^2 - 160*\pi^2*b^5*d^5*\log(\text{abs}(F)) \\
& ^3 + 80*I*\pi*b^5*d^5*\log(\text{abs}(F))^4 + 32*b^5*d^5*\log(\text{abs}(F))^5))*e^{(b*d*x*\log \\
& (\text{abs}(F)) + b*c*\log(\text{abs}(F)) + a*\log(\text{abs}(F)))}
\end{aligned}$$

### 3.67 $\int F^{a+b(c+dx)} x(e+fx)^2 dx$

**Optimal.** Leaf size=242

$$-\frac{e^2 F^{a+bc+bdx}}{b^2 d^2 \log^2(F)} - \frac{4efx F^{a+bc+bdx}}{b^2 d^2 \log^2(F)} + \frac{4ef F^{a+bc+bdx}}{b^3 d^3 \log^3(F)} - \frac{3f^2 x^2 F^{a+bc+bdx}}{b^2 d^2 \log^2(F)} + \frac{6f^2 x F^{a+bc+bdx}}{b^3 d^3 \log^3(F)} - \frac{6f^2 F^{a+bc+bdx}}{b^4 d^4 \log^4(F)} + \frac{e^2 x F^{a+bc+bdx}}{bd \log(F)} + \frac{2ef}{b^3 d^3 \log^3(F)}$$

[Out]  $(-6f^2 F^{a+bc+bdx})/(b^4 d^4 \text{Log}[F]^4) + (4e f F^{a+bc+bdx})/(b^3 d^3 \text{Log}[F]^3) + (6f^2 F^{a+bc+bdx} x)/(b^3 d^3 \text{Log}[F]^3) - (e^2 F^{a+bc+bdx})/(b^2 d^2 \text{Log}[F]^2) - (4e f F^{a+bc+bdx} x)/(b^2 d^2 \text{Log}[F]^2) - (3f^2 F^{a+bc+bdx} x^2)/(b^2 d^2 \text{Log}[F]^2) + (e^2 F^{a+bc+bdx} x)/(b d \text{Log}[F]) + (2e f F^{a+bc+bdx} x^2)/(b d \text{Log}[F]) + (f^2 F^{a+bc+bdx} x^3)/(b d \text{Log}[F])$

**Rubi [A]** time = 0.355587, antiderivative size = 242, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 3, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.15$ , Rules used = {2196, 2176, 2194}

$$-\frac{e^2 F^{a+bc+bdx}}{b^2 d^2 \log^2(F)} - \frac{4efx F^{a+bc+bdx}}{b^2 d^2 \log^2(F)} + \frac{4ef F^{a+bc+bdx}}{b^3 d^3 \log^3(F)} - \frac{3f^2 x^2 F^{a+bc+bdx}}{b^2 d^2 \log^2(F)} + \frac{6f^2 x F^{a+bc+bdx}}{b^3 d^3 \log^3(F)} - \frac{6f^2 F^{a+bc+bdx}}{b^4 d^4 \log^4(F)} + \frac{e^2 x F^{a+bc+bdx}}{bd \log(F)} + \frac{2ef}{b^3 d^3 \log^3(F)}$$

Antiderivative was successfully verified.

[In] Int[F^(a + b\*(c + d\*x))\*x\*(e + f\*x)^2, x]

[Out]  $(-6f^2 F^{a+bc+bdx})/(b^4 d^4 \text{Log}[F]^4) + (4e f F^{a+bc+bdx})/(b^3 d^3 \text{Log}[F]^3) + (6f^2 F^{a+bc+bdx} x)/(b^3 d^3 \text{Log}[F]^3) - (e^2 F^{a+bc+bdx})/(b^2 d^2 \text{Log}[F]^2) - (4e f F^{a+bc+bdx} x)/(b^2 d^2 \text{Log}[F]^2) - (3f^2 F^{a+bc+bdx} x^2)/(b^2 d^2 \text{Log}[F]^2) + (e^2 F^{a+bc+bdx} x)/(b d \text{Log}[F]) + (2e f F^{a+bc+bdx} x^2)/(b d \text{Log}[F]) + (f^2 F^{a+bc+bdx} x^3)/(b d \text{Log}[F])$

#### Rule 2196

Int[(F\_)^((c\_.)\*(v\_))\*(u\_), x\_Symbol] := Int[ExpandIntegrand[F^(c\*ExpandToSum[v, x]), u, x], x] /; FreeQ[{F, c}, x] && PolynomialQ[u, x] && LinearQ[v, x] && !\$UseGamma === True

#### Rule 2176

Int[((b\_.)\*(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[((c + d\*x)^m\*(b F^(g\*(e + f\*x)))^n)/(f\*g\*n\*Log[F]),

```
x] - Dist[(d*m)/(f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] && !$UseGamma === True
```

### Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

### Rubi steps

$$\begin{aligned}
\int F^{a+b(c+dx)} x(e+fx)^2 dx &= \int (e^2 F^{a+bc+bdx} x + 2ef F^{a+bc+bdx} x^2 + f^2 F^{a+bc+bdx} x^3) dx \\
&= e^2 \int F^{a+bc+bdx} x dx + (2ef) \int F^{a+bc+bdx} x^2 dx + f^2 \int F^{a+bc+bdx} x^3 dx \\
&= \frac{e^2 F^{a+bc+bdx} x}{bd \log(F)} + \frac{2ef F^{a+bc+bdx} x^2}{bd \log(F)} + \frac{f^2 F^{a+bc+bdx} x^3}{bd \log(F)} - \frac{e^2 \int F^{a+bc+bdx} dx}{bd \log(F)} - \frac{(4ef) \int F^{a+bc+bdx} x dx}{bd \log(F)} \\
&= -\frac{e^2 F^{a+bc+bdx}}{b^2 d^2 \log^2(F)} - \frac{4ef F^{a+bc+bdx} x}{b^2 d^2 \log^2(F)} - \frac{3f^2 F^{a+bc+bdx} x^2}{b^2 d^2 \log^2(F)} + \frac{e^2 F^{a+bc+bdx} x}{bd \log(F)} + \frac{2ef F^{a+bc+bdx} x^2}{bd \log(F)} + \frac{f^2 F^{a+bc+bdx} x^3}{bd \log(F)} \\
&= \frac{4ef F^{a+bc+bdx}}{b^3 d^3 \log^3(F)} + \frac{6f^2 F^{a+bc+bdx} x}{b^3 d^3 \log^3(F)} - \frac{e^2 F^{a+bc+bdx}}{b^2 d^2 \log^2(F)} - \frac{4ef F^{a+bc+bdx} x}{b^2 d^2 \log^2(F)} - \frac{3f^2 F^{a+bc+bdx} x^2}{b^2 d^2 \log^2(F)} + \frac{e^2 F^{a+bc+bdx} x}{bd \log(F)} \\
&= -\frac{6f^2 F^{a+bc+bdx}}{b^4 d^4 \log^4(F)} + \frac{4ef F^{a+bc+bdx}}{b^3 d^3 \log^3(F)} + \frac{6f^2 F^{a+bc+bdx} x}{b^3 d^3 \log^3(F)} - \frac{e^2 F^{a+bc+bdx}}{b^2 d^2 \log^2(F)} - \frac{4ef F^{a+bc+bdx} x}{b^2 d^2 \log^2(F)} - \frac{3f^2 F^{a+bc+bdx} x^2}{b^2 d^2 \log^2(F)} + \frac{e^2 F^{a+bc+bdx} x}{bd \log(F)}
\end{aligned}$$

**Mathematica [A]** time = 0.149704, size = 91, normalized size = 0.38

$$\frac{F^{a+b(c+dx)} \left( -b^2 d^2 \log^2(F) (e^2 + 4efx + 3f^2 x^2) + b^3 d^3 x \log^3(F) (e + fx)^2 + 2bdf \log(F) (2e + 3fx) - 6f^2 \right)}{b^4 d^4 \log^4(F)}$$

Antiderivative was successfully verified.

```
[In] Integrate[F^(a + b*(c + d*x))*x*(e + f*x)^2, x]
```

```
[Out] (F^(a + b*(c + d*x))*(-6*f^2 + 2*b*d*f*(2*e + 3*f*x)*Log[F] - b^2*d^2*(e^2
+ 4*e*f*x + 3*f^2*x^2)*Log[F]^2 + b^3*d^3*x*(e + f*x)^2*Log[F]^3))/(b^4*d^4
*Log[F]^4)
```

**Maple [A]** time = 0.007, size = 144, normalized size = 0.6

$$\frac{(\ln(F))^3 b^3 d^3 f^2 x^3 + 2 (\ln(F))^3 b^3 d^3 e f x^2 + (\ln(F))^3 b^3 d^3 e^2 x - 3 (\ln(F))^2 b^2 d^2 f^2 x^2 - 4 (\ln(F))^2 b^2 d^2 e f x - (\ln(F))^2 b^2 d^2 e^2 x}{(\ln(F))^4 b^4 d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(a+b\*(d\*x+c))\*x\*(f\*x+e)^2,x)

[Out]  $(\ln(F)^3 b^3 d^3 f^2 x^3 + 2 \ln(F)^3 b^3 d^3 e f x^2 + \ln(F)^3 b^3 d^3 e^2 x - 3 \ln(F)^2 b^2 d^2 f^2 x^2 - 4 \ln(F)^2 b^2 d^2 e f x - \ln(F)^2 b^2 d^2 e^2 x + 6 \ln(F) b^2 d^2 f^2 x + 4 \ln(F) b^2 d^2 e f x - 6 \ln(F) b^2 d^2 e^2 x) F^{(b*d*x+b*c+a)} / \ln(F)^4 / b^4 / d^4$

**Maxima [A]** time = 1.0494, size = 265, normalized size = 1.1

$$\frac{(F^{bc+a} b d x \log(F) - F^{bc+a}) F^{bdx} e^2}{b^2 d^2 \log(F)^2} + \frac{2(F^{bc+a} b^2 d^2 x^2 \log(F)^2 - 2 F^{bc+a} b d x \log(F) + 2 F^{bc+a}) F^{bdx} e f}{b^3 d^3 \log(F)^3} + \frac{(F^{bc+a} b^3 d^3 x^3 \log(F)^3 - 3 F^{bc+a} b^2 d^2 x^2 \log(F)^2 + 6 F^{bc+a} b d x \log(F) - 6 F^{bc+a}) F^{bdx} e^2}{b^4 d^4 \log(F)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*(d\*x+c))\*x\*(f\*x+e)^2,x, algorithm="maxima")

[Out]  $(F^{(b*c+a)} b^3 d^3 f^2 x^3 - F^{(b*c+a)} b^3 d^3 e f x^2 + F^{(b*c+a)} b^3 d^3 e^2 x) \log(F)^3 - (3 b^2 d^2 f^2 x^2 + 4 b^2 d^2 e f x + b^2 d^2 e^2) \log(F)^2 - 6 f^2 + 2(3 b d f^2 x + 2 b d e f) \log(F) - 6 F^{(b*c+a)} b^2 d^2 f^2 x^2 + 6 F^{(b*c+a)} b^2 d^2 e f x - 6 F^{(b*c+a)} b^2 d^2 e^2 x + 6 F^{(b*c+a)} b^3 d^3 f^2 x^3 - 6 F^{(b*c+a)} b^3 d^3 e f x^2 + 6 F^{(b*c+a)} b^3 d^3 e^2 x) / (b^4 d^4 \log(F)^4)$

**Fricas [A]** time = 1.57332, size = 288, normalized size = 1.19

$$\frac{((b^3 d^3 f^2 x^3 + 2 b^3 d^3 e f x^2 + b^3 d^3 e^2 x) \log(F)^3 - (3 b^2 d^2 f^2 x^2 + 4 b^2 d^2 e f x + b^2 d^2 e^2) \log(F)^2 - 6 f^2 + 2(3 b d f^2 x + 2 b d e f) \log(F) - 6 F^{(b*c+a)} b^2 d^2 f^2 x^2 + 6 F^{(b*c+a)} b^2 d^2 e f x - 6 F^{(b*c+a)} b^2 d^2 e^2 x + 6 F^{(b*c+a)} b^3 d^3 f^2 x^3 - 6 F^{(b*c+a)} b^3 d^3 e f x^2 + 6 F^{(b*c+a)} b^3 d^3 e^2 x) / (b^4 d^4 \log(F)^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*(d\*x+c))\*x\*(f\*x+e)^2,x, algorithm="fricas")

[Out]  $((b^3 d^3 f^2 x^3 + 2 b^3 d^3 e f x^2 + b^3 d^3 e^2 x) \log(F)^3 - (3 b^2 d^2 f^2 x^2 + 4 b^2 d^2 e f x + b^2 d^2 e^2) \log(F)^2 - 6 f^2 + 2(3 b d f^2 x + 2 b d e f) \log(F) - 6 F^{(b*c+a)} b^2 d^2 f^2 x^2 + 6 F^{(b*c+a)} b^2 d^2 e f x - 6 F^{(b*c+a)} b^2 d^2 e^2 x + 6 F^{(b*c+a)} b^3 d^3 f^2 x^3 - 6 F^{(b*c+a)} b^3 d^3 e f x^2 + 6 F^{(b*c+a)} b^3 d^3 e^2 x) / (b^4 d^4 \log(F)^4)$



$$x + 2*b*d*e*f*\log(F))*F^(b*d*x + b*c + a)/(b^4*d^4*\log(F)^4)$$

**Sympy [A]** time = 0.175929, size = 199, normalized size = 0.82

$$\left\{ \frac{F^{a+b(c+dx)}(b^3d^3e^2x\log(F)^3+2b^3d^3efx^2\log(F)^3+b^3d^3f^2x^3\log(F)^3-b^2d^2e^2\log(F)^2-4b^2d^2efx\log(F)^2-3b^2d^2f^2x^2\log(F)^2+4bdef\log(F)+6bdf^2x\log(F)-6f^2x^2}{b^4d^4\log(F)^4} + \frac{e^2x^2}{2} + \frac{2efx^3}{3} + \frac{f^2x^4}{4} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(a+b\*(d\*x+c))\*x\*(f\*x+e)\*\*2,x)

[Out] Piecewise((F\*\*(a + b\*(c + d\*x))\*(b\*\*3\*d\*\*3\*e\*\*2\*x\*log(F)\*\*3 + 2\*b\*\*3\*d\*\*3\*e\*f\*x\*\*2\*log(F)\*\*3 + b\*\*3\*d\*\*3\*f\*\*2\*x\*\*3\*log(F)\*\*3 - b\*\*2\*d\*\*2\*e\*\*2\*log(F)\*\*2 - 4\*b\*\*2\*d\*\*2\*e\*f\*x\*log(F)\*\*2 - 3\*b\*\*2\*d\*\*2\*f\*\*2\*x\*\*2\*log(F)\*\*2 + 4\*b\*d\*e\*f\*log(F) + 6\*b\*d\*f\*\*2\*x\*log(F) - 6\*f\*\*2)/(b\*\*4\*d\*\*4\*log(F)\*\*4), Ne(b\*\*4\*d\*\*4\*log(F)\*\*4, 0)), (e\*\*2\*x\*\*2/2 + 2\*e\*f\*x\*\*3/3 + f\*\*2\*x\*\*4/4, True))

**Giac [C]** time = 1.48567, size = 6340, normalized size = 26.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*(d\*x+c))\*x\*(f\*x+e)^2,x, algorithm="giac")

[Out] (2\*((pi\*b^2\*d^2\*log(abs(F))\*sgn(F) - pi\*b^2\*d^2\*log(abs(F)))\*(pi\*b\*d\*x\*sgn(F) - pi\*b\*d\*x)/((pi^2\*b^2\*d^2\*sgn(F) - pi^2\*b^2\*d^2 + 2\*b^2\*d^2\*log(abs(F))^2)^2 + 4\*(pi\*b^2\*d^2\*log(abs(F))\*sgn(F) - pi\*b^2\*d^2\*log(abs(F)))^2) + (pi^2\*b^2\*d^2\*sgn(F) - pi^2\*b^2\*d^2 + 2\*b^2\*d^2\*log(abs(F))^2)\*(b\*d\*x\*log(abs(F)) - 1)/((pi^2\*b^2\*d^2\*sgn(F) - pi^2\*b^2\*d^2 + 2\*b^2\*d^2\*log(abs(F))^2)^2 + 4\*(pi\*b^2\*d^2\*log(abs(F))\*sgn(F) - pi\*b^2\*d^2\*log(abs(F)))^2))\*cos(-1/2\*pi\*b\*d\*x\*sgn(F) + 1/2\*pi\*b\*d\*x - 1/2\*pi\*b\*c\*sgn(F) + 1/2\*pi\*b\*c - 1/2\*pi\*a\*sgn(F) + 1/2\*pi\*a) + ((pi^2\*b^2\*d^2\*sgn(F) - pi^2\*b^2\*d^2 + 2\*b^2\*d^2\*log(abs(F))^2)\*(pi\*b\*d\*x\*sgn(F) - pi\*b\*d\*x)/((pi^2\*b^2\*d^2\*sgn(F) - pi^2\*b^2\*d^2 + 2\*b^2\*d^2\*log(abs(F))^2)^2 + 4\*(pi\*b^2\*d^2\*log(abs(F))\*sgn(F) - pi\*b^2\*d^2\*log(abs(F)))^2) - 4\*(pi\*b^2\*d^2\*log(abs(F))\*sgn(F) - pi\*b^2\*d^2\*log(abs(F)))\*(b\*d\*x\*log(abs(F)) - 1)/((pi^2\*b^2\*d^2\*sgn(F) - pi^2\*b^2\*d^2 + 2\*b^2\*d^2\*log(abs(F))^2)^2 + 4\*(pi\*b^2\*d^2\*log(abs(F))\*sgn(F) - pi\*b^2\*d^2\*log(abs(F)))^2) + 4\*(pi\*b^2\*d^2\*log(abs(F))\*sgn(F) - pi\*b^2\*d^2\*log(abs(F)))^2)





$$\begin{aligned}
& *d*x*\log(\text{abs}(F)) + b*c*\log(\text{abs}(F)) + a*\log(\text{abs}(F))) - 1/2*I*((8*\pi^3*b^3*d^3*f^2*x^3*\text{sgn}(F) + 24*I*\pi^2*b^3*d^3*f^2*x^3*\log(\text{abs}(F))*\text{sgn}(F) - 24*\pi*b^3*d^3*f^2*x^3*\log(\text{abs}(F))^2*\text{sgn}(F) - 8*\pi^3*b^3*d^3*f^2*x^3 - 24*I*\pi^2*b^3*d^3*f^2*x^3*\log(\text{abs}(F)) + 24*\pi*b^3*d^3*f^2*x^3*\log(\text{abs}(F))^2 + 16*I*b^3*d^3*f^2*x^3*\log(\text{abs}(F))^3 - 24*I*\pi^2*b^2*d^2*f^2*x^2*\text{sgn}(F) + 48*\pi*b^2*d^2*f^2*x^2*\log(\text{abs}(F))*\text{sgn}(F) + 24*I*\pi^2*b^2*d^2*f^2*x^2 - 48*\pi*b^2*d^2*f^2*x^2*\log(\text{abs}(F)) - 48*I*b^2*d^2*f^2*x^2*\log(\text{abs}(F))^2 - 48*\pi*b*d*f^2*x*\text{sgn}(F) + 48*\pi*b*d*f^2*x + 96*I*b*d*f^2*x*\log(\text{abs}(F)) - 96*I*f^2)*e^{(1/2*I*\pi*b*d*x*\text{sgn}(F) - 1/2*I*\pi*b*d*x + 1/2*I*\pi*b*c*\text{sgn}(F) - 1/2*I*\pi*b*c + 1/2*I*\pi*a*\text{sgn}(F) - 1/2*I*\pi*a)/(8*\pi^4*b^4*d^4*\text{sgn}(F) + 32*I*\pi^3*b^4*d^4*\log(\text{abs}(F))*\text{sgn}(F) - 48*\pi^2*b^4*d^4*\log(\text{abs}(F))^2*\text{sgn}(F) - 32*I*\pi*b^4*d^4*\log(\text{abs}(F))^3*\text{sgn}(F) - 8*\pi^4*b^4*d^4 - 32*I*\pi^3*b^4*d^4*\log(\text{abs}(F)) + 48*\pi^2*b^4*d^4*\log(\text{abs}(F))^2 + 32*I*\pi*b^4*d^4*\log(\text{abs}(F))^3 - 16*b^4*d^4*\log(\text{abs}(F))^4) + (8*\pi^3*b^3*d^3*f^2*x^3*\text{sgn}(F) - 24*I*\pi^2*b^3*d^3*f^2*x^3*\log(\text{abs}(F))*\text{sgn}(F) - 24*\pi*b^3*d^3*f^2*x^3*\log(\text{abs}(F))^2*\text{sgn}(F) - 8*\pi^3*b^3*d^3*f^2*x^3 + 24*I*\pi^2*b^3*d^3*f^2*x^3*\log(\text{abs}(F)) + 24*\pi*b^3*d^3*f^2*x^3*\log(\text{abs}(F))^2 - 16*I*b^3*d^3*f^2*x^3*\log(\text{abs}(F))^3 + 24*I*\pi^2*b^2*d^2*f^2*x^2*\text{sgn}(F) + 48*\pi*b^2*d^2*f^2*x^2*\log(\text{abs}(F))*\text{sgn}(F) - 24*I*\pi^2*b^2*d^2*f^2*x^2 - 48*\pi*b^2*d^2*f^2*x^2*\log(\text{abs}(F)) + 48*I*b^2*d^2*f^2*x^2*\log(\text{abs}(F))^2 - 48*\pi*b*d*f^2*x*\text{sgn}(F) + 48*\pi*b*d*f^2*x - 96*I*b*d*f^2*x*\log(\text{abs}(F)) + 96*I*f^2)*e^{(-1/2*I*\pi*b*d*x*\text{sgn}(F) + 1/2*I*\pi*b*d*x - 1/2*I*\pi*b*c*\text{sgn}(F) + 1/2*I*\pi*b*c - 1/2*I*\pi*a*\text{sgn}(F) + 1/2*I*\pi*a)/(8*\pi^4*b^4*d^4*\text{sgn}(F) - 32*I*\pi^3*b^4*d^4*\log(\text{abs}(F))*\text{sgn}(F) - 48*\pi^2*b^4*d^4*\log(\text{abs}(F))^2*\text{sgn}(F) + 32*I*\pi*b^4*d^4*\log(\text{abs}(F))^3*\text{sgn}(F) - 8*\pi^4*b^4*d^4 + 32*I*\pi^3*b^4*d^4*\log(\text{abs}(F)) + 48*\pi^2*b^4*d^4*\log(\text{abs}(F))^2 - 32*I*\pi*b^4*d^4*\log(\text{abs}(F))^3 - 16*b^4*d^4*\log(\text{abs}(F))^4)}*e^{(b*d*x*\log(\text{abs}(F)) + b*c*\log(\text{abs}(F)) + a*\log(\text{abs}(F)))}
\end{aligned}$$

### 3.68 $\int F^{a+b(c+dx)}(e+fx)^2 dx$

**Optimal.** Leaf size=85

$$-\frac{2f(e+fx)F^{a+bc+bdx}}{b^2d^2\log^2(F)} + \frac{2f^2F^{a+bc+bdx}}{b^3d^3\log^3(F)} + \frac{(e+fx)^2F^{a+bc+bdx}}{bd\log(F)}$$

[Out] (2\*f^2\*F^(a + b\*c + b\*d\*x))/(b^3\*d^3\*Log[F]^3) - (2\*f\*F^(a + b\*c + b\*d\*x)\*(e + f\*x))/(b^2\*d^2\*Log[F]^2) + (F^(a + b\*c + b\*d\*x)\*(e + f\*x)^2)/(b\*d\*Log[F])

**Rubi [A]** time = 0.120364, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$ , Rules used = {2187, 2176, 2194}

$$-\frac{2f(e+fx)F^{a+bc+bdx}}{b^2d^2\log^2(F)} + \frac{2f^2F^{a+bc+bdx}}{b^3d^3\log^3(F)} + \frac{(e+fx)^2F^{a+bc+bdx}}{bd\log(F)}$$

Antiderivative was successfully verified.

[In] Int[F^(a + b\*(c + d\*x))\*(e + f\*x)^2,x]

[Out] (2\*f^2\*F^(a + b\*c + b\*d\*x))/(b^3\*d^3\*Log[F]^3) - (2\*f\*F^(a + b\*c + b\*d\*x)\*(e + f\*x))/(b^2\*d^2\*Log[F]^2) + (F^(a + b\*c + b\*d\*x)\*(e + f\*x)^2)/(b\*d\*Log[F])

#### Rule 2187

```
Int[((a_.) + (b_.)*((F_)^((g_.)*(v_)))^(n_.))^(p_.)*(u_)^(m_.), x_Symbol] :
> Int[NormalizePowerOfLinear[u, x]^m*(a + b*(F^(g*ExpandToSum[v, x]))^n)^p,
x] /; FreeQ[{F, a, b, g, n, p}, x] && LinearQ[v, x] && PowerOfLinearQ[u, x]
] && !(LinearMatchQ[v, x] && PowerOfLinearMatchQ[u, x]) && IntegerQ[m]
```

#### Rule 2176

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[((c + d*x)^m*(b*F^(g*(e + f*x)))^n)/(f*g*n*Log[F]),
x] - Dist[(d*m)/(f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m]
] && !$UseGamma === True
```

Rule 2194

`Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] := Simp[(F^(c*(a + b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]`

Rubi steps

$$\begin{aligned}
 \int F^{a+b(c+dx)}(e+fx)^2 dx &= \int F^{a+bc+bdx}(e+fx)^2 dx \\
 &= \frac{F^{a+bc+bdx}(e+fx)^2}{bd \log(F)} - \frac{(2f) \int F^{a+bc+bdx}(e+fx) dx}{bd \log(F)} \\
 &= -\frac{2fF^{a+bc+bdx}(e+fx)}{b^2d^2 \log^2(F)} + \frac{F^{a+bc+bdx}(e+fx)^2}{bd \log(F)} + \frac{(2f^2) \int F^{a+bc+bdx} dx}{b^2d^2 \log^2(F)} \\
 &= \frac{2f^2F^{a+bc+bdx}}{b^3d^3 \log^3(F)} - \frac{2fF^{a+bc+bdx}(e+fx)}{b^2d^2 \log^2(F)} + \frac{F^{a+bc+bdx}(e+fx)^2}{bd \log(F)}
 \end{aligned}$$

**Mathematica [A]** time = 0.0689011, size = 58, normalized size = 0.68

$$\frac{F^{a+b(c+dx)}(b^2d^2 \log^2(F)(e+fx)^2 - 2bdf \log(F)(e+fx) + 2f^2)}{b^3d^3 \log^3(F)}$$

Antiderivative was successfully verified.

[In] Integrate[F^(a + b\*(c + d\*x))\*(e + f\*x)^2,x]

[Out] (F^(a + b\*(c + d\*x))\*(2\*f^2 - 2\*b\*d\*f\*(e + f\*x)\*Log[F] + b^2\*d^2\*(e + f\*x)^2\*Log[F]^2))/(b^3\*d^3\*Log[F]^3)

**Maple [A]** time = 0.007, size = 93, normalized size = 1.1

$$\frac{((\ln(F))^2 b^2 d^2 f^2 x^2 + 2 (\ln(F))^2 b^2 d^2 e f x + (\ln(F))^2 b^2 d^2 e^2 - 2 \ln(F) b d f^2 x - 2 f e \ln(F) b d + 2 f^2) F^{bdx+bc+a}}{(\ln(F))^3 b^3 d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(a+b\*(d\*x+c))\*(f\*x+e)^2,x)

[Out]  $(\ln(F)^{2b^2d^2f^2x^2+2\ln(F)^{2b^2d^2e^2fx+\ln(F)^{2b^2d^2e^2-2\ln(F)}})*b^2d^2f^2x-2f^2e^2\ln(F)*b^2d^2f^2)*F^{(b^2d^2x+b^2c+a)}/b^3/d^3/\ln(F)^3$

**Maxima [A]** time = 1.02178, size = 181, normalized size = 2.13

$$\frac{F^{bdx+bc+a}e^2}{bd \log(F)} + \frac{2(F^{bc+a}bdx \log(F) - F^{bc+a})F^{bdx}ef}{b^2d^2 \log(F)^2} + \frac{(F^{bc+a}b^2d^2x^2 \log(F)^2 - 2F^{bc+a}bdx \log(F) + 2F^{bc+a})F^{bdx}f^2}{b^3d^3 \log(F)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(a+b*(d*x+c))*(f*x+e)^2,x, algorithm="maxima")`

[Out]  $F^{(b^2d^2x + b^2c + a)*e^2/(b^2d^2\log(F))} + 2*(F^{(b^2c + a)*b^2d^2x*\log(F)} - F^{(b^2c + a)})*F^{(b^2d^2x)*e^2f/(b^2d^2*\log(F)^2)} + (F^{(b^2c + a)*b^2d^2x^2*\log(F)^2} - 2*F^{(b^2c + a)*b^2d^2x*\log(F)} + 2*F^{(b^2c + a)})*F^{(b^2d^2x)*f^2/(b^3d^3*\log(F)^3)}$

**Fricas [A]** time = 1.48697, size = 192, normalized size = 2.26

$$\frac{((b^2d^2f^2x^2 + 2b^2d^2efx + b^2d^2e^2)\log(F)^2 + 2f^2 - 2(bdf^2x + bdef)\log(F))F^{bdx+bc+a}}{b^3d^3 \log(F)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(a+b*(d*x+c))*(f*x+e)^2,x, algorithm="fricas")`

[Out]  $((b^2d^2f^2x^2 + 2b^2d^2efx + b^2d^2e^2)*\log(F)^2 + 2f^2 - 2*(b^2d^2f^2x + b^2d^2ef)*\log(F))*F^{(b^2d^2x + b^2c + a)}/(b^3d^3*\log(F)^3)$

**Sympy [A]** time = 0.155241, size = 134, normalized size = 1.58

$$\begin{cases} \frac{F^{a+b(c+dx)}(b^2d^2e^2 \log(F)^2 + 2b^2d^2efx \log(F)^2 + b^2d^2f^2x^2 \log(F)^2 - 2bdef \log(F) - 2bdf^2x \log(F) + 2f^2)}{b^3d^3 \log(F)^3} & \text{for } b^3d^3 \log(F)^3 \neq 0 \\ e^2x + efx^2 + \frac{f^2x^3}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(a+b*(d*x+c))*(f*x+e)**2,x)
```

```
[Out] Piecewise((F**(a + b*(c + d*x))*(b**2*d**2*e**2*log(F)**2 + 2*b**2*d**2*e*f
*x*log(F)**2 + b**2*d**2*f**2*x**2*log(F)**2 - 2*b*d*e*f*log(F) - 2*b*d*f**
2*x*log(F) + 2*f**2)/(b**3*d**3*log(F)**3), Ne(b**3*d**3*log(F)**3, 0)), (e
**2*x + e*f*x**2 + f**2*x**3/3, True))
```

**Giac [C]** time = 1.38049, size = 3708, normalized size = 43.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(a+b*(d*x+c))*(f*x+e)^2,x, algorithm="giac")
```

```
[Out] 2*(2*b*d*cos(-1/2*pi*b*d*x*sgn(F) + 1/2*pi*b*d*x - 1/2*pi*b*c*sgn(F) + 1/2*
pi*b*c - 1/2*pi*a*sgn(F) + 1/2*pi*a)*log(abs(F))/(4*b^2*d^2*log(abs(F))^2 +
(pi*b*d*sgn(F) - pi*b*d)^2) - (pi*b*d*sgn(F) - pi*b*d)*sin(-1/2*pi*b*d*x*s
gn(F) + 1/2*pi*b*d*x - 1/2*pi*b*c*sgn(F) + 1/2*pi*b*c - 1/2*pi*a*sgn(F) + 1
/2*pi*a)/(4*b^2*d^2*log(abs(F))^2 + (pi*b*d*sgn(F) - pi*b*d)^2))*e^(b*d*x*l
og(abs(F)) + b*c*log(abs(F)) + a*log(abs(F)) + 2) - 1/2*I*(-2*I*e^(1/2*I*pi
*b*d*x*sgn(F) - 1/2*I*pi*b*d*x + 1/2*I*pi*b*c*sgn(F) - 1/2*I*pi*b*c + 1/2*I
*pi*a*sgn(F) - 1/2*I*pi*a)/(I*pi*b*d*sgn(F) - I*pi*b*d + 2*b*d*log(abs(F)))
+ 2*I*e^(-1/2*I*pi*b*d*x*sgn(F) + 1/2*I*pi*b*d*x - 1/2*I*pi*b*c*sgn(F) + 1
/2*I*pi*b*c - 1/2*I*pi*a*sgn(F) + 1/2*I*pi*a)/(-I*pi*b*d*sgn(F) + I*pi*b*d
+ 2*b*d*log(abs(F))))*e^(b*d*x*log(abs(F)) + b*c*log(abs(F)) + a*log(abs(F)
) + 2) + 2*(2*((pi*b^2*d^2*log(abs(F))*sgn(F) - pi*b^2*d^2*log(abs(F)))*(pi
*b*d*f*x*sgn(F) - pi*b*d*f*x)/((pi^2*b^2*d^2*sgn(F) - pi^2*b^2*d^2 + 2*b^2*
d^2*log(abs(F))^2)^2 + 4*(pi*b^2*d^2*log(abs(F))*sgn(F) - pi*b^2*d^2*log(ab
s(F)))^2) + (pi^2*b^2*d^2*sgn(F) - pi^2*b^2*d^2 + 2*b^2*d^2*log(abs(F))^2)*
(b*d*f*x*log(abs(F)) - f)/((pi^2*b^2*d^2*sgn(F) - pi^2*b^2*d^2 + 2*b^2*d^2*
log(abs(F))^2)^2 + 4*(pi*b^2*d^2*log(abs(F))*sgn(F) - pi*b^2*d^2*log(abs(F)
))^2))*cos(-1/2*pi*b*d*x*sgn(F) + 1/2*pi*b*d*x - 1/2*pi*b*c*sgn(F) + 1/2*pi
*b*c - 1/2*pi*a*sgn(F) + 1/2*pi*a) + ((pi^2*b^2*d^2*sgn(F) - pi^2*b^2*d^2 +
2*b^2*d^2*log(abs(F))^2)*(pi*b*d*f*x*sgn(F) - pi*b*d*f*x)/((pi^2*b^2*d^2*s
gn(F) - pi^2*b^2*d^2 + 2*b^2*d^2*log(abs(F))^2)^2 + 4*(pi*b^2*d^2*log(abs(F)
))*sgn(F) - pi*b^2*d^2*log(abs(F)))^2) - 4*(pi*b^2*d^2*log(abs(F))*sgn(F) -
pi*b^2*d^2*log(abs(F)))*(b*d*f*x*log(abs(F)) - f)/((pi^2*b^2*d^2*sgn(F) -
pi^2*b^2*d^2 + 2*b^2*d^2*log(abs(F))^2)^2 + 4*(pi*b^2*d^2*log(abs(F))*sgn(F)
) - pi*b^2*d^2*log(abs(F)))^2))*sin(-1/2*pi*b*d*x*sgn(F) + 1/2*pi*b*d*x - 1
/2*pi*b*c*sgn(F) + 1/2*pi*b*c - 1/2*pi*a*sgn(F) + 1/2*pi*a))*e^(b*d*x*log(a
bs(F)) + b*c*log(abs(F)) + a*log(abs(F)) + 1) - 1/2*I*((4*pi*b*d*f*x*sgn(F)
```



$$\begin{aligned}
& - 4\pi b d f x - 8I b d f x \log(\text{abs}(F)) + 8I f) e^{(1/2 I \pi b d x \text{sgn}(F)} \\
& - 1/2 I \pi b d x + 1/2 I \pi b c \text{sgn}(F) - 1/2 I \pi b c + 1/2 I \pi a \text{sgn}(F) \\
& - 1/2 I \pi a) / (2 \pi^2 b^2 d^2 \text{sgn}(F) + 4 I \pi b^2 d^2 \log(\text{abs}(F)) \text{sgn}(F) - \\
& 2 \pi^2 b^2 d^2 - 4 I \pi b^2 d^2 \log(\text{abs}(F)) + 4 b^2 d^2 \log(\text{abs}(F))^2) + (4 \\
& \pi b d f x \text{sgn}(F) - 4 \pi b d f x + 8 I b d f x \log(\text{abs}(F)) - 8 I f) e^{(-1/ \\
& 2 I \pi b d x \text{sgn}(F) + 1/2 I \pi b d x - 1/2 I \pi b c \text{sgn}(F) + 1/2 I \pi b c - \\
& 1/2 I \pi a \text{sgn}(F) + 1/2 I \pi a) / (2 \pi^2 b^2 d^2 \text{sgn}(F) - 4 I \pi b^2 d^2 \log(\text{abs}(F)) \text{sgn}(F) - \\
& 2 \pi^2 b^2 d^2 + 4 I \pi b^2 d^2 \log(\text{abs}(F)) + 4 b^2 d^2 \log(\text{abs}(F))^2) * e^{(b d x \log(\text{abs}(F)) + b c \log(\text{abs}(F)) + a \log(\text{abs}(F)) + 1) \\
& - ((2 (\pi b^2 d^2 f^2 x^2 \log(\text{abs}(F)) \text{sgn}(F) - \pi b^2 d^2 f^2 x^2 \log(\text{abs}(F))) - \pi b d f^2 x \text{sgn}(F) + \pi b d f^2 x) * (\pi^3 b^3 d^3 \text{sgn}(F) - 3 \pi b^3 d^3 \\
& \log(\text{abs}(F))^2 \text{sgn}(F) - \pi^3 b^3 d^3 + 3 \pi b^3 d^3 \log(\text{abs}(F))^2) / ((\pi^3 b^3 d^3 \text{sgn}(F) - 3 \pi b^3 d^3 \log(\text{abs}(F))^2 \text{sgn}(F) - \pi^3 b^3 d^3 + 3 \pi b^3 \\
& \log(\text{abs}(F))^2)^2 + (3 \pi^2 b^3 d^3 \log(\text{abs}(F)) \text{sgn}(F) - 3 \pi^2 b^3 d^3 \log(\text{abs}(F)) + 2 b^3 d^3 \log(\text{abs}(F))^3)^2) - (\pi^2 b^2 d^2 f^2 x^2 \text{sgn}(F) \\
& - \pi^2 b^2 d^2 f^2 x^2 + 2 b^2 d^2 f^2 x^2 \log(\text{abs}(F))^2 - 4 b d f^2 x \log(\text{abs}(F)) + 4 f^2) * (3 \pi^2 b^3 d^3 \log(\text{abs}(F)) \text{sgn}(F) - 3 \pi^2 b^3 d^3 \log(\text{abs}(F)) + 2 b^3 d^3 \log(\text{abs}(F))^3) / ((\pi^3 b^3 d^3 \text{sgn}(F) - 3 \pi b^3 d^3 \log(\text{abs}(F))^2 \text{sgn}(F) - \pi^3 b^3 d^3 + 3 \pi b^3 d^3 \log(\text{abs}(F))^2)^2 + (3 \pi^2 b^3 d^3 \log(\text{abs}(F)) \text{sgn}(F) - 3 \pi^2 b^3 d^3 \log(\text{abs}(F)) + 2 b^3 d^3 \log(\text{abs}(F))^3)^2) * \cos(-1/2 \pi b d x \text{sgn}(F) + 1/2 \pi b d x - 1/2 \pi b c \text{sgn}(F) + 1/2 \pi b c - 1/2 \pi a \text{sgn}(F) + 1/2 \pi a) - ((\pi^2 b^2 d^2 f^2 x^2 \text{sgn}(F) - \pi^2 b^2 d^2 f^2 x^2 + 2 b^2 d^2 f^2 x^2 \log(\text{abs}(F))^2 - 4 b d f^2 x \log(\text{abs}(F)) + 4 f^2) * (\pi^3 b^3 d^3 \text{sgn}(F) - 3 \pi b^3 d^3 \log(\text{abs}(F))^2 \text{sgn}(F) - \pi^3 b^3 d^3 + 3 \pi b^3 d^3 \log(\text{abs}(F))^2) / ((\pi^3 b^3 d^3 \text{sgn}(F) - 3 \pi b^3 d^3 \log(\text{abs}(F))^2 \text{sgn}(F) - \pi^3 b^3 d^3 + 3 \pi b^3 d^3 \log(\text{abs}(F))^2)^2 + (3 \pi^2 b^3 d^3 \log(\text{abs}(F)) \text{sgn}(F) - 3 \pi^2 b^3 d^3 \log(\text{abs}(F)) + 2 b^3 d^3 \log(\text{abs}(F))^3)^2) * \sin(-1/2 \pi b d x \text{sgn}(F) + 1/2 \pi b d x - 1/2 \pi b c \text{sgn}(F) + 1/2 \pi b c - 1/2 \pi a \text{sgn}(F) + 1/2 \pi a) * e^{(b d x \log(\text{abs}(F)) + b c \log(\text{abs}(F)) + a \log(\text{abs}(F)))} + 1/2 I * ((4 I \pi^2 b^2 d^2 f^2 x^2 \text{sgn}(F) - 8 \pi b^2 d^2 f^2 x^2 \log(\text{abs}(F)) \text{sgn}(F) - 4 I \pi^2 b^2 d^2 f^2 x^2 + 8 \pi b^2 d^2 f^2 x^2 \log(\text{abs}(F)) + 8 I b^2 d^2 f^2 x^2 \log(\text{abs}(F))^2 + 8 \pi b d f^2 x \text{sgn}(F) - 8 \pi b d f^2 x - 16 I b d f^2 x \log(\text{abs}(F)) + 16 I f^2) * e^{(1/2 I \pi b d x \text{sgn}(F) - 1/2 I \pi b d x + 1/2 I \pi b c \text{sgn}(F) - 1/2 I \pi b c + 1/2 I \pi a \text{sgn}(F) - 1/2 I \pi a) / (-4 I \pi^3 b^3 d^3 \text{sgn}(F) + 12 \pi^2 b^3 d^3 \log(\text{abs}(F)) \text{sgn}(F) + 12 I \pi b^3 d^3 \log(\text{abs}(F)))^2 \text{sgn}(F) + 4 I \pi^3 b^3 d^3 - 12 \pi^2 b^3 d^3 \log(\text{abs}(F)) - 12 I \pi b^3 d^3 \log(\text{abs}(F))^2 + 8 b^3 d^3 \log(\text{abs}(F))^3) - (4 I \pi^2 b^2 d^2 f^2 x^2 \text{sgn}(F) + 8 \pi b^2 d^2 f^2 x^2 \log(\text{abs}(F)) \text{sgn}(F) - 4 I \pi^2 b^2 d^2 f^2 x^2 - 8 \pi b^2 d^2 f^2 x^2 \log(\text{abs}(F)) + 8 I b^2 d^2 f^2 x^2 \log(\text{abs}(F))^2 - 8 \pi
\end{aligned}$$

$$\begin{aligned}
& i*b*d*f^2*x*sgn(F) + 8*\pi*b*d*f^2*x - 16*I*b*d*f^2*x*\log(\text{abs}(F)) + 16*I*f^2 \\
& )*e^{(-1/2*I*\pi*b*d*x*sgn(F) + 1/2*I*\pi*b*d*x - 1/2*I*\pi*b*c*sgn(F) + 1/2*I* \\
& \pi*b*c - 1/2*I*\pi*a*sgn(F) + 1/2*I*\pi*a)/(4*I*\pi^3*b^3*d^3*sgn(F) + 12*\pi^2 \\
& *b^3*d^3*\log(\text{abs}(F))*sgn(F) - 12*I*\pi*b^3*d^3*\log(\text{abs}(F))^2*sgn(F) - 4*I*\pi \\
& ^3*b^3*d^3 - 12*\pi^2*b^3*d^3*\log(\text{abs}(F)) + 12*I*\pi*b^3*d^3*\log(\text{abs}(F))^2 + \\
& 8*b^3*d^3*\log(\text{abs}(F))^3)}*e^{(b*d*x*\log(\text{abs}(F)) + b*c*\log(\text{abs}(F)) + a*\log(\text{ab} \\
& s(F)))}
\end{aligned}$$

$$3.69 \quad \int \frac{F^{a+b(c+dx)}(e+fx)^2}{x} dx$$

**Optimal.** Leaf size=96

$$-\frac{f^2 F^{a+bc+bdx}}{b^2 d^2 \log^2(F)} + e^2 F^{a+bc} \text{Ei}(bdx \log(F)) + \frac{2ef F^{a+bc+bdx}}{bd \log(F)} + \frac{f^2 x F^{a+bc+bdx}}{bd \log(F)}$$

[Out]  $e^{2*F^{(a + b*c)}*ExpIntegralEi[b*d*x*Log[F]] - (f^2*F^{(a + b*c + b*d*x)})/(b^2*d^2*Log[F]^2) + (2*e*f*F^{(a + b*c + b*d*x)})/(b*d*Log[F]) + (f^2*F^{(a + b*c + b*d*x)*x})/(b*d*Log[F])$

**Rubi [A]** time = 0.257749, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$ , Rules used = {2199, 2194, 2178, 2176}

$$-\frac{f^2 F^{a+bc+bdx}}{b^2 d^2 \log^2(F)} + e^2 F^{a+bc} \text{Ei}(bdx \log(F)) + \frac{2ef F^{a+bc+bdx}}{bd \log(F)} + \frac{f^2 x F^{a+bc+bdx}}{bd \log(F)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(F^{(a + b*(c + d*x))}*(e + f*x)^2)/x, x]$

[Out]  $e^{2*F^{(a + b*c)}*ExpIntegralEi[b*d*x*Log[F]] - (f^2*F^{(a + b*c + b*d*x)})/(b^2*d^2*Log[F]^2) + (2*e*f*F^{(a + b*c + b*d*x)})/(b*d*Log[F]) + (f^2*F^{(a + b*c + b*d*x)*x})/(b*d*Log[F])$

### Rule 2199

$\text{Int}[(F_{-})^{((c_{-})*(v_{-}))*(u_{-})^{(m_{-})}*(w_{-})}, x\_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[F^{(c*\text{ExpandToSum}[v, x]), w*\text{NormalizePowerOfLinear}[u, x]^m, x}], x] /; \text{FreeQ}\{F, c\}, x \} \&\& \text{PolynomialQ}[w, x] \&\& \text{LinearQ}[v, x] \&\& \text{PowerOfLinearQ}[u, x] \&\& \text{IntegerQ}[m] \&\& !\$UseGamma == True$

### Rule 2194

$\text{Int}[(F_{-})^{((c_{-})*((a_{-}) + (b_{-})*(x_{-})))^{(n_{-})}, x\_Symbol] \rightarrow \text{Simp}[(F^{(c*(a + b*x)))^n/(b*c*n*Log[F]), x] /; \text{FreeQ}\{F, a, b, c, n\}, x]$

### Rule 2178

$\text{Int}[(F_{-})^{((g_{-})*((e_{-}) + (f_{-})*(x_{-})))^{(c_{-}) + (d_{-})*(x_{-})}, x\_Symbol] \rightarrow \text{Simp}[(F^{(g*(e - (c*f)/d)}*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; F$

```
reeQ[{F, c, d, e, f, g}, x] && !$UseGamma === True
```

### Rule 2176

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[((c + d*x)^m*(b*F^(g*(e + f*x)))^n)/(f*g*n*Log[F]), x] - Dist[(d*m)/(f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m] && !$UseGamma === True
```

### Rubi steps

$$\begin{aligned} \int \frac{F^{a+b(c+dx)}(e+fx)^2}{x} dx &= \int \left( 2efF^{a+bc+bdx} + \frac{e^2F^{a+bc+bdx}}{x} + f^2F^{a+bc+bdx}x \right) dx \\ &= e^2 \int \frac{F^{a+bc+bdx}}{x} dx + (2ef) \int F^{a+bc+bdx} dx + f^2 \int F^{a+bc+bdx}x dx \\ &= e^2F^{a+bc}\text{Ei}(bdx \log(F)) + \frac{2efF^{a+bc+bdx}}{bd \log(F)} + \frac{f^2F^{a+bc+bdx}x}{bd \log(F)} - \frac{f^2 \int F^{a+bc+bdx} dx}{bd \log(F)} \\ &= e^2F^{a+bc}\text{Ei}(bdx \log(F)) - \frac{f^2F^{a+bc+bdx}}{b^2d^2 \log^2(F)} + \frac{2efF^{a+bc+bdx}}{bd \log(F)} + \frac{f^2F^{a+bc+bdx}x}{bd \log(F)} \end{aligned}$$

**Mathematica [A]** time = 0.125699, size = 54, normalized size = 0.56

$$F^{a+bc} \left( \frac{fF^{bdx}(bd \log(F)(2e+fx) - f)}{b^2d^2 \log^2(F)} + e^2\text{Ei}(bdx \log(F)) \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(F^(a + b*(c + d*x))*(e + f*x)^2)/x,x]
```

```
[Out] F^(a + b*c)*(e^2*ExpIntegralEi[b*d*x*Log[F]] + (f*F^(b*d*x)*(-f + b*d*(2*e + f*x)*Log[F]))/(b^2*d^2*Log[F]^2))
```

**Maple [A]** time = 0.045, size = 126, normalized size = 1.3

$$-e^2F^{bc}F^a\text{Ei}(1, bc \ln(F) + \ln(F)a - bdx \ln(F) - (bc + a) \ln(F)) + \frac{f^2F^{bdx}F^{bc+a}x}{bd \ln(F)} - \frac{f^2F^{bdx}F^{bc+a}}{b^2d^2 (\ln(F))^2} + 2 \frac{feF^{bdx}F^{bc+a}}{bd \ln(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(F^(a+b*(d*x+c))*(f*x+e)^2/x,x)`

[Out]  $-e^{2*F^{(b*c)}}*F^a*Ei(1,b*c*\ln(F)+\ln(F)*a-b*d*x*\ln(F)-(b*c+a)*\ln(F))+f^2/\ln(F)/b/d*F^{(b*d*x)}*F^{(b*c+a)}*x-f^2/\ln(F)^2/b^2/d^2*F^{(b*d*x)}*F^{(b*c+a)}+2*f/\ln(F)/b/d*e*F^{(b*d*x)}*F^{(b*c+a)}$

**Maxima [A]** time = 1.15666, size = 117, normalized size = 1.22

$$F^{bc+a}e^2Ei(bdx \log(F)) + \frac{2F^{bdx+bc+a}ef}{bd \log(F)} + \frac{(F^{bc+a}bdx \log(F) - F^{bc+a})F^{bdx}f^2}{b^2d^2 \log(F)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(a+b*(d*x+c))*(f*x+e)^2/x,x, algorithm="maxima")`

[Out]  $F^{(b*c + a)}*e^{2*Ei(b*d*x*\log(F))} + 2*F^{(b*d*x + b*c + a)}*e*f/(b*d*\log(F)) + (F^{(b*c + a)}*b*d*x*\log(F) - F^{(b*c + a)})*F^{(b*d*x)}*f^2/(b^2*d^2*\log(F)^2)$

**Fricas [A]** time = 1.52764, size = 180, normalized size = 1.88

$$\frac{F^{bc+a}b^2d^2e^2Ei(bdx \log(F)) \log(F)^2 - (f^2 - (bdf^2x + 2bdef) \log(F))F^{bdx+bc+a}}{b^2d^2 \log(F)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(a+b*(d*x+c))*(f*x+e)^2/x,x, algorithm="fricas")`

[Out]  $(F^{(b*c + a)}*b^2*d^2*e^2*Ei(b*d*x*\log(F))*\log(F)^2 - (f^2 - (b*d*f^2*x + 2*b*d*e*f)*\log(F))*F^{(b*d*x + b*c + a)})/(b^2*d^2*\log(F)^2)$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{a+b(c+dx)}(e+fx)^2}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(a+b*(d*x+c))*(f*x+e)**2/x,x)
```

```
[Out] Integral(F**(a + b*(c + d*x))*(e + f*x)**2/x, x)
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx + e)^2 F^{(dx+c)b+a}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(a+b*(d*x+c))*(f*x+e)^2/x,x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^2*F^((d*x + c)*b + a)/x, x)
```

$$3.70 \quad \int \frac{F^{a+b(c+dx)}(e+fx)^2}{x^2} dx$$

**Optimal.** Leaf size=85

$$bde^2 \log(F)F^{a+bc} \text{Ei}(bdx \log(F)) - \frac{e^2 F^{a+bc+bdx}}{x} + 2efF^{a+bc} \text{Ei}(bdx \log(F)) + \frac{f^2 F^{a+bc+bdx}}{bd \log(F)}$$

[Out]  $-\left(\frac{e^{2F^{a+b(c+dx)}}}{x}\right) + 2efF^{a+bc} \text{ExpIntegralEi}[b*d*x*\text{Log}[F]] + \frac{f^2 F^{a+b(c+dx)}}{b*d*\text{Log}[F]} + b*d*e^{2F^{a+b(c+dx)}} \text{ExpIntegralEi}[b*d*x*\text{Log}[F]]*\text{Log}[F]$

**Rubi [A]** time = 0.276619, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$ , Rules used = {2199, 2194, 2177, 2178}

$$bde^2 \log(F)F^{a+bc} \text{Ei}(bdx \log(F)) - \frac{e^2 F^{a+bc+bdx}}{x} + 2efF^{a+bc} \text{Ei}(bdx \log(F)) + \frac{f^2 F^{a+bc+bdx}}{bd \log(F)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(F^{a+b(c+dx)})(e+fx)^2/x^2, x]$

[Out]  $-\left(\frac{e^{2F^{a+b(c+dx)}}}{x}\right) + 2efF^{a+bc} \text{ExpIntegralEi}[b*d*x*\text{Log}[F]] + \frac{f^2 F^{a+b(c+dx)}}{b*d*\text{Log}[F]} + b*d*e^{2F^{a+b(c+dx)}} \text{ExpIntegralEi}[b*d*x*\text{Log}[F]]*\text{Log}[F]$

### Rule 2199

$\text{Int}[(F_{-})^{((c_{-})*(v_{-}))*(u_{-})^{(m_{-})}*(w_{-})}, x_{\text{Symbol}}] \rightarrow \text{Int}[\text{ExpandIntegrand}[F^{(c*\text{ExpandToSum}[v, x])}, w*\text{NormalizePowerOfLinear}[u, x]^m, x], x] /; \text{FreeQ}\{F, c\}, x \&\& \text{PolynomialQ}[w, x] \&\& \text{LinearQ}[v, x] \&\& \text{PowerOfLinearQ}[u, x] \&\& \text{IntegerQ}[m] \&\& !\$UseGamma == True$

### Rule 2194

$\text{Int}[(F_{-})^{((c_{-})*((a_{-}) + (b_{-})*(x_{-})))^{(n_{-})}, x_{\text{Symbol}}] \rightarrow \text{Simp}[(F^{(c*(a + b*x))})^n/(b*c*n*\text{Log}[F]), x] /; \text{FreeQ}\{F, a, b, c, n\}, x]$

### Rule 2177

$\text{Int}[(b_{-})*(F_{-})^{((g_{-})*((e_{-}) + (f_{-})*(x_{-})))^{(n_{-})}*((c_{-}) + (d_{-})*(x_{-}))^{(m_{-})}, x_{\text{Symbol}}] \rightarrow \text{Simp}[(c + d*x)^{(m+1)}*(b*F^{(g*(e + f*x))})^n/(d*(m+1))$

```
, x] - Dist[(f*g*n*Log[F])/(d*(m + 1)), Int[(c + d*x)^(m + 1)*(b*F^(g*(e + f*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && LtQ[m, -1] && IntegerQ[2*m] && !$UseGamma == True
```

### Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; FreeQ[{F, c, d, e, f, g}, x] && !$UseGamma == True
```

### Rubi steps

$$\begin{aligned} \int \frac{F^{a+b(c+dx)}(e+fx)^2}{x^2} dx &= \int \left( f^2 F^{a+bc+bdx} + \frac{e^2 F^{a+bc+bdx}}{x^2} + \frac{2ef F^{a+bc+bdx}}{x} \right) dx \\ &= e^2 \int \frac{F^{a+bc+bdx}}{x^2} dx + (2ef) \int \frac{F^{a+bc+bdx}}{x} dx + f^2 \int F^{a+bc+bdx} dx \\ &= -\frac{e^2 F^{a+bc+bdx}}{x} + 2ef F^{a+bc} \text{Ei}(bdx \log(F)) + \frac{f^2 F^{a+bc+bdx}}{bd \log(F)} + (bde^2 \log(F)) \int \frac{F^{a+bc+bdx}}{x} dx \\ &= -\frac{e^2 F^{a+bc+bdx}}{x} + 2ef F^{a+bc} \text{Ei}(bdx \log(F)) + \frac{f^2 F^{a+bc+bdx}}{bd \log(F)} + bde^2 F^{a+bc} \text{Ei}(bdx \log(F)) \log(F) \end{aligned}$$

**Mathematica [A]** time = 0.149333, size = 58, normalized size = 0.68

$$F^{a+bc} \left( F^{bdx} \left( \frac{f^2}{bd \log(F)} - \frac{e^2}{x} \right) + e(bde \log(F) + 2f) \text{Ei}(bdx \log(F)) \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(F^(a + b*(c + d*x))*(e + f*x)^2)/x^2,x]
```

```
[Out] F^(a + b*c)*(F^(b*d*x)*(-(e^2/x) + f^2/(b*d*Log[F]))) + e*ExpIntegralEi[b*d*x*Log[F]]*(2*f + b*d*e*Log[F])
```

**Maple [A]** time = 0.053, size = 135, normalized size = 1.6

$$-\frac{e^2 F^{bdx} F^{bc+a}}{x} + \frac{f^2 F^{bdx} F^{bc+a}}{bd \ln(F)} - \ln(F) bde^2 F^{bc} F^a \text{Ei}(1, bc \ln(F) + \ln(F) a - bdx \ln(F) - (bc + a) \ln(F)) - 2 f e F^{bc} F^a \text{Ei}(1, bc \ln(F) + \ln(F) a - bdx \ln(F) - (bc + a) \ln(F))$$

Verification of antiderivative is not currently implemented for this CAS.



[In] `int(F^(a+b*(d*x+c))*(f*x+e)^2/x^2,x)`

[Out]  $-e^{2*F^{(b*d*x)}*F^{(b*c+a)}/x+1/\ln(F)}/b/d*f^2*F^{(b*d*x)}*F^{(b*c+a)}-\ln(F)*b*d*e^{2*F^{(b*c)}*F^a*Ei(1,b*c*\ln(F)+\ln(F)*a-b*d*x*\ln(F)-(b*c+a)*\ln(F))}-2*f*e*F^{(b*c)}*F^a*Ei(1,b*c*\ln(F)+\ln(F)*a-b*d*x*\ln(F)-(b*c+a)*\ln(F))$

**Maxima [A]** time = 1.19312, size = 92, normalized size = 1.08

$$F^{bc+a}bde^2\Gamma(-1,-bdx\log(F))\log(F) + 2F^{bc+a}efEi(bdx\log(F)) + \frac{F^{bdx+bc+a}f^2}{bd\log(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(a+b*(d*x+c))*(f*x+e)^2/x^2,x, algorithm="maxima")`

[Out]  $F^{(b*c + a)*b*d*e^2*\gamma(-1, -b*d*x*\log(F))*\log(F) + 2*F^{(b*c + a)*e*f*Ei(b*d*x*\log(F)) + F^{(b*d*x + b*c + a)*f^2/(b*d*\log(F))}$

**Fricas [A]** time = 1.49197, size = 189, normalized size = 2.22

$$\frac{(b^2d^2e^2x\log(F)^2 + 2bdefx\log(F))F^{bc+a}Ei(bdx\log(F)) - (bde^2\log(F) - f^2x)F^{bdx+bc+a}}{bdx\log(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(a+b*(d*x+c))*(f*x+e)^2/x^2,x, algorithm="fricas")`

[Out]  $((b^2*d^2*e^2*x*\log(F)^2 + 2*b*d*e*f*x*\log(F))*F^{(b*c + a)*Ei(b*d*x*\log(F))} - (b*d*e^2*\log(F) - f^2*x)*F^{(b*d*x + b*c + a)})/(b*d*x*\log(F))$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{a+b(c+dx)}(e+fx)^2}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(a+b*(d*x+c))*(f*x+e)**2/x**2,x)
```

```
[Out] Integral(F**(a + b*(c + d*x))*(e + f*x)**2/x**2, x)
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx + e)^2 F^{(dx+c)b+a}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(a+b*(d*x+c))*(f*x+e)^2/x^2,x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^2*F^((d*x + c)*b + a)/x^2, x)
```

$$3.71 \quad \int \frac{F^{a+b(c+dx)}(e+fx)^2}{x^3} dx$$

**Optimal.** Leaf size=136

$$\frac{1}{2}b^2d^2e^2 \log^2(F)F^{a+bc} \text{Ei}(bdx \log(F)) - \frac{e^2F^{a+bc+bdx}}{2x^2} - \frac{bde^2 \log(F)F^{a+bc+bdx}}{2x} + 2bdef \log(F)F^{a+bc} \text{Ei}(bdx \log(F)) - \frac{2efF^{a+b(c+dx)}}{x}$$

[Out]  $-(e^2F^{a+b(c+dx)})/(2x^2) - (2efF^{a+b(c+dx)})/x + f^2F^{a+b(c+dx)} \text{ExpIntegralEi}[b*d*x*\text{Log}[F]] - (b*d*e^2F^{a+b(c+dx)}*\text{Log}[F])/ (2*x) + 2*b*d*e*fF^{a+b(c+dx)}*\text{ExpIntegralEi}[b*d*x*\text{Log}[F]]*\text{Log}[F] + (b^2*d^2*e^2F^{a+b(c+dx)}*\text{ExpIntegralEi}[b*d*x*\text{Log}[F]]*\text{Log}[F]^2)/2$

**Rubi [A]** time = 0.362979, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 3, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$ , Rules used = {2199, 2177, 2178}

$$\frac{1}{2}b^2d^2e^2 \log^2(F)F^{a+bc} \text{Ei}(bdx \log(F)) - \frac{e^2F^{a+bc+bdx}}{2x^2} - \frac{bde^2 \log(F)F^{a+bc+bdx}}{2x} + 2bdef \log(F)F^{a+bc} \text{Ei}(bdx \log(F)) - \frac{2efF^{a+b(c+dx)}}{x}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(F^{a+b(c+dx)})(e+fx)^2/x^3, x]$

[Out]  $-(e^2F^{a+b(c+dx)})/(2x^2) - (2efF^{a+b(c+dx)})/x + f^2F^{a+b(c+dx)} \text{ExpIntegralEi}[b*d*x*\text{Log}[F]] - (b*d*e^2F^{a+b(c+dx)}*\text{Log}[F])/ (2*x) + 2*b*d*e*fF^{a+b(c+dx)}*\text{ExpIntegralEi}[b*d*x*\text{Log}[F]]*\text{Log}[F] + (b^2*d^2*e^2F^{a+b(c+dx)}*\text{ExpIntegralEi}[b*d*x*\text{Log}[F]]*\text{Log}[F]^2)/2$

### Rule 2199

$\text{Int}[(F_)^{((c_.)*(v_))*(u_)^{(m_)}*(w_)}, x\_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[F^{(c*\text{ExpandToSum}[v, x])}, w*\text{NormalizePowerOfLinear}[u, x]^m, x], x] /;$  FreeQ[{F, c}, x] && PolynomialQ[w, x] && LinearQ[v, x] && PowerOfLinearQ[u, x] && IntegerQ[m] && !\$UseGamma == True

### Rule 2177

$\text{Int}[(b_.)*(F_)^{((g_.)*((e_.) + (f_.)*(x_)))^{(n_)}*((c_.) + (d_.)*(x_))^{(m_)}}, x\_Symbol] \rightarrow \text{Simp}[(c + dx)^{(m+1)}*(bF^{(g*(e+fx))})^n / (d*(m+1)), x] - \text{Dist}[(f*g*n*\text{Log}[F]) / (d*(m+1)), \text{Int}[(c + dx)^{(m+1)}*(bF^{(g*(e+fx))})^n, x], x] /;$  FreeQ[{F, b, c, d, e, f, g, n}, x] && LtQ[m, -1] && Int

egerQ[2\*m] && !\$UseGamma == True

### Rule 2178

Int[(F\_)^((g\_)\*(e\_) + (f\_)\*(x\_)))/((c\_) + (d\_)\*(x\_)), x\_Symbol] := Simp[(F^(g\*(e - (c\*f)/d))\*ExpIntegralEi[(f\*g\*(c + d\*x)\*Log[F])/d])/d, x] /; FreeQ[{F, c, d, e, f, g}, x] && !\$UseGamma == True

### Rubi steps

$$\begin{aligned} \int \frac{F^{a+b(c+dx)}(e+fx)^2}{x^3} dx &= \int \left( \frac{e^2 F^{a+bc+bdx}}{x^3} + \frac{2ef F^{a+bc+bdx}}{x^2} + \frac{f^2 F^{a+bc+bdx}}{x} \right) dx \\ &= e^2 \int \frac{F^{a+bc+bdx}}{x^3} dx + (2ef) \int \frac{F^{a+bc+bdx}}{x^2} dx + f^2 \int \frac{F^{a+bc+bdx}}{x} dx \\ &= -\frac{e^2 F^{a+bc+bdx}}{2x^2} - \frac{2ef F^{a+bc+bdx}}{x} + f^2 F^{a+bc} \text{Ei}(bdx \log(F)) + \frac{1}{2} (bde^2 \log(F)) \int \frac{F^{a+bc+bdx}}{x^2} dx + (2ef) \int \frac{F^{a+bc+bdx}}{x} dx \\ &= -\frac{e^2 F^{a+bc+bdx}}{2x^2} - \frac{2ef F^{a+bc+bdx}}{x} + f^2 F^{a+bc} \text{Ei}(bdx \log(F)) - \frac{bde^2 F^{a+bc+bdx} \log(F)}{2x} + 2bdef F^{a+bc} \text{Ei}(bdx \log(F)) \\ &= -\frac{e^2 F^{a+bc+bdx}}{2x^2} - \frac{2ef F^{a+bc+bdx}}{x} + f^2 F^{a+bc} \text{Ei}(bdx \log(F)) - \frac{bde^2 F^{a+bc+bdx} \log(F)}{2x} + 2bdef F^{a+bc} \text{Ei}(bdx \log(F)) \end{aligned}$$

**Mathematica [A]** time = 0.14826, size = 76, normalized size = 0.56

$$\frac{F^{a+bc} \left( x^2 \left( b^2 d^2 e^2 \log^2(F) + 4bdef \log(F) + 2f^2 \right) \text{Ei}(bdx \log(F)) - eF^{bdx} (bdex \log(F) + e + 4fx) \right)}{2x^2}$$

Antiderivative was successfully verified.

[In] Integrate[(F^(a + b\*(c + d\*x))\*(e + f\*x)^2)/x^3,x]

[Out] (F^(a + b\*c)\*(-(e\*F^(b\*d\*x)\*(e + 4\*f\*x + b\*d\*e\*x\*Log[F])) + x^2\*ExpIntegralEi[b\*d\*x\*Log[F]]\*(2\*f^2 + 4\*b\*d\*e\*f\*Log[F] + b^2\*d^2\*e^2\*Log[F]^2)))/(2\*x^2)

**Maple [A]** time = 0.059, size = 204, normalized size = 1.5

$$-\frac{b^2 d^2 (\ln(F))^2 e^2 F^{bc} F^a \text{Ei}(1, bc \ln(F) + \ln(F) a - bdx \ln(F) - (bc + a) \ln(F))}{2} - f^2 F^{bc} F^a \text{Ei}(1, bc \ln(F) + \ln(F) a - bdx \ln(F))$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}(F^{(a+b*(d*x+c))}*(f*x+e)^2/x^3,x)$

[Out]  $-1/2*b^2*d^2*\ln(F)^2*e^2*F^{(b*c)}*F^a*Ei(1,b*c*\ln(F)+\ln(F)*a-b*d*x*\ln(F)-(b*c+a)*\ln(F))-f^2*F^{(b*c)}*F^a*Ei(1,b*c*\ln(F)+\ln(F)*a-b*d*x*\ln(F)-(b*c+a)*\ln(F))-2*f*e*F^{(b*d*x)}*F^{(b*c+a)}/x-2*b*d*\ln(F)*f*e*F^{(b*c)}*F^a*Ei(1,b*c*\ln(F)+\ln(F)*a-b*d*x*\ln(F)-(b*c+a)*\ln(F))-1/2*b*d*\ln(F)*e^2*F^{(b*d*x)}*F^{(b*c+a)}/x-1/2*e^2*F^{(b*d*x)}*F^{(b*c+a)}/x^2$

**Maxima [A]** time = 1.32388, size = 100, normalized size = 0.74

$-F^{bc+a}b^2d^2e^2\Gamma(-2,-bdx \log(F)) \log(F)^2 + 2F^{bc+a}bdef\Gamma(-1,-bdx \log(F)) \log(F) + F^{bc+a}f^2Ei(bdx \log(F))$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}(F^{(a+b*(d*x+c))}*(f*x+e)^2/x^3,x, \text{algorithm}="maxima")$

[Out]  $-F^{(b*c + a)}*b^2*d^2*e^2*\text{gamma}(-2, -b*d*x*\log(F))*\log(F)^2 + 2*F^{(b*c + a)}*b*d*e*f*\text{gamma}(-1, -b*d*x*\log(F))*\log(F) + F^{(b*c + a)}*f^2*Ei(b*d*x*\log(F))$

**Fricas [A]** time = 1.50513, size = 215, normalized size = 1.58

$$\frac{(b^2d^2e^2x^2 \log(F)^2 + 4bdefx^2 \log(F) + 2f^2x^2)F^{bc+a}Ei(bdx \log(F)) - (bde^2x \log(F) + 4efx + e^2)F^{bdx+bc+a}}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}(F^{(a+b*(d*x+c))}*(f*x+e)^2/x^3,x, \text{algorithm}="fricas")$

[Out]  $1/2*((b^2*d^2*e^2*x^2*\log(F)^2 + 4*b*d*e*f*x^2*\log(F) + 2*f^2*x^2)*F^{(b*c + a)}*Ei(b*d*x*\log(F)) - (b*d*e^2*x*\log(F) + 4*e*f*x + e^2)*F^{(b*d*x + b*c + a)})/x^2$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{a+b(c+dx)}(e+fx)^2}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(a+b\*(d\*x+c))\*(f\*x+e)\*\*2/x\*\*3,x)

[Out] Integral(F\*\*(a + b\*(c + d\*x))\*(e + f\*x)\*\*2/x\*\*3, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx + e)^2 F^{(dx+c)b+a}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*(d\*x+c))\*(f\*x+e)^2/x^3,x, algorithm="giac")

[Out] integrate((f\*x + e)^2\*F^((d\*x + c)\*b + a)/x^3, x)

$$3.72 \quad \int \frac{F^{a+b(c+dx)}(e+fx)^2}{x^4} dx$$

**Optimal.** Leaf size=217

$$\frac{1}{6}b^3d^3e^2 \log^3(F)F^{a+bc} \text{Ei}(bdx \log(F)) - \frac{b^2d^2e^2 \log^2(F)F^{a+bc+bdx}}{6x} + b^2d^2ef \log^2(F)F^{a+bc} \text{Ei}(bdx \log(F)) - \frac{e^2F^{a+bc+bdx}}{3x^3} - \frac{bd}{6}$$

[Out]  $-(e^2F^{a+b*c+b*d*x})/(3*x^3) - (e*f*F^{a+b*c+b*d*x})/x^2 - (f^2*F^{a+b*c+b*d*x})/x - (b*d*e^2*F^{a+b*c+b*d*x}*Log[F])/(6*x^2) - (b*d*e*f*F^{a+b*c+b*d*x}*Log[F])/x + b*d*f^2*F^{a+b*c}*ExpIntegralEi[b*d*x*Log[F]]*Log[F] - (b^2*d^2*e^2*F^{a+b*c+b*d*x}*Log[F]^2)/(6*x) + b^2*d^2*e*f*F^{a+b*c}*ExpIntegralEi[b*d*x*Log[F]]*Log[F]^2 + (b^3*d^3*e^2*F^{a+b*c}*ExpIntegralEi[b*d*x*Log[F]]*Log[F]^3)/6$

**Rubi [A]** time = 0.458988, antiderivative size = 217, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 3, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$ , Rules used = {2199, 2177, 2178}

$$\frac{1}{6}b^3d^3e^2 \log^3(F)F^{a+bc} \text{Ei}(bdx \log(F)) - \frac{b^2d^2e^2 \log^2(F)F^{a+bc+bdx}}{6x} + b^2d^2ef \log^2(F)F^{a+bc} \text{Ei}(bdx \log(F)) - \frac{e^2F^{a+bc+bdx}}{3x^3} - \frac{bd}{6}$$

Antiderivative was successfully verified.

[In] Int[(F^(a + b\*(c + d\*x))\*(e + f\*x)^2)/x^4,x]

[Out]  $-(e^2*F^{a+b*c+b*d*x})/(3*x^3) - (e*f*F^{a+b*c+b*d*x})/x^2 - (f^2*F^{a+b*c+b*d*x})/x - (b*d*e^2*F^{a+b*c+b*d*x}*Log[F])/(6*x^2) - (b*d*e*f*F^{a+b*c+b*d*x}*Log[F])/x + b*d*f^2*F^{a+b*c}*ExpIntegralEi[b*d*x*Log[F]]*Log[F] - (b^2*d^2*e^2*F^{a+b*c+b*d*x}*Log[F]^2)/(6*x) + b^2*d^2*e*f*F^{a+b*c}*ExpIntegralEi[b*d*x*Log[F]]*Log[F]^2 + (b^3*d^3*e^2*F^{a+b*c}*ExpIntegralEi[b*d*x*Log[F]]*Log[F]^3)/6$

### Rule 2199

```
Int[(F_)^((c_.)*(v_))*(u_)^(m_.)*(w_), x_Symbol] :> Int[ExpandIntegrand[F^(c*ExpandToSum[v, x]), w*NormalizePowerOfLinear[u, x]^m, x] /; FreeQ[{F, c}, x] && PolynomialQ[w, x] && LinearQ[v, x] && PowerOfLinearQ[u, x] && IntegerQ[m] && !$UseGamma === True
```

### Rule 2177

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_)))^(n_.)*((c_.) + (d_.)*(x_))^(m
_), x_Symbol] := Simp[((c + d*x)^(m + 1)*(b*F^(g*(e + f*x)))^n)/(d*(m + 1))
, x] - Dist[(f*g*n*Log[F])/(d*(m + 1)), Int[(c + d*x)^(m + 1)*(b*F^(g*(e +
f*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && LtQ[m, -1] && Int
egerQ[2*m] && !$UseGamma == True
```

### Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] := Si
mp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; F
reeQ[{F, c, d, e, f, g}, x] && !$UseGamma == True
```

### Rubi steps

$$\begin{aligned} \int \frac{F^{a+b(c+dx)}(e+fx)^2}{x^4} dx &= \int \left( \frac{e^2 F^{a+bc+bdx}}{x^4} + \frac{2ef F^{a+bc+bdx}}{x^3} + \frac{f^2 F^{a+bc+bdx}}{x^2} \right) dx \\ &= e^2 \int \frac{F^{a+bc+bdx}}{x^4} dx + (2ef) \int \frac{F^{a+bc+bdx}}{x^3} dx + f^2 \int \frac{F^{a+bc+bdx}}{x^2} dx \\ &= -\frac{e^2 F^{a+bc+bdx}}{3x^3} - \frac{ef F^{a+bc+bdx}}{x^2} - \frac{f^2 F^{a+bc+bdx}}{x} + \frac{1}{3} (bde^2 \log(F)) \int \frac{F^{a+bc+bdx}}{x^3} dx + (bdef \log(F)) \int \frac{F^{a+bc+bdx}}{x^2} dx \\ &= -\frac{e^2 F^{a+bc+bdx}}{3x^3} - \frac{ef F^{a+bc+bdx}}{x^2} - \frac{f^2 F^{a+bc+bdx}}{x} - \frac{bde^2 F^{a+bc+bdx} \log(F)}{6x^2} - \frac{bdef F^{a+bc+bdx} \log(F)}{x} + b \\ &= -\frac{e^2 F^{a+bc+bdx}}{3x^3} - \frac{ef F^{a+bc+bdx}}{x^2} - \frac{f^2 F^{a+bc+bdx}}{x} - \frac{bde^2 F^{a+bc+bdx} \log(F)}{6x^2} - \frac{bdef F^{a+bc+bdx} \log(F)}{x} + b \\ &= -\frac{e^2 F^{a+bc+bdx}}{3x^3} - \frac{ef F^{a+bc+bdx}}{x^2} - \frac{f^2 F^{a+bc+bdx}}{x} - \frac{bde^2 F^{a+bc+bdx} \log(F)}{6x^2} - \frac{bdef F^{a+bc+bdx} \log(F)}{x} + b \end{aligned}$$

**Mathematica [A]** time = 0.223677, size = 116, normalized size = 0.53

$$\frac{F^{a+bc} \left( bdx^3 \log(F) \left( b^2 d^2 e^2 \log^2(F) + 6bdef \log(F) + 6f^2 \right) \text{Ei}(bdx \log(F)) - F^{bdx} \left( b^2 d^2 e^2 x^2 \log^2(F) + bdx \log(F)(e + 6fx) \right) \right)}{6x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(F^(a + b*(c + d*x))*(e + f*x)^2)/x^4, x]
```

```
[Out] (F^(a + b*c)*(b*d*x^3*ExpIntegralEi[b*d*x*Log[F]]*Log[F]*(6*f^2 + 6*b*d*e*f
*Log[F] + b^2*d^2*e^2*Log[F]^2) - F^(b*d*x)*(2*(e^2 + 3*e*f*x + 3*f^2*x^2)
+ b*d*e*x*(e + 6*f*x)*Log[F] + b^2*d^2*e^2*x^2*Log[F]^2))/(6*x^3)
```



---

**Maple [A]** time = 0.065, size = 290, normalized size = 1.3

$$\frac{(\ln(F))^3 b^3 d^3 e^2 F^{bc} F^a \text{Ei}(1, bc \ln(F) + \ln(F) a - bdx \ln(F) - (bc + a) \ln(F))}{6} - \frac{fe F^{bdx} F^{bc+a}}{x^2} - \frac{fe \ln(F) b d F^{bdx} F^{bc+a}}{x} - (1$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(F^(a+b*(d*x+c))*(f*x+e)^2/x^4,x)`

[Out]  $-1/6 * \ln(F)^3 * b^3 * d^3 * e^2 * F^{(b*c)} * F^a * \text{Ei}(1, b*c*\ln(F) + \ln(F)*a - b*d*x*\ln(F) - (b*c+a)*\ln(F)) - f*e * F^{(b*d*x)} * F^{(b*c+a)} / x^2 - \ln(F) * b*d*f*e * F^{(b*d*x)} * F^{(b*c+a)} / x - \ln(F)^2 * b^2 * d^2 * f * e * F^{(b*c)} * F^a * \text{Ei}(1, b*c*\ln(F) + \ln(F)*a - b*d*x*\ln(F) - (b*c+a)*\ln(F)) - 1/3 * e^2 * F^{(b*d*x)} * F^{(b*c+a)} / x^3 - 1/6 * \ln(F) * b*d * e^2 * F^{(b*d*x)} * F^{(b*c+a)} / x^2 - 1/6 * \ln(F)^2 * b^2 * d^2 * e^2 * F^{(b*d*x)} * F^{(b*c+a)} / x - f^2 * F^{(b*d*x)} * F^{(b*c+a)} / x - \ln(F) * b*d*f^2 * F^{(b*c)} * F^a * \text{Ei}(1, b*c*\ln(F) + \ln(F)*a - b*d*x*\ln(F) - (b*c+a)*\ln(F))$

---

**Maxima [A]** time = 1.21015, size = 115, normalized size = 0.53

$$F^{bc+a} b^3 d^3 e^2 \Gamma(-3, -bdx \log(F)) \log(F)^3 - 2 F^{bc+a} b^2 d^2 e f \Gamma(-2, -bdx \log(F)) \log(F)^2 + F^{bc+a} b d f^2 \Gamma(-1, -bdx \log(F)) \log(F)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(a+b*(d*x+c))*(f*x+e)^2/x^4,x, algorithm="maxima")`

[Out]  $F^{(b*c + a)} * b^3 * d^3 * e^2 * \text{gamma}(-3, -b*d*x*\log(F)) * \log(F)^3 - 2 * F^{(b*c + a)} * b^2 * d^2 * e * f * \text{gamma}(-2, -b*d*x*\log(F)) * \log(F)^2 + F^{(b*c + a)} * b * d * f^2 * \text{gamma}(-1, -b*d*x*\log(F)) * \log(F)$

---

**Fricas [A]** time = 1.53015, size = 317, normalized size = 1.46

$$\frac{(b^3 d^3 e^2 x^3 \log(F)^3 + 6 b^2 d^2 e f x^3 \log(F)^2 + 6 b d f^2 x^3 \log(F)) F^{bc+a} \text{Ei}(bdx \log(F)) - (b^2 d^2 e^2 x^2 \log(F)^2 + 6 f^2 x^2 + 6 e f x - 6 x^3)}{6 x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(a+b*(d*x+c))*(f*x+e)^2/x^4,x, algorithm="fricas")`

```
[Out] 1/6*((b^3*d^3*e^2*x^3*log(F)^3 + 6*b^2*d^2*e*f*x^3*log(F)^2 + 6*b*d*f^2*x^3
*log(F))*F^(b*c + a)*Ei(b*d*x*log(F)) - (b^2*d^2*e^2*x^2*log(F)^2 + 6*f^2*x
^2 + 6*e*f*x + 2*e^2 + (6*b*d*e*f*x^2 + b*d*e^2*x)*log(F))*F^(b*d*x + b*c +
a))/x^3
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{a+b(c+dx)} (e+fx)^2}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(a+b*(d*x+c))*(f*x+e)**2/x**4,x)
```

```
[Out] Integral(F**(a + b*(c + d*x))*(e + f*x)**2/x**4, x)
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)^2 F^{(dx+c)b+a}}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(a+b*(d*x+c))*(f*x+e)^2/x^4,x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^2*F^((d*x + c)*b + a)/x^4, x)
```

$$3.73 \quad \int \frac{F^{a+b(c+dx)}(e+fx)^2}{x^5} dx$$

**Optimal.** Leaf size=321

$$\frac{1}{24}b^4d^4e^2 \log^4(F)F^{a+bc} \text{Ei}(bdx \log(F)) - \frac{b^2d^2e^2 \log^2(F)F^{a+bc+bdx}}{24x^2} - \frac{b^3d^3e^2 \log^3(F)F^{a+bc+bdx}}{24x} + \frac{1}{3}b^3d^3ef \log^3(F)F^{a+bc} \text{Ei}(bdx \log(F))$$

[Out]  $-(e^2F^{a+bc+bdx})/(4x^4) - (2efF^{a+bc+bdx})/(3x^3) - (f^2F^{a+bc+bdx})/(2x^2) - (bde^2F^{a+bc+bdx} \text{Log}[F])/(12x^3) - (bde^2F^{a+bc+bdx} \text{Log}[F])/(3x^2) - (bdf^2F^{a+bc+bdx} \text{Log}[F])/(2x) - (b^2d^2e^2F^{a+bc+bdx} \text{Log}[F]^2)/(24x^2) - (b^2d^2e^2F^{a+bc+bdx} \text{Log}[F]^2)/(3x) + (b^2d^2f^2F^{a+bc+bdx} \text{ExpIntegralEi}[b^2dx \text{Log}[F]] \text{Log}[F]^2)/2 - (b^3d^3e^2F^{a+bc+bdx} \text{Log}[F]^3)/(24x) + (b^3d^3efF^{a+bc+bdx} \text{ExpIntegralEi}[bdx \text{Log}[F]] \text{Log}[F]^3)/3 + (b^4d^4e^2F^{a+bc+bdx} \text{ExpIntegralEi}[bdx \text{Log}[F]] \text{Log}[F]^4)/24$

**Rubi [A]** time = 0.577845, antiderivative size = 321, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 3, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$ , Rules used = {2199, 2177, 2178}

$$\frac{1}{24}b^4d^4e^2 \log^4(F)F^{a+bc} \text{Ei}(bdx \log(F)) - \frac{b^2d^2e^2 \log^2(F)F^{a+bc+bdx}}{24x^2} - \frac{b^3d^3e^2 \log^3(F)F^{a+bc+bdx}}{24x} + \frac{1}{3}b^3d^3ef \log^3(F)F^{a+bc} \text{Ei}(bdx \log(F))$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(F^{a+b(c+dx)})(e+fx)^2]/x^5, x]$

[Out]  $-(e^2F^{a+bc+bdx})/(4x^4) - (2efF^{a+bc+bdx})/(3x^3) - (f^2F^{a+bc+bdx})/(2x^2) - (bde^2F^{a+bc+bdx} \text{Log}[F])/(12x^3) - (bde^2F^{a+bc+bdx} \text{Log}[F])/(3x^2) - (bdf^2F^{a+bc+bdx} \text{Log}[F])/(2x) - (b^2d^2e^2F^{a+bc+bdx} \text{Log}[F]^2)/(24x^2) - (b^2d^2e^2F^{a+bc+bdx} \text{Log}[F]^2)/(3x) + (b^2d^2f^2F^{a+bc+bdx} \text{ExpIntegralEi}[b^2dx \text{Log}[F]] \text{Log}[F]^2)/2 - (b^3d^3e^2F^{a+bc+bdx} \text{Log}[F]^3)/(24x) + (b^3d^3efF^{a+bc+bdx} \text{ExpIntegralEi}[bdx \text{Log}[F]] \text{Log}[F]^3)/3 + (b^4d^4e^2F^{a+bc+bdx} \text{ExpIntegralEi}[bdx \text{Log}[F]] \text{Log}[F]^4)/24$

### Rule 2199

$\text{Int}[(F_+)^{(c_+)}(v_+)(u_+)^{(m_+)}(w_+), x\_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[F_+^{(c_+)} \text{ExpandToSum}[v_+, x]], w_+ \text{NormalizePowerOfLinear}[u_+, x]^m, x], x] /; \text{FreeQ}\{F,$

c}, x] && PolynomialQ[w, x] && LinearQ[v, x] && PowerOfLinearQ[u, x] && IntegerQ[m] && !\$UseGamma === True

### Rule 2177

Int[((b\_)\*(F\_)^((g\_)\*((e\_) + (f\_)\*(x\_)))^(n\_))\*((c\_) + (d\_)\*(x\_))^(m\_), x\_Symbol] := Simp[((c + d\*x)^(m + 1)\*(b\*F^(g\*(e + f\*x)))^n)/(d\*(m + 1)), x] - Dist[(f\*g\*n\*Log[F])/(d\*(m + 1)), Int[(c + d\*x)^(m + 1)\*(b\*F^(g\*(e + f\*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && LtQ[m, -1] && IntegerQ[2\*m] && !\$UseGamma === True

### Rule 2178

Int[(F\_)^((g\_)\*((e\_) + (f\_)\*(x\_)))/((c\_) + (d\_)\*(x\_)), x\_Symbol] := Simp[(F^(g\*(e - (c\*f)/d))\*ExpIntegralEi[(f\*g\*(c + d\*x)\*Log[F])/d])/d, x] /; FreeQ[{F, c, d, e, f, g}, x] && !\$UseGamma === True

### Rubi steps

$$\begin{aligned}
 \int \frac{F^{a+b(c+dx)}(e+fx)^2}{x^5} dx &= \int \left( \frac{e^2 F^{a+bc+bdx}}{x^5} + \frac{2ef F^{a+bc+bdx}}{x^4} + \frac{f^2 F^{a+bc+bdx}}{x^3} \right) dx \\
 &= e^2 \int \frac{F^{a+bc+bdx}}{x^5} dx + (2ef) \int \frac{F^{a+bc+bdx}}{x^4} dx + f^2 \int \frac{F^{a+bc+bdx}}{x^3} dx \\
 &= -\frac{e^2 F^{a+bc+bdx}}{4x^4} - \frac{2ef F^{a+bc+bdx}}{3x^3} - \frac{f^2 F^{a+bc+bdx}}{2x^2} + \frac{1}{4} (bde^2 \log(F)) \int \frac{F^{a+bc+bdx}}{x^4} dx + \frac{1}{3} (2bdef \log(F)) \int \frac{F^{a+bc+bdx}}{x^3} dx \\
 &= -\frac{e^2 F^{a+bc+bdx}}{4x^4} - \frac{2ef F^{a+bc+bdx}}{3x^3} - \frac{f^2 F^{a+bc+bdx}}{2x^2} - \frac{bde^2 F^{a+bc+bdx} \log(F)}{12x^3} - \frac{bdef F^{a+bc+bdx} \log(F)}{3x^2} \\
 &= -\frac{e^2 F^{a+bc+bdx}}{4x^4} - \frac{2ef F^{a+bc+bdx}}{3x^3} - \frac{f^2 F^{a+bc+bdx}}{2x^2} - \frac{bde^2 F^{a+bc+bdx} \log(F)}{12x^3} - \frac{bdef F^{a+bc+bdx} \log(F)}{3x^2} \\
 &= -\frac{e^2 F^{a+bc+bdx}}{4x^4} - \frac{2ef F^{a+bc+bdx}}{3x^3} - \frac{f^2 F^{a+bc+bdx}}{2x^2} - \frac{bde^2 F^{a+bc+bdx} \log(F)}{12x^3} - \frac{bdef F^{a+bc+bdx} \log(F)}{3x^2} \\
 &= -\frac{e^2 F^{a+bc+bdx}}{4x^4} - \frac{2ef F^{a+bc+bdx}}{3x^3} - \frac{f^2 F^{a+bc+bdx}}{2x^2} - \frac{bde^2 F^{a+bc+bdx} \log(F)}{12x^3} - \frac{bdef F^{a+bc+bdx} \log(F)}{3x^2}
 \end{aligned}$$

**Mathematica [A]** time = 0.296526, size = 156, normalized size = 0.49

$$\frac{F^{a+bc} \left( b^2 d^2 x^4 \log^2(F) \left( b^2 d^2 e^2 \log^2(F) + 8bdef \log(F) + 12f^2 \right) \text{Ei}(bdx \log(F)) - F^{bdx} \left( b^3 d^3 e^2 x^3 \log^3(F) + b^2 d^2 e x^2 \log^2(F) \right) \right)}{24x^4}$$

Antiderivative was successfully verified.

[In] Integrate[(F^(a + b\*(c + d\*x))\*(e + f\*x)^2)/x^5,x]

[Out] (F^(a + b\*c)\*(b^2\*d^2\*x^4\*ExpIntegralEi[b\*d\*x\*Log[F]]\*Log[F]^2\*(12\*f^2 + 8\*b\*d\*e\*f\*Log[F] + b^2\*d^2\*e^2\*Log[F]^2) - F^(b\*d\*x)\*(2\*(3\*e^2 + 8\*e\*f\*x + 6\*f^2\*x^2) + 2\*b\*d\*x\*(e^2 + 4\*e\*f\*x + 6\*f^2\*x^2)\*Log[F] + b^2\*d^2\*e\*x^2\*(e + 8\*f\*x)\*Log[F]^2 + b^3\*d^3\*e^2\*x^3\*Log[F]^3)))/(24\*x^4)

**Maple [A]** time = 0.066, size = 382, normalized size = 1.2

$$\frac{(\ln(F))^4 b^4 d^4 e^2 F^{bc} F^a \text{Ei}(1, bc \ln(F) + \ln(F) a - bdx \ln(F) - (bc + a) \ln(F))}{24} - \frac{2 f e F^{bdx} F^{bc+a}}{3 x^3} - \frac{f e \ln(F) b d F^{bdx} F^{bc+a}}{3 x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(a+b\*(d\*x+c))\*(f\*x+e)^2/x^5,x)

[Out] 
$$\begin{aligned} & -1/24*\ln(F)^4*b^4*d^4*e^2*F^{(b*c)*F^a*\text{Ei}(1,b*c*\ln(F)+\ln(F)*a-b*d*x*\ln(F)-(b \\ & *c+a)*\ln(F))-2/3*f*e*F^{(b*d*x)*F^{(b*c+a)}/x^3-1/3*\ln(F)*b*d*f*e*F^{(b*d*x)*F^{ \\ & (b*c+a)}/x^2-1/3*\ln(F)^2*b^2*d^2*f*e*F^{(b*d*x)*F^{(b*c+a)}/x-1/3*\ln(F)^3*b^3*d \\ & ^3*f*e*F^{(b*c)*F^a*\text{Ei}(1,b*c*\ln(F)+\ln(F)*a-b*d*x*\ln(F)-(b*c+a)*\ln(F))-1/2*\ln \\ & (F)*b*d*f^2*F^{(b*d*x)*F^{(b*c+a)}/x-1/2*\ln(F)^2*b^2*d^2*f^2*F^{(b*c)*F^a*\text{Ei}(1, \\ & b*c*\ln(F)+\ln(F)*a-b*d*x*\ln(F)-(b*c+a)*\ln(F))-1/2*f^2*F^{(b*d*x)*F^{(b*c+a)}/x^ \\ & 2-1/4*e^2*F^{(b*d*x)*F^{(b*c+a)}/x^4-1/12*\ln(F)*b*d*e^2*F^{(b*d*x)*F^{(b*c+a)}/x^ \\ & 3-1/24*\ln(F)^2*b^2*d^2*e^2*F^{(b*d*x)*F^{(b*c+a)}/x^2-1/24*\ln(F)^3*b^3*d^3*e^2 \\ & *F^{(b*d*x)*F^{(b*c+a)}/x} \end{aligned}$$

**Maxima [A]** time = 1.24327, size = 126, normalized size = 0.39

$$-F^{bc+a} b^4 d^4 e^2 \Gamma(-4, -bdx \log(F)) \log(F)^4 + 2 F^{bc+a} b^3 d^3 e f \Gamma(-3, -bdx \log(F)) \log(F)^3 - F^{bc+a} b^2 d^2 f^2 \Gamma(-2, -bdx \log(F))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*(d\*x+c))\*(f\*x+e)^2/x^5,x, algorithm="maxima")

[Out] 
$$\begin{aligned} & -F^{(b*c + a)*b^4*d^4*e^2*\text{gamma}(-4, -b*d*x*\log(F))*\log(F)^4 + 2*F^{(b*c + a)* \\ & b^3*d^3*e*f*\text{gamma}(-3, -b*d*x*\log(F))*\log(F)^3 - F^{(b*c + a)*b^2*d^2*f^2*\text{gam} \\ & \text{ma}(-2, -b*d*x*\log(F))*\log(F)^2 \end{aligned}$$

**Fricas [A]** time = 1.50121, size = 421, normalized size = 1.31

$$\frac{(b^4 d^4 e^2 x^4 \log(F)^4 + 8 b^3 d^3 e f x^4 \log(F)^3 + 12 b^2 d^2 f^2 x^4 \log(F)^2) F^{bc+a} \operatorname{Ei}(bdx \log(F)) - (b^3 d^3 e^2 x^3 \log(F)^3 + 12 f^2 x^2 + 16 e f x + 6 e^2) F^{bc+a}}{24 x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*(d\*x+c))\*(f\*x+e)^2/x^5,x, algorithm="fricas")

[Out] 1/24\*((b^4\*d^4\*e^2\*x^4\*log(F)^4 + 8\*b^3\*d^3\*e\*f\*x^4\*log(F)^3 + 12\*b^2\*d^2\*f^2\*x^4\*log(F)^2)\*F^(b\*c + a)\*Ei(b\*d\*x\*log(F)) - (b^3\*d^3\*e^2\*x^3\*log(F)^3 + 12\*f^2\*x^2 + 16\*e\*f\*x + (8\*b^2\*d^2\*e\*f\*x^3 + b^2\*d^2\*e^2\*x^2)\*log(F)^2 + 6\*e^2 + 2\*(6\*b\*d\*f^2\*x^3 + 4\*b\*d\*e\*f\*x^2 + b\*d\*e^2\*x)\*log(F))\*F^(b\*d\*x + b\*c + a))/x^4

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{a+b(c+dx)} (e + fx)^2}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(a+b\*(d\*x+c))\*(f\*x+e)\*\*2/x\*\*5,x)

[Out] Integral(F\*\*(a + b\*(c + d\*x))\*(e + f\*x)\*\*2/x\*\*5, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx + e)^2 F^{(dx+c)b+a}}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*(d\*x+c))\*(f\*x+e)^2/x^5,x, algorithm="giac")

[Out] integrate((f\*x + e)^2\*F^((d\*x + c)\*b + a)/x^5, x)

### 3.74 $\int e^{-a-bx}(a+bx)^4(c+dx)^3 dx$

**Optimal.** Leaf size=754

$$\frac{3d^2e^{-a-bx}(a+bx)^6(bc-ad)}{b^4} - \frac{18d^2e^{-a-bx}(a+bx)^5(bc-ad)}{b^4} - \frac{90d^2e^{-a-bx}(a+bx)^4(bc-ad)}{b^4} - \frac{360d^2e^{-a-bx}(a+bx)^3(bc-ad)}{b^4}$$

[Out]  $(-5040*d^3*E^{-a-b*x})/b^4 - (2160*d^2*(b*c-a*d)*E^{-a-b*x})/b^4 - (360*d*(b*c-a*d)^2*E^{-a-b*x})/b^4 - (24*(b*c-a*d)^3*E^{-a-b*x})/b^4 - (5040*d^3*E^{-a-b*x}*(a+b*x))/b^4 - (2160*d^2*(b*c-a*d)*E^{-a-b*x}*(a+b*x))/b^4 - (360*d*(b*c-a*d)^2*E^{-a-b*x}*(a+b*x))/b^4 - (24*(b*c-a*d)^3*E^{-a-b*x}*(a+b*x))/b^4 - (2520*d^3*E^{-a-b*x}*(a+b*x)^2)/b^4 - (1080*d^2*(b*c-a*d)*E^{-a-b*x}*(a+b*x)^2)/b^4 - (180*d*(b*c-a*d)^2*E^{-a-b*x}*(a+b*x)^2)/b^4 - (12*(b*c-a*d)^3*E^{-a-b*x}*(a+b*x)^2)/b^4 - (840*d^3*E^{-a-b*x}*(a+b*x)^3)/b^4 - (360*d^2*(b*c-a*d)*E^{-a-b*x}*(a+b*x)^3)/b^4 - (60*d*(b*c-a*d)^2*E^{-a-b*x}*(a+b*x)^3)/b^4 - (4*(b*c-a*d)^3*E^{-a-b*x}*(a+b*x)^3)/b^4 - (210*d^3*E^{-a-b*x}*(a+b*x)^4)/b^4 - (90*d^2*(b*c-a*d)*E^{-a-b*x}*(a+b*x)^4)/b^4 - (15*d*(b*c-a*d)^2*E^{-a-b*x}*(a+b*x)^4)/b^4 - ((b*c-a*d)^3*E^{-a-b*x}*(a+b*x)^4)/b^4 - (42*d^3*E^{-a-b*x}*(a+b*x)^5)/b^4 - (18*d^2*(b*c-a*d)*E^{-a-b*x}*(a+b*x)^5)/b^4 - (3*d*(b*c-a*d)^2*E^{-a-b*x}*(a+b*x)^5)/b^4 - (7*d^3*E^{-a-b*x}*(a+b*x)^6)/b^4 - (3*d^2*(b*c-a*d)*E^{-a-b*x}*(a+b*x)^6)/b^4 - (d^3*E^{-a-b*x}*(a+b*x)^7)/b^4$

**Rubi [A]** time = 0.915864, antiderivative size = 754, normalized size of antiderivative = 1., number of steps used = 28, number of rules used = 3, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.12$ , Rules used = {2196, 2176, 2194}

$$\frac{3d^2e^{-a-bx}(a+bx)^6(bc-ad)}{b^4} - \frac{18d^2e^{-a-bx}(a+bx)^5(bc-ad)}{b^4} - \frac{90d^2e^{-a-bx}(a+bx)^4(bc-ad)}{b^4} - \frac{360d^2e^{-a-bx}(a+bx)^3(bc-ad)}{b^4}$$

Antiderivative was successfully verified.

[In] Int[E^{-a-b\*x}\*(a+b\*x)^4\*(c+d\*x)^3,x]

[Out]  $(-5040*d^3*E^{-a-b*x})/b^4 - (2160*d^2*(b*c-a*d)*E^{-a-b*x})/b^4 - (360*d*(b*c-a*d)^2*E^{-a-b*x})/b^4 - (24*(b*c-a*d)^3*E^{-a-b*x})/b^4 - (5040*d^3*E^{-a-b*x}*(a+b*x))/b^4 - (2160*d^2*(b*c-a*d)*E^{-a-b*x}*(a+b*x))/b^4 - (360*d*(b*c-a*d)^2*E^{-a-b*x}*(a+b*x))/b^4 - (24*(b*c-a*d)^3*E^{-a-b*x}*(a+b*x))/b^4 - (2520*d^3*E^{-a-b*x}*(a+b*x)^2)/b^4 - (1080*d^2*(b*c-a*d)*E^{-a-b*x}*(a+b*x)^2)/b^4 - (180*d*(b*c-a*d)^2*E^{-a-b*x}*(a+b*x)^2)/b^4 - (12*(b*c-a*d)^3*E^{-a-b*x}*(a+b*x)^2)/b^4 - (840*d^3*E^{-a-b*x}*(a+b*x)^3)/b^4 - (360*d^2*(b*c-a*d)*E^{-a-b*x}*(a+b*x)^3)/b^4 - (60*d*(b*c-a*d)^2*E^{-a-b*x}*(a+b*x)^3)/b^4 - (4*(b*c-a*d)^3*E^{-a-b*x}*(a+b*x)^3)/b^4 - (210*d^3*E^{-a-b*x}*(a+b*x)^4)/b^4 - (90*d^2*(b*c-a*d)*E^{-a-b*x}*(a+b*x)^4)/b^4 - (15*d*(b*c-a*d)^2*E^{-a-b*x}*(a+b*x)^4)/b^4 - ((b*c-a*d)^3*E^{-a-b*x}*(a+b*x)^4)/b^4 - (42*d^3*E^{-a-b*x}*(a+b*x)^5)/b^4 - (18*d^2*(b*c-a*d)*E^{-a-b*x}*(a+b*x)^5)/b^4 - (3*d*(b*c-a*d)^2*E^{-a-b*x}*(a+b*x)^5)/b^4 - (7*d^3*E^{-a-b*x}*(a+b*x)^6)/b^4 - (3*d^2*(b*c-a*d)*E^{-a-b*x}*(a+b*x)^6)/b^4 - (d^3*E^{-a-b*x}*(a+b*x)^7)/b^4$

$$\begin{aligned}
& + b*x)^2)/b^4 - (840*d^3*E^{(-a - b*x)*(a + b*x)^3})/b^4 - (360*d^2*(b*c - a \\
& *d)*E^{(-a - b*x)*(a + b*x)^3})/b^4 - (60*d*(b*c - a*d)^2*E^{(-a - b*x)*(a + b \\
& *x)^3})/b^4 - (4*(b*c - a*d)^3*E^{(-a - b*x)*(a + b*x)^3})/b^4 - (210*d^3*E^{(- \\
& a - b*x)*(a + b*x)^4})/b^4 - (90*d^2*(b*c - a*d)*E^{(-a - b*x)*(a + b*x)^4})/b \\
& ^4 - (15*d*(b*c - a*d)^2*E^{(-a - b*x)*(a + b*x)^4})/b^4 - ((b*c - a*d)^3*E^{( \\
& -a - b*x)*(a + b*x)^4})/b^4 - (42*d^3*E^{(-a - b*x)*(a + b*x)^5})/b^4 - (18*d^ \\
& 2*(b*c - a*d)*E^{(-a - b*x)*(a + b*x)^5})/b^4 - (3*d*(b*c - a*d)^2*E^{(-a - b* \\
& x)*(a + b*x)^5})/b^4 - (7*d^3*E^{(-a - b*x)*(a + b*x)^6})/b^4 - (3*d^2*(b*c - \\
& a*d)*E^{(-a - b*x)*(a + b*x)^6})/b^4 - (d^3*E^{(-a - b*x)*(a + b*x)^7})/b^4
\end{aligned}$$

### Rule 2196

```

Int[(F_)^((c_.)*(v_))*(u_), x_Symbol] := Int[ExpandIntegrand[F^(c*ExpandToS
um[v, x]), u, x], x] /; FreeQ[{F, c}, x] && PolynomialQ[u, x] && LinearQ[v,
x] && !$UseGamma === True

```

### Rule 2176

```

Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] := Simp[((c + d*x)^m*(b*F^(g*(e + f*x)))^n)/(f*g*n*Log[F]),
x] - Dist[(d*m)/(f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] && !$UseGamma === True

```

### Rule 2194

```

Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] := Simp[(F^(c*(a +
b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]

```

### Rubi steps



$$\begin{aligned}
\int e^{-a-bx}(a+bx)^4(c+dx)^3 dx &= \int \left( \frac{(bc-ad)^3 e^{-a-bx}(a+bx)^4}{b^3} + \frac{3d(bc-ad)^2 e^{-a-bx}(a+bx)^5}{b^3} + \frac{3d^2(bc-ad) e^{-a-bx}(a+bx)^6}{b^3} + \frac{d^3 e^{-a-bx}(a+bx)^7}{b^3} \right) dx \\
&= \frac{d^3 \int e^{-a-bx}(a+bx)^7 dx}{b^3} + \frac{(3d^2(bc-ad)) \int e^{-a-bx}(a+bx)^6 dx}{b^3} + \frac{(3d(bc-ad)^2) \int e^{-a-bx}(a+bx)^5 dx}{b^3} + \frac{(bc-ad)^3 \int e^{-a-bx}(a+bx)^4 dx}{b^3} \\
&= -\frac{(bc-ad)^3 e^{-a-bx}(a+bx)^4}{b^4} - \frac{3d(bc-ad)^2 e^{-a-bx}(a+bx)^5}{b^4} - \frac{3d^2(bc-ad) e^{-a-bx}(a+bx)^6}{b^4} - \frac{d^3 e^{-a-bx}(a+bx)^7}{b^4} \\
&= -\frac{4(bc-ad)^3 e^{-a-bx}(a+bx)^3}{b^4} - \frac{15d(bc-ad)^2 e^{-a-bx}(a+bx)^4}{b^4} - \frac{(bc-ad)^3 e^{-a-bx}(a+bx)^5}{b^4} - \frac{d^3 e^{-a-bx}(a+bx)^6}{b^4} \\
&= -\frac{12(bc-ad)^3 e^{-a-bx}(a+bx)^2}{b^4} - \frac{60d(bc-ad)^2 e^{-a-bx}(a+bx)^3}{b^4} - \frac{4(bc-ad)^3 e^{-a-bx}(a+bx)^4}{b^4} - \frac{d^3 e^{-a-bx}(a+bx)^5}{b^4} \\
&= -\frac{24(bc-ad)^3 e^{-a-bx}(a+bx)}{b^4} - \frac{180d(bc-ad)^2 e^{-a-bx}(a+bx)^2}{b^4} - \frac{12(bc-ad)^3 e^{-a-bx}(a+bx)^3}{b^4} - \frac{d^3 e^{-a-bx}(a+bx)^4}{b^4} \\
&= -\frac{24(bc-ad)^3 e^{-a-bx}}{b^4} - \frac{360d(bc-ad)^2 e^{-a-bx}(a+bx)}{b^4} - \frac{24(bc-ad)^3 e^{-a-bx}(a+bx)}{b^4} - \frac{10d^3 e^{-a-bx}(a+bx)}{b^4} \\
&= -\frac{360d(bc-ad)^2 e^{-a-bx}}{b^4} - \frac{24(bc-ad)^3 e^{-a-bx}}{b^4} - \frac{2160d^2(bc-ad) e^{-a-bx}(a+bx)}{b^4} - \frac{360d^3 e^{-a-bx}(a+bx)}{b^4} \\
&= -\frac{2160d^2(bc-ad) e^{-a-bx}}{b^4} - \frac{360d(bc-ad)^2 e^{-a-bx}}{b^4} - \frac{24(bc-ad)^3 e^{-a-bx}}{b^4} - \frac{5040d^3 e^{-a-bx}(a+bx)}{b^4} \\
&= -\frac{5040d^3 e^{-a-bx}}{b^4} - \frac{2160d^2(bc-ad) e^{-a-bx}}{b^4} - \frac{360d(bc-ad)^2 e^{-a-bx}}{b^4} - \frac{24(bc-ad)^3 e^{-a-bx}}{b^4}
\end{aligned}$$

**Mathematica [A]** time = 0.695949, size = 458, normalized size = 0.61

$$\frac{e^{-a-bx} \left( -6b^5 x^2 (c+dx) \left( (a^2+2a+2)c^2 + 2(a^2+3a+4)cdx + (a^2+4a+7)d^2x^2 \right) - 2b^4 x \left( 3(2a^3+9a^2+24a+30)c^2 + 6(2a^2+3a+2)cdx + (a^2+4a+7)d^2x^2 \right) \right)}{b^4}$$

Antiderivative was successfully verified.

[In] Integrate[E^(-a - b\*x)\*(a + b\*x)^4\*(c + d\*x)^3,x]

[Out] (E^(-a - b\*x))\*(-6\*(840 + 480\*a + 120\*a^2 + 16\*a^3 + a^4)\*d^3 - b^7\*x^4\*(c + d\*x)^3 - b^6\*x^3\*(c + d\*x)^2\*(4\*(1 + a)\*c + (7 + 4\*a)\*d\*x) - 6\*b\*d^2\*((360 + 240\*a + 72\*a^2 + 12\*a^3 + a^4)\*c + (840 + 480\*a + 120\*a^2 + 16\*a^3 + a^4)\*d\*x) - 6\*b^5\*x^2\*(c + d\*x)\*((2 + 2\*a + a^2)\*c^2 + 2\*(4 + 3\*a + a^2)\*c\*d\*x + (7 + 4\*a + a^2)\*d^2\*x^2) - 3\*b^2\*d\*((120 + 96\*a + 36\*a^2 + 8\*a^3 + a^4)\*c^2 + 2\*(360 + 240\*a + 72\*a^2 + 12\*a^3 + a^4)\*c\*d\*x + (840 + 480\*a + 120\*a^2 + 16\*a^3 + a^4)\*d^2\*x^2) - 2\*b^4\*x\*(2\*(6 + 6\*a + 3\*a^2 + a^3)\*c^3 + 3\*(30 + 24\*a + 9\*a^2 + 2\*a^3)\*c^2\*d\*x + 6\*(30 + 20\*a + 6\*a^2 + a^3)\*c\*d^2\*x^2 + (105 + 60\*a + 15\*a^2 + 2\*a^3)\*d^3\*x^3) - b^3\*((24 + 24\*a + 12\*a^2 + 4\*a^3 +

$$\frac{a^4*c^3 + 3*(120 + 96*a + 36*a^2 + 8*a^3 + a^4)*c^2*d*x + 3*(360 + 240*a + 72*a^2 + 12*a^3 + a^4)*c*d^2*x^2 + (840 + 480*a + 120*a^2 + 16*a^3 + a^4)*d^3*x^3)}{b^4}$$

**Maple [A]** time = 0.005, size = 1062, normalized size = 1.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(exp(-b*x-a)*(b*x+a)^4*(d*x+c)^3,x)`

[Out] 
$$-(b^7*d^3*x^7+4*a*b^6*d^3*x^6+3*b^7*c*d^2*x^6+6*a^2*b^5*d^3*x^5+12*a*b^6*c*d^2*x^5+3*b^7*c^2*d*x^5+7*b^6*d^3*x^6+4*a^3*b^4*d^3*x^4+18*a^2*b^5*c*d^2*x^4+12*a*b^6*c^2*d*x^4+24*a*b^5*d^3*x^5+b^7*c^3*x^4+18*b^6*c*d^2*x^5+a^4*b^3*d^3*x^3+12*a^3*b^4*c*d^2*x^3+18*a^2*b^5*c^2*d*x^3+30*a^2*b^4*d^3*x^4+4*a*b^6*c^3*x^3+60*a*b^5*c*d^2*x^4+15*b^6*c^2*d*x^4+42*b^5*d^3*x^5+3*a^4*b^3*c*d^2*x^2+12*a^3*b^4*c^2*d*x^2+16*a^3*b^3*d^3*x^3+6*a^2*b^5*c^3*x^2+72*a^2*b^4*c*d^2*x^3+48*a*b^5*c^2*d*x^3+120*a*b^4*d^3*x^4+4*b^6*c^3*x^3+90*b^5*c*d^2*x^4+3*a^4*b^3*c^2*d*x+3*a^4*b^2*d^3*x^2+4*a^3*b^4*c^3*x+36*a^3*b^3*c*d^2*x^2+54*a^2*b^4*c^2*d*x^2+120*a^2*b^3*d^3*x^3+12*a*b^5*c^3*x^2+240*a*b^4*c*d^2*x^3+60*b^5*c^2*d*x^3+210*b^4*d^3*x^4+a^4*b^3*c^3+6*a^4*b^2*c*d^2*x+24*a^3*b^3*c^2*d*x+48*a^3*b^2*d^3*x^2+12*a^2*b^4*c^3*x+216*a^2*b^3*c*d^2*x^2+144*a*b^4*c^2*d*x^2+480*a*b^3*d^3*x^3+12*b^5*c^3*x^2+360*b^4*c*d^2*x^3+3*a^4*b^2*c^2*d+6*a^4*b*d^3*x+4*a^3*b^3*c^3+72*a^3*b^2*c*d^2*x+108*a^2*b^3*c^2*d*x+360*a^2*b^2*d^3*x^2+24*a*b^4*c^3*x+720*a*b^3*c*d^2*x^2+180*b^4*c^2*d*x^2+840*b^3*d^3*x^3+6*a^4*b*c*d^2+24*a^3*b^2*c^2*d+96*a^3*b*d^3*x+12*a^2*b^3*c^3+432*a^2*b^2*c*d^2*x+288*a*b^3*c^2*d*x+1440*a*b^2*d^3*x^2+24*b^4*c^3*x+1080*b^3*c*d^2*x^2+6*a^4*d^3+72*a^3*b*c*d^2+108*a^2*b^2*c^2*d+720*a^2*b*d^3*x+24*a*b^3*c^3+1440*a*b^2*c*d^2*x+360*b^3*c^2*d*x+2520*b^2*d^3*x^2+96*a^3*d^3+432*a^2*b*c*d^2+288*a*b^2*c^2*d+2880*a*b*d^3*x+24*b^3*c^3+2160*b^2*c*d^2*x+720*a^2*d^3+1440*a*b*c*d^2+360*b^2*c^2*d+5040*b*d^3*x+2880*a*d^3+2160*b*c*d^2+5040*d^3)*exp(-b*x-a)/b^4$$

**Maxima [A]** time = 1.1762, size = 1207, normalized size = 1.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(-b\*x-a)\*(b\*x+a)^4\*(d\*x+c)^3,x, algorithm="maxima")

[Out] 
$$\begin{aligned} & -4*(b*x + 1)*a^3*c^3*e^{(-b*x - a)}/b - a^4*c^3*e^{(-b*x - a)}/b - 3*(b*x + 1)* \\ & a^4*c^2*d*e^{(-b*x - a)}/b^2 - 6*(b^2*x^2 + 2*b*x + 2)*a^2*c^3*e^{(-b*x - a)}/b \\ & - 12*(b^2*x^2 + 2*b*x + 2)*a^3*c^2*d*e^{(-b*x - a)}/b^2 - 3*(b^2*x^2 + 2*b*x \\ & + 2)*a^4*c*d^2*e^{(-b*x - a)}/b^3 - 4*(b^3*x^3 + 3*b^2*x^2 + 6*b*x + 6)*a*c^ \\ & 3*e^{(-b*x - a)}/b - 18*(b^3*x^3 + 3*b^2*x^2 + 6*b*x + 6)*a^2*c^2*d*e^{(-b*x - \\ & a)}/b^2 - 12*(b^3*x^3 + 3*b^2*x^2 + 6*b*x + 6)*a^3*c*d^2*e^{(-b*x - a)}/b^3 - \\ & (b^3*x^3 + 3*b^2*x^2 + 6*b*x + 6)*a^4*d^3*e^{(-b*x - a)}/b^4 - (b^4*x^4 + 4* \\ & b^3*x^3 + 12*b^2*x^2 + 24*b*x + 24)*c^3*e^{(-b*x - a)}/b - 12*(b^4*x^4 + 4*b^ \\ & 3*x^3 + 12*b^2*x^2 + 24*b*x + 24)*a*c^2*d*e^{(-b*x - a)}/b^2 - 18*(b^4*x^4 + \\ & 4*b^3*x^3 + 12*b^2*x^2 + 24*b*x + 24)*a^2*c*d^2*e^{(-b*x - a)}/b^3 - 4*(b^4*x \\ & ^4 + 4*b^3*x^3 + 12*b^2*x^2 + 24*b*x + 24)*a^3*d^3*e^{(-b*x - a)}/b^4 - 3*(b^ \\ & 5*x^5 + 5*b^4*x^4 + 20*b^3*x^3 + 60*b^2*x^2 + 120*b*x + 120)*c^2*d*e^{(-b*x \\ & - a)}/b^2 - 12*(b^5*x^5 + 5*b^4*x^4 + 20*b^3*x^3 + 60*b^2*x^2 + 120*b*x + 12 \\ & 0)*a*c*d^2*e^{(-b*x - a)}/b^3 - 6*(b^5*x^5 + 5*b^4*x^4 + 20*b^3*x^3 + 60*b^2* \\ & x^2 + 120*b*x + 120)*a^2*d^3*e^{(-b*x - a)}/b^4 - 3*(b^6*x^6 + 6*b^5*x^5 + 30 \\ & *b^4*x^4 + 120*b^3*x^3 + 360*b^2*x^2 + 720*b*x + 720)*c*d^2*e^{(-b*x - a)}/b^ \\ & 3 - 4*(b^6*x^6 + 6*b^5*x^5 + 30*b^4*x^4 + 120*b^3*x^3 + 360*b^2*x^2 + 720*b \\ & *x + 720)*a*d^3*e^{(-b*x - a)}/b^4 - (b^7*x^7 + 7*b^6*x^6 + 42*b^5*x^5 + 210* \\ & b^4*x^4 + 840*b^3*x^3 + 2520*b^2*x^2 + 5040*b*x + 5040)*d^3*e^{(-b*x - a)}/b^ \\ & 4 \end{aligned}$$

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**Fricas [A]** time = 1.50603, size = 1310, normalized size = 1.74

$$\frac{(b^7 d^3 x^7 + (a^4 + 4 a^3 + 12 a^2 + 24 a + 24) b^3 c^3 + (3 b^7 c d^2 + (4 a + 7) b^6 d^3) x^6 + 3 (a^4 + 8 a^3 + 36 a^2 + 96 a + 120) b^2 c^2 d}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(-b\*x-a)\*(b\*x+a)^4\*(d\*x+c)^3,x, algorithm="fricas")

[Out] 
$$\begin{aligned} & -(b^7*d^3*x^7 + (a^4 + 4*a^3 + 12*a^2 + 24*a + 24)*b^3*c^3 + (3*b^7*c*d^2 + \\ & (4*a + 7)*b^6*d^3)*x^6 + 3*(a^4 + 8*a^3 + 36*a^2 + 96*a + 120)*b^2*c^2*d + \\ & 3*(b^7*c^2*d + 2*(2*a + 3)*b^6*c*d^2 + 2*(a^2 + 4*a + 7)*b^5*d^3)*x^5 + 6* \\ & (a^4 + 12*a^3 + 72*a^2 + 240*a + 360)*b*c*d^2 + (b^7*c^3 + 3*(4*a + 5)*b^6* \\ & c^2*d + 6*(3*a^2 + 10*a + 15)*b^5*c*d^2 + 2*(2*a^3 + 15*a^2 + 60*a + 105)*b \\ & ^4*d^3)*x^4 + 6*(a^4 + 16*a^3 + 120*a^2 + 480*a + 840)*d^3 + (4*(a + 1)*b^6 \\ & *c^3 + 6*(3*a^2 + 8*a + 10)*b^5*c^2*d + 12*(a^3 + 6*a^2 + 20*a + 30)*b^4*c* \\ & d^2 + (a^4 + 16*a^3 + 120*a^2 + 480*a + 840)*b^3*d^3)*x^3 + 3*(2*(a^2 + 2*a \\ & + 2)*b^5*c^3 + 2*(2*a^3 + 9*a^2 + 24*a + 30)*b^4*c^2*d + (a^4 + 12*a^3 + 7 \\ & 2*a^2 + 240*a + 360)*b^3*c*d^2 + (a^4 + 16*a^3 + 120*a^2 + 480*a + 840)*b^2 \end{aligned}$$

$$*d^3)*x^2 + (4*(a^3 + 3*a^2 + 6*a + 6)*b^4*c^3 + 3*(a^4 + 8*a^3 + 36*a^2 + 96*a + 120)*b^3*c^2*d + 6*(a^4 + 12*a^3 + 72*a^2 + 240*a + 360)*b^2*c*d^2 + 6*(a^4 + 16*a^3 + 120*a^2 + 480*a + 840)*b*d^3)*x)*e^{(-b*x - a)/b^4}$$

**Sympy [A]** time = 0.491955, size = 1445, normalized size = 1.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(-b\*x-a)\*(b\*x+a)\*\*4\*(d\*x+c)\*\*3,x)

[Out] Piecewise((( -a\*\*4\*b\*\*3\*c\*\*3 - 3\*a\*\*4\*b\*\*3\*c\*\*2\*d\*x - 3\*a\*\*4\*b\*\*3\*c\*d\*\*2\*x\*\*2 - a\*\*4\*b\*\*3\*d\*\*3\*x\*\*3 - 3\*a\*\*4\*b\*\*2\*c\*\*2\*d - 6\*a\*\*4\*b\*\*2\*c\*d\*\*2\*x - 3\*a\*\*4\*b\*\*2\*d\*\*3\*x\*\*2 - 6\*a\*\*4\*b\*c\*d\*\*2 - 6\*a\*\*4\*b\*d\*\*3\*x - 6\*a\*\*4\*d\*\*3 - 4\*a\*\*3\*b\*\*4\*c\*\*3\*x - 12\*a\*\*3\*b\*\*4\*c\*\*2\*d\*x\*\*2 - 12\*a\*\*3\*b\*\*4\*c\*d\*\*2\*x\*\*3 - 4\*a\*\*3\*b\*\*4\*d\*\*3\*x\*\*4 - 4\*a\*\*3\*b\*\*3\*c\*\*3 - 24\*a\*\*3\*b\*\*3\*c\*\*2\*d\*x - 36\*a\*\*3\*b\*\*3\*c\*d\*\*2\*x\*\*2 - 16\*a\*\*3\*b\*\*3\*d\*\*3\*x\*\*3 - 24\*a\*\*3\*b\*\*2\*c\*\*2\*d - 72\*a\*\*3\*b\*\*2\*c\*d\*\*2\*x - 48\*a\*\*3\*b\*\*2\*d\*\*3\*x\*\*2 - 72\*a\*\*3\*b\*c\*d\*\*2 - 96\*a\*\*3\*b\*d\*\*3\*x - 96\*a\*\*3\*d\*\*3 - 6\*a\*\*2\*b\*\*5\*c\*\*3\*x\*\*2 - 18\*a\*\*2\*b\*\*5\*c\*\*2\*d\*x\*\*3 - 18\*a\*\*2\*b\*\*5\*c\*d\*\*2\*x\*\*4 - 6\*a\*\*2\*b\*\*5\*d\*\*3\*x\*\*5 - 12\*a\*\*2\*b\*\*4\*c\*\*3\*x - 54\*a\*\*2\*b\*\*4\*c\*\*2\*d\*x\*\*2 - 72\*a\*\*2\*b\*\*4\*c\*d\*\*2\*x\*\*3 - 30\*a\*\*2\*b\*\*4\*d\*\*3\*x\*\*4 - 12\*a\*\*2\*b\*\*3\*c\*\*3 - 108\*a\*\*2\*b\*\*3\*c\*\*2\*d\*x - 216\*a\*\*2\*b\*\*3\*c\*d\*\*2\*x\*\*2 - 120\*a\*\*2\*b\*\*3\*d\*\*3\*x\*\*3 - 108\*a\*\*2\*b\*\*2\*c\*\*2\*d - 432\*a\*\*2\*b\*\*2\*c\*d\*\*2\*x - 360\*a\*\*2\*b\*\*2\*d\*\*3\*x\*\*2 - 432\*a\*\*2\*b\*c\*d\*\*2 - 720\*a\*\*2\*b\*d\*\*3\*x - 720\*a\*\*2\*d\*\*3 - 4\*a\*b\*\*6\*c\*\*3\*x\*\*3 - 12\*a\*b\*\*6\*c\*\*2\*d\*x\*\*4 - 12\*a\*b\*\*6\*c\*d\*\*2\*x\*\*5 - 4\*a\*b\*\*6\*d\*\*3\*x\*\*6 - 12\*a\*b\*\*5\*c\*\*3\*x\*\*2 - 48\*a\*b\*\*5\*c\*\*2\*d\*x\*\*3 - 60\*a\*b\*\*5\*c\*d\*\*2\*x\*\*4 - 24\*a\*b\*\*5\*d\*\*3\*x\*\*5 - 24\*a\*b\*\*4\*c\*\*3\*x - 144\*a\*b\*\*4\*c\*\*2\*d\*x\*\*2 - 240\*a\*b\*\*4\*c\*d\*\*2\*x\*\*3 - 120\*a\*b\*\*4\*d\*\*3\*x\*\*4 - 24\*a\*b\*\*3\*c\*\*3 - 288\*a\*b\*\*3\*c\*\*2\*d\*x - 720\*a\*b\*\*3\*c\*d\*\*2\*x\*\*2 - 480\*a\*b\*\*3\*d\*\*3\*x\*\*3 - 288\*a\*b\*\*2\*c\*\*2\*d - 1440\*a\*b\*\*2\*c\*d\*\*2\*x - 1440\*a\*b\*\*2\*d\*\*3\*x\*\*2 - 1440\*a\*b\*c\*d\*\*2 - 2880\*a\*b\*d\*\*3\*x - 2880\*a\*d\*\*3 - b\*\*7\*c\*\*3\*x\*\*4 - 3\*b\*\*7\*c\*\*2\*d\*x\*\*5 - 3\*b\*\*7\*c\*d\*\*2\*x\*\*6 - b\*\*7\*d\*\*3\*x\*\*7 - 4\*b\*\*6\*c\*\*3\*x\*\*3 - 15\*b\*\*6\*c\*\*2\*d\*x\*\*4 - 18\*b\*\*6\*c\*d\*\*2\*x\*\*5 - 7\*b\*\*6\*d\*\*3\*x\*\*6 - 12\*b\*\*5\*c\*\*3\*x\*\*2 - 60\*b\*\*5\*c\*\*2\*d\*x\*\*3 - 90\*b\*\*5\*c\*d\*\*2\*x\*\*4 - 42\*b\*\*5\*d\*\*3\*x\*\*5 - 24\*b\*\*4\*c\*\*3\*x - 180\*b\*\*4\*c\*\*2\*d\*x\*\*2 - 360\*b\*\*4\*c\*d\*\*2\*x\*\*3 - 210\*b\*\*4\*d\*\*3\*x\*\*4 - 24\*b\*\*3\*c\*\*3 - 360\*b\*\*3\*c\*\*2\*d\*x - 1080\*b\*\*3\*c\*d\*\*2\*x\*\*2 - 840\*b\*\*3\*d\*\*3\*x\*\*3 - 360\*b\*\*2\*c\*\*2\*d - 2160\*b\*\*2\*c\*d\*\*2\*x - 2520\*b\*\*2\*d\*\*3\*x\*\*2 - 2160\*b\*c\*d\*\*2 - 5040\*b\*d\*\*3\*x - 5040\*d\*\*3)\*exp(-a - b\*x)/b\*\*4, Ne(b\*\*4, 0)), (a\*\*4\*c\*\*3\*x + b\*\*4\*d\*\*3\*x\*\*8/8 + x\*\*7\*(4\*a\*b\*\*3\*d\*\*3/7 + 3\*b\*\*4\*c\*d\*\*2/7) + x\*\*6\*(a\*\*2\*b\*\*2\*d\*\*3 + 2\*a\*b\*\*3\*c\*d\*\*2 + b\*\*4\*c\*\*2\*d/2) + x\*\*5\*(4\*a\*\*3\*b\*d\*\*3/5 + 18\*a\*\*2\*b\*\*2\*c\*d\*\*2/5 + 12\*a\*b\*\*3\*c\*\*2\*d/5 + b\*\*4\*c\*\*3/5) + x\*\*4\*(a\*\*4\*d\*\*3/4 + 3\*a\*\*3\*b\*c\*d\*\*2 + 9\*

$a^{**2}b^{**2}c^{**2}d/2 + a*b^{**3}c^{**3}) + x^{**3}(a^{**4}c*d^{**2} + 4*a^{**3}b*c^{**2}d + 2$   
 $*a^{**2}b^{**2}c^{**3}) + x^{**2}(3*a^{**4}c^{**2}d/2 + 2*a^{**3}b*c^{**3}), True))$

**Giac [A]** time = 1.2921, size = 1480, normalized size = 1.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(-b\*x-a)\*(b\*x+a)^4\*(d\*x+c)^3,x, algorithm="giac")

[Out]  $-(b^{11}d^3x^7 + 3b^{11}cd^2x^6 + 4a^2b^{10}d^3x^6 + 3b^{11}c^2dx^5 + 12a^2b^{10}cd^2x^5 + 6a^2b^9d^3x^5 + 7b^{10}d^3x^6 + b^{11}c^3x^4 + 12a^2b^{10}c^2dx^4 + 18a^2b^9cd^2x^4 + 4a^3b^8d^3x^4 + 18b^{10}cd^2x^5 + 24a^2b^9d^3x^5 + 4a^2b^{10}c^3x^3 + 18a^2b^9c^2dx^3 + 12a^3b^8cd^2x^3 + a^4b^7d^3x^3 + 15b^{10}c^2dx^4 + 60a^2b^9cd^2x^4 + 30a^2b^8d^3x^4 + 42b^9d^3x^5 + 6a^2b^9c^3x^2 + 12a^3b^8c^2dx^2 + 3a^4b^7cd^2x^2 + 4b^{10}c^3x^3 + 48a^2b^9c^2dx^3 + 72a^2b^8cd^2x^3 + 16a^3b^7d^3x^3 + 90b^9cd^2x^4 + 120a^2b^8d^3x^4 + 4a^3b^8c^3x + 3a^4b^7c^2dx + 12a^2b^9c^3x^2 + 54a^2b^8c^2dx^2 + 36a^3b^7cd^2x^2 + 3a^4b^6d^3x^2 + 60b^9c^2dx^3 + 240a^2b^8cd^2x^3 + 120a^2b^7d^3x^3 + 210b^8d^3x^4 + a^4b^7c^3 + 12a^2b^8c^3x + 24a^3b^7c^2dx + 6a^4b^6cd^2x + 12b^9c^3x^2 + 144a^2b^8c^2dx^2 + 216a^2b^7cd^2x^2 + 48a^3b^6d^3x^2 + 360b^8cd^2x^3 + 480a^2b^7d^3x^3 + 4a^3b^7c^3 + 3a^4b^6c^2d + 24a^2b^8c^3x + 108a^2b^7c^2dx + 72a^3b^6cd^2x + 6a^4b^5d^3x + 180b^8c^2dx^2 + 720a^2b^7cd^2x^2 + 360a^2b^6d^3x^2 + 840b^7d^3x^3 + 12a^2b^7c^3 + 24a^3b^6c^2d + 6a^4b^5cd^2 + 24b^8c^3x + 288a^2b^7c^2dx + 432a^2b^6cd^2x + 96a^3b^5d^3x + 1080b^7cd^2x^2 + 1440a^2b^6d^3x^2 + 24a^2b^7c^3 + 108a^2b^6c^2d + 72a^3b^5cd^2 + 6a^4b^4d^3 + 360b^7c^2dx + 1440a^2b^6cd^2x + 720a^2b^5d^3x + 2520b^6d^3x^2 + 24b^7c^3 + 288a^2b^6c^2d + 432a^2b^5cd^2 + 96a^3b^4d^3 + 2160b^6cd^2x + 2880a^2b^5d^3x + 360b^6c^2d + 1440a^2b^5cd^2 + 720a^2b^4d^3 + 5040b^5d^3x + 2160b^5cd^2 + 2880a^2b^4d^3 + 5040b^4d^3)*e^(-b*x - a)/b^8$

### 3.75 $\int e^{-a-bx}(a+bx)^4(c+dx)^2 dx$

**Optimal.** Leaf size=495

$$\frac{2de^{-a-bx}(a+bx)^5(bc-ad)}{b^3} - \frac{e^{-a-bx}(a+bx)^4(bc-ad)^2}{b^3} - \frac{10de^{-a-bx}(a+bx)^4(bc-ad)}{b^3} - \frac{4e^{-a-bx}(a+bx)^3(bc-ad)^2}{b^3} - \frac{40de^{-a-bx}(a+bx)^3(bc-ad)}{b^3} - \frac{20de^{-a-bx}(a+bx)^2(bc-ad)^2}{b^3} - \frac{20de^{-a-bx}(a+bx)^2(bc-ad)}{b^3} - \frac{20de^{-a-bx}(a+bx)(bc-ad)^2}{b^3} - \frac{20de^{-a-bx}(a+bx)(bc-ad)}{b^3} - \frac{20de^{-a-bx}(bc-ad)^2}{b^3} - \frac{20de^{-a-bx}(bc-ad)}{b^3} - \frac{20de^{-a-bx}}{b^3}$$

[Out]  $(-720*d^2*E^{-a-b*x})/b^3 - (240*d*(b*c-a*d)*E^{-a-b*x})/b^3 - (24*(b*c-a*d)^2*E^{-a-b*x})/b^3 - (720*d^2*E^{-a-b*x}*(a+b*x))/b^3 - (240*d*(b*c-a*d)*E^{-a-b*x}*(a+b*x))/b^3 - (24*(b*c-a*d)^2*E^{-a-b*x}*(a+b*x))/b^3 - (360*d^2*E^{-a-b*x}*(a+b*x)^2)/b^3 - (120*d*(b*c-a*d)*E^{-a-b*x}*(a+b*x)^2)/b^3 - (12*(b*c-a*d)^2*E^{-a-b*x}*(a+b*x)^2)/b^3 - (120*d^2*E^{-a-b*x}*(a+b*x)^3)/b^3 - (40*d*(b*c-a*d)*E^{-a-b*x}*(a+b*x)^3)/b^3 - (4*(b*c-a*d)^2*E^{-a-b*x}*(a+b*x)^3)/b^3 - (30*d^2*E^{-a-b*x}*(a+b*x)^4)/b^3 - (10*d*(b*c-a*d)*E^{-a-b*x}*(a+b*x)^4)/b^3 - ((b*c-a*d)^2*E^{-a-b*x}*(a+b*x)^4)/b^3 - (6*d^2*E^{-a-b*x}*(a+b*x)^5)/b^3 - (2*d*(b*c-a*d)*E^{-a-b*x}*(a+b*x)^5)/b^3 - (d^2*E^{-a-b*x}*(a+b*x)^6)/b^3$

**Rubi [A]** time = 0.636341, antiderivative size = 495, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 3, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.12$ , Rules used = {2196, 2176, 2194}

$$\frac{2de^{-a-bx}(a+bx)^5(bc-ad)}{b^3} - \frac{e^{-a-bx}(a+bx)^4(bc-ad)^2}{b^3} - \frac{10de^{-a-bx}(a+bx)^4(bc-ad)}{b^3} - \frac{4e^{-a-bx}(a+bx)^3(bc-ad)^2}{b^3} - \frac{40de^{-a-bx}(a+bx)^3(bc-ad)}{b^3} - \frac{20de^{-a-bx}(a+bx)^2(bc-ad)^2}{b^3} - \frac{20de^{-a-bx}(a+bx)^2(bc-ad)}{b^3} - \frac{20de^{-a-bx}(a+bx)(bc-ad)^2}{b^3} - \frac{20de^{-a-bx}(a+bx)(bc-ad)}{b^3} - \frac{20de^{-a-bx}(bc-ad)^2}{b^3} - \frac{20de^{-a-bx}(bc-ad)}{b^3} - \frac{20de^{-a-bx}}{b^3}$$

Antiderivative was successfully verified.

[In] Int[E^{-a-b\*x}\*(a+b\*x)^4\*(c+d\*x)^2,x]

[Out]  $(-720*d^2*E^{-a-b*x})/b^3 - (240*d*(b*c-a*d)*E^{-a-b*x})/b^3 - (24*(b*c-a*d)^2*E^{-a-b*x})/b^3 - (720*d^2*E^{-a-b*x}*(a+b*x))/b^3 - (240*d*(b*c-a*d)*E^{-a-b*x}*(a+b*x))/b^3 - (24*(b*c-a*d)^2*E^{-a-b*x}*(a+b*x))/b^3 - (360*d^2*E^{-a-b*x}*(a+b*x)^2)/b^3 - (120*d*(b*c-a*d)*E^{-a-b*x}*(a+b*x)^2)/b^3 - (12*(b*c-a*d)^2*E^{-a-b*x}*(a+b*x)^2)/b^3 - (120*d^2*E^{-a-b*x}*(a+b*x)^3)/b^3 - (40*d*(b*c-a*d)*E^{-a-b*x}*(a+b*x)^3)/b^3 - (4*(b*c-a*d)^2*E^{-a-b*x}*(a+b*x)^3)/b^3 - (30*d^2*E^{-a-b*x}*(a+b*x)^4)/b^3 - (10*d*(b*c-a*d)*E^{-a-b*x}*(a+b*x)^4)/b^3 - ((b*c-a*d)^2*E^{-a-b*x}*(a+b*x)^4)/b^3 - (6*d^2*E^{-a-b*x}*(a+b*x)^5)/b^3 - (2*d*(b*c-a*d)*E^{-a-b*x}*(a+b*x)^5)/b^3 - (d^2*E^{-a-b*x}*(a+b*x)^6)/b^3$

Rule 2196

```
Int[(F_)^((c_)*(v_))*(u_), x_Symbol] := Int[ExpandIntegrand[F^(c*ExpandToSum[v, x]), u, x], x] /; FreeQ[{F, c}, x] && PolynomialQ[u, x] && LinearQ[v, x] && !$UseGamma === True
```

Rule 2176

```
Int[((b_)*(F_)^((g_)*((e_) + (f_)*(x_))))^(n_)*((c_) + (d_)*(x_))^(m_), x_Symbol] := Simp[((c + d*x)^m*(b*F^(g*(e + f*x)))^n)/(f*g*n*Log[F]), x] - Dist[(d*m)/(f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m] && !$UseGamma === True
```

Rule 2194

```
Int[((F_)^((c_)*((a_) + (b_)*(x_))))^(n_), x_Symbol] := Simp[(F^(c*(a + b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned}
\int e^{-a-bx}(a+bx)^4(c+dx)^2 dx &= \int \left( \frac{(bc-ad)^2 e^{-a-bx}(a+bx)^4}{b^2} + \frac{2d(bc-ad)e^{-a-bx}(a+bx)^5}{b^2} + \frac{d^2 e^{-a-bx}(a+bx)^6}{b^2} \right) dx \\
&= \frac{d^2 \int e^{-a-bx}(a+bx)^6 dx}{b^2} + \frac{(2d(bc-ad)) \int e^{-a-bx}(a+bx)^5 dx}{b^2} + \frac{(bc-ad)^2 \int e^{-a-bx}(a+bx)^4 dx}{b^2} \\
&= -\frac{(bc-ad)^2 e^{-a-bx}(a+bx)^4}{b^3} - \frac{2d(bc-ad)e^{-a-bx}(a+bx)^5}{b^3} - \frac{d^2 e^{-a-bx}(a+bx)^6}{b^3} + \frac{(6d^2) \int e^{-a-bx}(a+bx)^3 dx}{b^2} \\
&= -\frac{4(bc-ad)^2 e^{-a-bx}(a+bx)^3}{b^3} - \frac{10d(bc-ad)e^{-a-bx}(a+bx)^4}{b^3} - \frac{(bc-ad)^2 e^{-a-bx}(a+bx)^4}{b^3} \\
&= -\frac{12(bc-ad)^2 e^{-a-bx}(a+bx)^2}{b^3} - \frac{40d(bc-ad)e^{-a-bx}(a+bx)^3}{b^3} - \frac{4(bc-ad)^2 e^{-a-bx}(a+bx)^4}{b^3} \\
&= -\frac{24(bc-ad)^2 e^{-a-bx}(a+bx)}{b^3} - \frac{120d(bc-ad)e^{-a-bx}(a+bx)^2}{b^3} - \frac{12(bc-ad)^2 e^{-a-bx}(a+bx)^4}{b^3} \\
&= -\frac{24(bc-ad)^2 e^{-a-bx}}{b^3} - \frac{240d(bc-ad)e^{-a-bx}(a+bx)}{b^3} - \frac{24(bc-ad)^2 e^{-a-bx}(a+bx)}{b^3} - \frac{360d^2 e^{-a-bx}(a+bx)^2}{b^3} \\
&= -\frac{240d(bc-ad)e^{-a-bx}}{b^3} - \frac{24(bc-ad)^2 e^{-a-bx}}{b^3} - \frac{720d^2 e^{-a-bx}(a+bx)}{b^3} - \frac{240d(bc-ad)e^{-a-bx}(a+bx)^2}{b^3} \\
&= -\frac{720d^2 e^{-a-bx}}{b^3} - \frac{240d(bc-ad)e^{-a-bx}}{b^3} - \frac{24(bc-ad)^2 e^{-a-bx}}{b^3} - \frac{720d^2 e^{-a-bx}(a+bx)}{b^3} - \frac{240d(bc-ad)e^{-a-bx}(a+bx)^2}{b^3}
\end{aligned}$$





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**Maxima [A]** time = 1.18123, size = 809, normalized size = 1.63

$$\frac{4(bx+1)a^3c^2e^{(-bx-a)}}{b} - \frac{a^4c^2e^{(-bx-a)}}{b} - \frac{2(bx+1)a^4cde^{(-bx-a)}}{b^2} - \frac{6(b^2x^2+2bx+2)a^2c^2e^{(-bx-a)}}{b} - \frac{8(b^2x^2+2bx+2)a^3}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(-b\*x-a)\*(b\*x+a)^4\*(d\*x+c)^2,x, algorithm="maxima")

[Out]  $-4*(b*x + 1)*a^3*c^2*e^{(-b*x - a)}/b - a^4*c^2*e^{(-b*x - a)}/b - 2*(b*x + 1)*a^4*c*d*e^{(-b*x - a)}/b^2 - 6*(b^2*x^2 + 2*b*x + 2)*a^2*c^2*e^{(-b*x - a)}/b - 8*(b^2*x^2 + 2*b*x + 2)*a^3*c*d*e^{(-b*x - a)}/b^2 - (b^2*x^2 + 2*b*x + 2)*a^4*d^2*e^{(-b*x - a)}/b^3 - 4*(b^3*x^3 + 3*b^2*x^2 + 6*b*x + 6)*a*c^2*e^{(-b*x - a)}/b - 12*(b^3*x^3 + 3*b^2*x^2 + 6*b*x + 6)*a^2*c*d*e^{(-b*x - a)}/b^2 - 4*(b^3*x^3 + 3*b^2*x^2 + 6*b*x + 6)*a^3*d^2*e^{(-b*x - a)}/b^3 - (b^4*x^4 + 4*b^3*x^3 + 12*b^2*x^2 + 24*b*x + 24)*c^2*e^{(-b*x - a)}/b - 8*(b^4*x^4 + 4*b^3*x^3 + 12*b^2*x^2 + 24*b*x + 24)*a*c*d*e^{(-b*x - a)}/b^2 - 6*(b^4*x^4 + 4*b^3*x^3 + 12*b^2*x^2 + 24*b*x + 24)*a^2*d^2*e^{(-b*x - a)}/b^3 - 2*(b^5*x^5 + 5*b^4*x^4 + 20*b^3*x^3 + 60*b^2*x^2 + 120*b*x + 120)*c*d*e^{(-b*x - a)}/b^2 - 4*(b^5*x^5 + 5*b^4*x^4 + 20*b^3*x^3 + 60*b^2*x^2 + 120*b*x + 120)*a*d^2*e^{(-b*x - a)}/b^3 - (b^6*x^6 + 6*b^5*x^5 + 30*b^4*x^4 + 120*b^3*x^3 + 360*b^2*x^2 + 720*b*x + 720)*d^2*e^{(-b*x - a)}/b^3$

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**Fricas [A]** time = 1.43459, size = 844, normalized size = 1.71

$$\frac{(b^6d^2x^6 + 2(b^6cd + (2a + 3)b^5d^2)x^5 + (a^4 + 4a^3 + 12a^2 + 24a + 24)b^2c^2 + (b^6c^2 + 2(4a + 5)b^5cd + 2(3a^2 + 10a + 15)b^4d^2)x^4 + 2(a^4 + 8a^3 + 36a^2 + 96a + 120)*b*c*d + 4*((a + 1)*b^5*c^2 + (3*a^2 + 8*a + 10)*b^4*c*d + (a^3 + 6*a^2 + 20*a + 30)*b^3*d^2)*x^3 + 2*(a^4 + 12*a^3 + 72*a^2 + 240*a + 360)*d^2 + (6*(a^2 + 2*a + 2)*b^4*c^2 + 4*(2*a^3 + 9*a^2 + 24*a + 30)*b^3*c*d + (a^4 + 12*a^3 + 72*a^2 + 240*a + 360)*b^2*d^2)*x^2 + 2*(2*(a^3 + 3*a^2 + 6*a + 6)*b^3*c^2 + (a^4 + 8*a^3 + 36*a^2 + 96*a + 120)*b^2*c*d + (a^4 + 12*a^3 + 72*a^2 + 240*a + 360)*b*d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(-b\*x-a)\*(b\*x+a)^4\*(d\*x+c)^2,x, algorithm="fricas")

[Out]  $-(b^6*d^2*x^6 + 2*(b^6*c*d + (2*a + 3)*b^5*d^2)*x^5 + (a^4 + 4*a^3 + 12*a^2 + 24*a + 24)*b^2*c^2 + (b^6*c^2 + 2*(4*a + 5)*b^5*c*d + 2*(3*a^2 + 10*a + 15)*b^4*d^2)*x^4 + 2*(a^4 + 8*a^3 + 36*a^2 + 96*a + 120)*b*c*d + 4*((a + 1)*b^5*c^2 + (3*a^2 + 8*a + 10)*b^4*c*d + (a^3 + 6*a^2 + 20*a + 30)*b^3*d^2)*x^3 + 2*(a^4 + 12*a^3 + 72*a^2 + 240*a + 360)*d^2 + (6*(a^2 + 2*a + 2)*b^4*c^2 + 4*(2*a^3 + 9*a^2 + 24*a + 30)*b^3*c*d + (a^4 + 12*a^3 + 72*a^2 + 240*a + 360)*b^2*d^2)*x^2 + 2*(2*(a^3 + 3*a^2 + 6*a + 6)*b^3*c^2 + (a^4 + 8*a^3 + 36*a^2 + 96*a + 120)*b^2*c*d + (a^4 + 12*a^3 + 72*a^2 + 240*a + 360)*b*d$

$$^2)*x)*e^{(-b*x - a)/b^3}$$

**Sympy [A]** time = 0.356124, size = 899, normalized size = 1.82

$$\left\{ \begin{array}{l} (-a^4b^2c^2-2a^4b^2cdx-a^4b^2d^2x^2-2a^4bcd-2a^4bd^2x-2a^4d^2-4a^3b^3c^2x-8a^3b^3cdx^2-4a^3b^3d^2x^3-4a^3b^2c^2-16a^3b^2cdx-12a^3b^2d^2x^2-16a^3bcd-24a^3bd^2x-24a^3d^2-6a^2b^4c^2x+\frac{b^4d^2x^7}{7}+x^6\left(\frac{2ab^3d^2}{3}+\frac{b^4cd}{3}\right)+x^5\left(\frac{6a^2b^2d^2}{5}+\frac{8ab^3cd}{5}+\frac{b^4c^2}{5}\right)+x^4\left(a^3bd^2+3a^2b^2cd+ab^3c^2\right)+x^3\left(\frac{a^4d^2}{3}+\frac{8a^3bcd}{3}+2a^2b^4c^2\right) \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(-b\*x-a)\*(b\*x+a)\*\*4\*(d\*x+c)\*\*2,x)

[Out] Piecewise((( -a\*\*4\*b\*\*2\*c\*\*2 - 2\*a\*\*4\*b\*\*2\*c\*d\*x - a\*\*4\*b\*\*2\*d\*\*2\*x\*\*2 - 2\*a\*\*4\*b\*c\*d - 2\*a\*\*4\*b\*d\*\*2\*x - 2\*a\*\*4\*d\*\*2 - 4\*a\*\*3\*b\*\*3\*c\*\*2\*x - 8\*a\*\*3\*b\*\*3\*c\*d\*x\*\*2 - 4\*a\*\*3\*b\*\*3\*d\*\*2\*x\*\*3 - 4\*a\*\*3\*b\*\*2\*c\*\*2 - 16\*a\*\*3\*b\*\*2\*c\*d\*x - 12\*a\*\*3\*b\*\*2\*d\*\*2\*x\*\*2 - 16\*a\*\*3\*b\*c\*d - 24\*a\*\*3\*b\*d\*\*2\*x - 24\*a\*\*3\*d\*\*2 - 6\*a\*\*2\*b\*\*4\*c\*\*2\*x\*\*2 - 12\*a\*\*2\*b\*\*4\*c\*d\*x\*\*3 - 6\*a\*\*2\*b\*\*4\*d\*\*2\*x\*\*4 - 12\*a\*\*2\*b\*\*3\*c\*\*2\*x - 36\*a\*\*2\*b\*\*3\*c\*d\*x\*\*2 - 24\*a\*\*2\*b\*\*3\*d\*\*2\*x\*\*3 - 12\*a\*\*2\*b\*\*2\*c\*\*2 - 72\*a\*\*2\*b\*\*2\*c\*d\*x - 72\*a\*\*2\*b\*\*2\*d\*\*2\*x\*\*2 - 72\*a\*\*2\*b\*c\*d - 144\*a\*\*2\*b\*d\*\*2\*x - 144\*a\*\*2\*d\*\*2 - 4\*a\*b\*\*5\*c\*\*2\*x\*\*3 - 8\*a\*b\*\*5\*c\*d\*x\*\*4 - 4\*a\*b\*\*5\*d\*\*2\*x\*\*5 - 12\*a\*b\*\*4\*c\*\*2\*x\*\*2 - 32\*a\*b\*\*4\*c\*d\*x\*\*3 - 20\*a\*b\*\*4\*d\*\*2\*x\*\*4 - 24\*a\*b\*\*3\*c\*\*2\*x - 96\*a\*b\*\*3\*c\*d\*x\*\*2 - 80\*a\*b\*\*3\*d\*\*2\*x\*\*3 - 24\*a\*b\*\*2\*c\*\*2 - 192\*a\*b\*\*2\*c\*d\*x - 240\*a\*b\*\*2\*d\*\*2\*x\*\*2 - 192\*a\*b\*c\*d - 480\*a\*b\*d\*\*2\*x - 480\*a\*d\*\*2 - b\*\*6\*c\*\*2\*x\*\*4 - 2\*b\*\*6\*c\*d\*x\*\*5 - b\*\*6\*d\*\*2\*x\*\*6 - 4\*b\*\*5\*c\*\*2\*x\*\*3 - 10\*b\*\*5\*c\*d\*x\*\*4 - 6\*b\*\*5\*d\*\*2\*x\*\*5 - 12\*b\*\*4\*c\*\*2\*x\*\*2 - 40\*b\*\*4\*c\*d\*x\*\*3 - 30\*b\*\*4\*d\*\*2\*x\*\*4 - 24\*b\*\*3\*c\*\*2\*x - 120\*b\*\*3\*c\*d\*x\*\*2 - 120\*b\*\*3\*d\*\*2\*x\*\*3 - 24\*b\*\*2\*c\*\*2 - 240\*b\*\*2\*c\*d\*x - 360\*b\*\*2\*d\*\*2\*x\*\*2 - 240\*b\*c\*d - 720\*b\*d\*\*2\*x - 720\*d\*\*2)\*exp(-a - b\*x)/b\*\*3, Ne(b\*\*3, 0)), (a\*\*4\*c\*\*2\*x + b\*\*4\*d\*\*2\*x\*\*7/7 + x\*\*6\*(2\*a\*b\*\*3\*d\*\*2/3 + b\*\*4\*c\*d/3) + x\*\*5\*(6\*a\*\*2\*b\*\*2\*d\*\*2/5 + 8\*a\*b\*\*3\*c\*d/5 + b\*\*4\*c\*\*2/5) + x\*\*4\*(a\*\*3\*b\*d\*\*2 + 3\*a\*\*2\*b\*\*2\*c\*d + a\*b\*\*3\*c\*\*2) + x\*\*3\*(a\*\*4\*d\*\*2/3 + 8\*a\*\*3\*b\*c\*d/3 + 2\*a\*\*2\*b\*\*2\*c\*\*2) + x\*\*2\*(a\*\*4\*c\*d + 2\*a\*\*3\*b\*c\*\*2), True))

**Giac [A]** time = 1.25861, size = 910, normalized size = 1.84

$$\left( b^{10}d^2x^6 + 2b^{10}cdx^5 + 4ab^9d^2x^5 + b^{10}c^2x^4 + 8ab^9cdx^4 + 6a^2b^8d^2x^4 + 6b^9d^2x^5 + 4ab^9c^2x^3 + 12a^2b^8cdx^3 + 4a^3b^7d^2x^2 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(-b\*x-a)\*(b\*x+a)^4\*(d\*x+c)^2,x, algorithm="giac")

[Out]  $-(b^{10}d^2x^6 + 2b^{10}cdx^5 + 4a^2b^9d^2x^5 + b^{10}c^2x^4 + 8ab^9cdx^4 + 6a^2b^8d^2x^4 + 6b^9d^2x^5 + 4ab^9c^2x^3 + 12a^2b^8cdx^3 + 4a^3b^7d^2x^3 + 10b^9cdx^4 + 20ab^8d^2x^4 + 6a^2b^8c^2x^2 + 8a^3b^7cdx^2 + a^4b^6d^2x^2 + 4b^9c^2x^3 + 32ab^8cdx^3 + 24a^2b^7d^2x^3 + 30b^8d^2x^4 + 4a^3b^7c^2x + 2a^4b^6cdx + 12ab^8c^2x^2 + 36a^2b^7cdx^2 + 12a^3b^6d^2x^2 + 40b^8cdx^3 + 80ab^7d^2x^3 + a^4b^6c^2 + 12a^2b^7c^2x + 16a^3b^6cdx + 2a^4b^5d^2x + 12b^8c^2x^2 + 96ab^7cdx^2 + 72a^2b^6d^2x^2 + 120b^7d^2x^3 + 4a^3b^6c^2 + 2a^4b^5cd + 24ab^7c^2x + 72a^2b^6cdx + 24a^3b^5d^2x + 120b^7cdx^2 + 240ab^6d^2x^2 + 12a^2b^6c^2 + 16a^3b^5cd + 2a^4b^4d^2 + 24b^7c^2x + 192ab^6cdx + 144a^2b^5d^2x + 360b^6d^2x^2 + 24ab^6c^2 + 72a^2b^5cd + 24a^3b^4d^2 + 240b^6cdx + 480ab^5d^2x + 24b^6c^2 + 192ab^5cd + 144a^2b^4d^2 + 720b^5d^2x + 240b^5cd + 480ab^4d^2 + 720b^4d^2)e^{(-bx - a)}/b^7$

### 3.76 $\int e^{-a-bx}(a+bx)^4(c+dx) dx$

**Optimal.** Leaf size=271

$$\frac{e^{-a-bx}(a+bx)^4(bc-ad)}{b^2} - \frac{4e^{-a-bx}(a+bx)^3(bc-ad)}{b^2} - \frac{12e^{-a-bx}(a+bx)^2(bc-ad)}{b^2} - \frac{24e^{-a-bx}(a+bx)(bc-ad)}{b^2} - \frac{24e^{-a-bx}(a+bx)^4}{b^2}$$

[Out]  $(-120*d*E^{-a-b*x})/b^2 - (24*(b*c - a*d)*E^{-a-b*x})/b^2 - (120*d*E^{-a-b*x}*(a+b*x))/b^2 - (24*(b*c - a*d)*E^{-a-b*x}*(a+b*x))/b^2 - (60*d*E^{-a-b*x}*(a+b*x)^2)/b^2 - (12*(b*c - a*d)*E^{-a-b*x}*(a+b*x)^2)/b^2 - (20*d*E^{-a-b*x}*(a+b*x)^3)/b^2 - (4*(b*c - a*d)*E^{-a-b*x}*(a+b*x)^3)/b^2 - (5*d*E^{-a-b*x}*(a+b*x)^4)/b^2 - ((b*c - a*d)*E^{-a-b*x}*(a+b*x)^4)/b^2 - (d*E^{-a-b*x}*(a+b*x)^5)/b^2$

**Rubi [A]** time = 0.338639, antiderivative size = 271, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 3, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.13$ , Rules used = {2196, 2176, 2194}

$$\frac{e^{-a-bx}(a+bx)^4(bc-ad)}{b^2} - \frac{4e^{-a-bx}(a+bx)^3(bc-ad)}{b^2} - \frac{12e^{-a-bx}(a+bx)^2(bc-ad)}{b^2} - \frac{24e^{-a-bx}(a+bx)(bc-ad)}{b^2} - \frac{24e^{-a-bx}(a+bx)^4}{b^2}$$

Antiderivative was successfully verified.

[In] Int[E<sup>-a - b\*x</sup>\*(a + b\*x)<sup>4</sup>\*(c + d\*x), x]

[Out]  $(-120*d*E^{-a-b*x})/b^2 - (24*(b*c - a*d)*E^{-a-b*x})/b^2 - (120*d*E^{-a-b*x}*(a+b*x))/b^2 - (24*(b*c - a*d)*E^{-a-b*x}*(a+b*x))/b^2 - (60*d*E^{-a-b*x}*(a+b*x)^2)/b^2 - (12*(b*c - a*d)*E^{-a-b*x}*(a+b*x)^2)/b^2 - (20*d*E^{-a-b*x}*(a+b*x)^3)/b^2 - (4*(b*c - a*d)*E^{-a-b*x}*(a+b*x)^3)/b^2 - (5*d*E^{-a-b*x}*(a+b*x)^4)/b^2 - ((b*c - a*d)*E^{-a-b*x}*(a+b*x)^4)/b^2 - (d*E^{-a-b*x}*(a+b*x)^5)/b^2$

#### Rule 2196

Int[(F\_)^((c\_.)\*(v\_))\*(u\_), x\_Symbol] :> Int[ExpandIntegrand[F^(c\*ExpandToSum[v, x]), u, x], x] /; FreeQ[{F, c}, x] && PolynomialQ[u, x] && LinearQ[v, x] && !\$UseGamma == True

#### Rule 2176

Int[((b\_.)\*(F\_)^(g\_.)\*((e\_.) + (f\_.)\*(x\_)))^(n\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[((c + d\*x)^m\*(b\*F^(g\*(e + f\*x)))^n)/(f\*g\*n\*Log[F]),

```
x] - Dist[(d*m)/(f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] && !$UseGamma === True
```

### Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

### Rubi steps

$$\begin{aligned}
\int e^{-a-bx}(a+bx)^4(c+dx) dx &= \int \left( \frac{(bc-ad)e^{-a-bx}(a+bx)^4}{b} + \frac{de^{-a-bx}(a+bx)^5}{b} \right) dx \\
&= \frac{d \int e^{-a-bx}(a+bx)^5 dx}{b} + \frac{(bc-ad) \int e^{-a-bx}(a+bx)^4 dx}{b} \\
&= -\frac{(bc-ad)e^{-a-bx}(a+bx)^4}{b^2} - \frac{de^{-a-bx}(a+bx)^5}{b^2} + \frac{(5d) \int e^{-a-bx}(a+bx)^4 dx}{b} + \frac{(4(bc-ad))}{b^2} \\
&= -\frac{4(bc-ad)e^{-a-bx}(a+bx)^3}{b^2} - \frac{5de^{-a-bx}(a+bx)^4}{b^2} - \frac{(bc-ad)e^{-a-bx}(a+bx)^4}{b^2} - \frac{de^{-a-bx}(a+bx)^5}{b^2} \\
&= -\frac{12(bc-ad)e^{-a-bx}(a+bx)^2}{b^2} - \frac{20de^{-a-bx}(a+bx)^3}{b^2} - \frac{4(bc-ad)e^{-a-bx}(a+bx)^3}{b^2} - \frac{5de^{-a-bx}(a+bx)^4}{b^2} \\
&= -\frac{24(bc-ad)e^{-a-bx}(a+bx)}{b^2} - \frac{60de^{-a-bx}(a+bx)^2}{b^2} - \frac{12(bc-ad)e^{-a-bx}(a+bx)^2}{b^2} - \frac{20de^{-a-bx}(a+bx)^3}{b^2} \\
&= -\frac{24(bc-ad)e^{-a-bx}}{b^2} - \frac{120de^{-a-bx}(a+bx)}{b^2} - \frac{24(bc-ad)e^{-a-bx}(a+bx)}{b^2} - \frac{60de^{-a-bx}(a+bx)^2}{b^2} \\
&= -\frac{120de^{-a-bx}}{b^2} - \frac{24(bc-ad)e^{-a-bx}}{b^2} - \frac{120de^{-a-bx}(a+bx)}{b^2} - \frac{24(bc-ad)e^{-a-bx}(a+bx)}{b^2} - \frac{60de^{-a-bx}(a+bx)^2}{b^2}
\end{aligned}$$

**Mathematica [A]** time = 0.252916, size = 191, normalized size = 0.7

$$\frac{e^{-a-bx} \left( -2b^3x^2 \left( 3(a^2 + 2a + 2)c + (3a^2 + 8a + 10)dx \right) - 2b^2x \left( 2(a^3 + 3a^2 + 6a + 6)c + (2a^3 + 9a^2 + 24a + 30)dx \right) - \right)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(-a - b*x)*(a + b*x)^4*(c + d*x), x]
```

```
[Out] (E^(-a - b*x))*(-((120 + 96*a + 36*a^2 + 8*a^3 + a^4)*d) - b^5*x^4*(c + d*x)
- b^4*x^3*(4*(1 + a)*c + (5 + 4*a)*d*x) - 2*b^3*x^2*(3*(2 + 2*a + a^2)*c +
(10 + 8*a + 3*a^2)*d*x) - 2*b^2*x*(2*(6 + 6*a + 3*a^2 + a^3)*c + (30 + 24*
```

$$a + 9a^2 + 2a^3)dx) - b((24 + 24a + 12a^2 + 4a^3 + a^4)c + (120 + 96a + 36a^2 + 8a^3 + a^4)dx))/b^2$$

**Maple [A]** time = 0.005, size = 297, normalized size = 1.1

$$(b^5 dx^5 + 4 ab^4 dx^4 + b^5 cx^4 + 6 a^2 b^3 dx^3 + 4 ab^4 cx^3 + 5 b^4 dx^4 + 4 a^3 b^2 dx^2 + 6 a^2 b^3 cx^2 + 16 ab^3 dx^3 + 4 b^4 cx^3 + a^4 b dx + 4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(exp(-b*x-a)*(b*x+a)^4*(d*x+c), x)`

[Out]  $-(b^5 d x^5 + 4 a b^4 d x^4 + b^5 c x^4 + 6 a^2 b^3 d x^3 + 4 a b^4 c x^3 + 5 b^4 d x^4 + 4 a^3 b^2 d x^2 + 6 a^2 b^3 c x^2 + 16 a b^3 d x^3 + 4 b^4 c x^3 + a^4 b d x + 4 a^3 b^2 c x + 18 a^2 b^2 d x^2 + 12 a b^3 c x^2 + 20 b^3 d x^3 + a^4 b c + 8 a^3 b d x + 12 a^2 b^2 c x + 48 a b^2 d x^2 + 12 b^3 c x^2 + a^4 d + 4 a^3 b c + 36 a^2 b d x + 24 a b^2 c x + 60 b^2 d x^2 + 8 a^3 d + 12 a^2 b c + 96 a b d x + 24 b^2 c x + 36 a^2 d + 24 a b c + 120 b d x + 96 a d + 24 b c + 120 d) \exp(-b x - a) / b^2$

**Maxima [A]** time = 1.08597, size = 464, normalized size = 1.71

$$\frac{4(bx+1)a^3ce^{(-bx-a)}}{b} - \frac{a^4ce^{(-bx-a)}}{b} - \frac{(bx+1)a^4de^{(-bx-a)}}{b^2} - \frac{6(b^2x^2+2bx+2)a^2ce^{(-bx-a)}}{b} - \frac{4(b^2x^2+2bx+2)a^3de^{(-bx-a)}}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(-b*x-a)*(b*x+a)^4*(d*x+c), x, algorithm="maxima")`

[Out]  $-4*(b*x + 1)*a^3*c*e^{(-b*x - a)}/b - a^4*c*e^{(-b*x - a)}/b - (b*x + 1)*a^4*d*e^{(-b*x - a)}/b^2 - 6*(b^2*x^2 + 2*b*x + 2)*a^2*c*e^{(-b*x - a)}/b - 4*(b^2*x^2 + 2*b*x + 2)*a^3*d*e^{(-b*x - a)}/b^2 - 4*(b^3*x^3 + 3*b^2*x^2 + 6*b*x + 6)*a*c*e^{(-b*x - a)}/b - 6*(b^3*x^3 + 3*b^2*x^2 + 6*b*x + 6)*a^2*d*e^{(-b*x - a)}/b^2 - (b^4*x^4 + 4*b^3*x^3 + 12*b^2*x^2 + 24*b*x + 24)*c*e^{(-b*x - a)}/b - 4*(b^4*x^4 + 4*b^3*x^3 + 12*b^2*x^2 + 24*b*x + 24)*a*d*e^{(-b*x - a)}/b^2 - (b^5*x^5 + 5*b^4*x^4 + 20*b^3*x^3 + 60*b^2*x^2 + 120*b*x + 120)*d*e^{(-b*x - a)}/b^2$

**Fricas [A]** time = 1.50285, size = 468, normalized size = 1.73

$$\frac{(b^5 dx^5 + (b^5 c + (4a + 5)b^4 d)x^4 + 2(2(a + 1)b^4 c + (3a^2 + 8a + 10)b^3 d)x^3 + (a^4 + 4a^3 + 12a^2 + 24a + 24)bc + 2(3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(-b\*x-a)\*(b\*x+a)^4\*(d\*x+c), x, algorithm="fricas")

[Out]  $-(b^5 d x^5 + (b^5 c + (4a + 5)b^4 d)x^4 + 2(2(a + 1)b^4 c + (3a^2 + 8a + 10)b^3 d)x^3 + (a^4 + 4a^3 + 12a^2 + 24a + 24)bc + 2(3(a^2 + 2a + 2)b^3 c + (2a^3 + 9a^2 + 24a + 30)b^2 d)x^2 + (a^4 + 8a^3 + 36a^2 + 96a + 120)d + (4(a^3 + 3a^2 + 6a + 6)b^2 c + (a^4 + 8a^3 + 36a^2 + 96a + 120)b d)x)e^{-(b*x - a)}/b^2$

**Sympy [A]** time = 0.245281, size = 447, normalized size = 1.65

$$\frac{(-a^4 bc - a^4 b dx - a^4 d - 4a^3 b^2 cx - 4a^3 b^2 dx^2 - 4a^3 bc - 8a^3 b dx - 8a^3 d - 6a^2 b^3 cx^2 - 6a^2 b^3 dx^3 - 12a^2 b^2 cx - 18a^2 b^2 dx^2 - 12a^2 bc - 36a^2 b dx - 36a^2 d - 4ab^4 cx^3 - 4ab^4 dx^4 - 12ab^3 c}{a^4 cx + \frac{b^4 dx^6}{6} + x^5 \left( \frac{4ab^3 d}{5} + \frac{b^4 c}{5} \right) + x^4 \left( \frac{3a^2 b^2 d}{2} + ab^3 c \right) + x^3 \left( \frac{4a^3 b d}{3} + 2a^2 b^2 c \right) + x^2 \left( \frac{a^4 d}{2} + 2a^3 bc \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(-b\*x-a)\*(b\*x+a)\*\*4\*(d\*x+c), x)

[Out] Piecewise((( $-a**4*b*c - a**4*b*d*x - a**4*d - 4*a**3*b**2*c*x - 4*a**3*b**2*d*x**2 - 4*a**3*b*c - 8*a**3*b*d*x - 8*a**3*d - 6*a**2*b**3*c*x**2 - 6*a**2*b**3*d*x**3 - 12*a**2*b**2*c*x - 18*a**2*b**2*d*x**2 - 12*a**2*b*c - 36*a**2*b*d*x - 36*a**2*d - 4*a*b**4*c*x**3 - 4*a*b**4*d*x**4 - 12*a*b**3*c*x**2 - 16*a*b**3*d*x**3 - 24*a*b**2*c*x - 48*a*b**2*d*x**2 - 24*a*b*c - 96*a*b*d*x - 96*a*d - b**5*c*x**4 - b**5*d*x**5 - 4*b**4*c*x**3 - 5*b**4*d*x**4 - 12*b**3*c*x**2 - 20*b**3*d*x**3 - 24*b**2*c*x - 60*b**2*d*x**2 - 24*b*c - 120*b*d*x - 120*d$ )\*exp(-a - b\*x)/b\*\*2, Ne(b\*\*2, 0)), (a\*\*4\*c\*x + b\*\*4\*d\*x\*\*6/6 + x\*\*5\*(4\*a\*b\*\*3\*d/5 + b\*\*4\*c/5) + x\*\*4\*(3\*a\*\*2\*b\*\*2\*d/2 + a\*b\*\*3\*c) + x\*\*3\*(4\*a\*\*3\*b\*d/3 + 2\*a\*\*2\*b\*\*2\*c) + x\*\*2\*(a\*\*4\*d/2 + 2\*a\*\*3\*b\*c), True))

**Giac [A]** time = 1.24089, size = 447, normalized size = 1.65

$$(b^9 dx^5 + b^9 cx^4 + 4ab^8 dx^4 + 4ab^8 cx^3 + 6a^2 b^7 dx^3 + 5b^8 dx^4 + 6a^2 b^7 cx^2 + 4a^3 b^6 dx^2 + 4b^8 cx^3 + 16ab^7 dx^3 + 4a^3 b^6 cx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(-b\*x-a)\*(b\*x+a)^4\*(d\*x+c),x, algorithm="giac")

[Out]  $-(b^9 d x^5 + b^9 c x^4 + 4 a b^8 d x^4 + 4 a b^8 c x^3 + 6 a^2 b^7 d x^3 + 5 b^8 d x^4 + 6 a^2 b^7 c x^2 + 4 a^3 b^6 d x^2 + 4 b^8 c x^3 + 16 a b^7 d x^3 + 4 a^3 b^6 c x + a^4 b^5 d x + 12 a b^7 c x^2 + 18 a^2 b^6 d x^2 + 20 b^7 d x^3 + a^4 b^5 c + 12 a^2 b^6 c x + 8 a^3 b^5 d x + 12 b^7 c x^2 + 48 a b^6 d x^2 + 4 a^3 b^5 c + a^4 b^4 d + 24 a b^6 c x + 36 a^2 b^5 d x + 60 b^6 d x^2 + 12 a^2 b^5 c + 8 a^3 b^4 d + 24 b^6 c x + 96 a b^5 d x + 24 a b^5 c + 36 a^2 b^4 d + 120 b^5 d x + 24 b^5 c + 96 a b^4 d + 120 b^4 d) e^{(-b x - a) / b^6}$



### 3.77 $\int e^{-a-bx}(a+bx)^4 dx$

**Optimal.** Leaf size=102

$$-\frac{e^{-a-bx}(a+bx)^4}{b} - \frac{4e^{-a-bx}(a+bx)^3}{b} - \frac{12e^{-a-bx}(a+bx)^2}{b} - \frac{24e^{-a-bx}(a+bx)}{b} - \frac{24e^{-a-bx}}{b}$$

[Out]  $(-24E^{-a-bx})/b - (24E^{-a-bx})(a+bx)/b - (12E^{-a-bx})(a+bx)^2/b - (4E^{-a-bx})(a+bx)^3/b - (E^{-a-bx})(a+bx)^4/b$

**Rubi [A]** time = 0.0964918, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$ , Rules used = {2176, 2194}

$$-\frac{e^{-a-bx}(a+bx)^4}{b} - \frac{4e^{-a-bx}(a+bx)^3}{b} - \frac{12e^{-a-bx}(a+bx)^2}{b} - \frac{24e^{-a-bx}(a+bx)}{b} - \frac{24e^{-a-bx}}{b}$$

Antiderivative was successfully verified.

[In] Int[E<sup>-(a+bx)</sup>(a+bx)<sup>4</sup>, x]

[Out]  $(-24E^{-a-bx})/b - (24E^{-a-bx})(a+bx)/b - (12E^{-a-bx})(a+bx)^2/b - (4E^{-a-bx})(a+bx)^3/b - (E^{-a-bx})(a+bx)^4/b$

#### Rule 2176

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[((c + d*x)^m*(b*F^(g*(e + f*x)))^n)/(f*g*n*Log[F]), x] - Dist[(d*m)/(f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m] && !$UseGamma == True
```

#### Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a + b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

#### Rubi steps

$$\begin{aligned}
\int e^{-a-bx}(a+bx)^4 dx &= -\frac{e^{-a-bx}(a+bx)^4}{b} + 4 \int e^{-a-bx}(a+bx)^3 dx \\
&= -\frac{4e^{-a-bx}(a+bx)^3}{b} - \frac{e^{-a-bx}(a+bx)^4}{b} + 12 \int e^{-a-bx}(a+bx)^2 dx \\
&= -\frac{12e^{-a-bx}(a+bx)^2}{b} - \frac{4e^{-a-bx}(a+bx)^3}{b} - \frac{e^{-a-bx}(a+bx)^4}{b} + 24 \int e^{-a-bx}(a+bx) dx \\
&= -\frac{24e^{-a-bx}(a+bx)}{b} - \frac{12e^{-a-bx}(a+bx)^2}{b} - \frac{4e^{-a-bx}(a+bx)^3}{b} - \frac{e^{-a-bx}(a+bx)^4}{b} + 24 \int e^{-a-bx} dx \\
&= -\frac{24e^{-a-bx}}{b} - \frac{24e^{-a-bx}(a+bx)}{b} - \frac{12e^{-a-bx}(a+bx)^2}{b} - \frac{4e^{-a-bx}(a+bx)^3}{b} - \frac{e^{-a-bx}(a+bx)^4}{b}
\end{aligned}$$

**Mathematica [A]** time = 0.061442, size = 50, normalized size = 0.49

$$\frac{e^{-a-bx} \left( -(a+bx)^4 - 4(a+bx)^3 - 12(a+bx)^2 - 24(a+bx) - 24 \right)}{b}$$

Antiderivative was successfully verified.

[In] Integrate[E^(-a - b\*x)\*(a + b\*x)^4,x]

[Out] (E^(-a - b\*x)\*(-24 - 24\*(a + b\*x) - 12\*(a + b\*x)^2 - 4\*(a + b\*x)^3 - (a + b\*x)^4))/b

**Maple [A]** time = 0.004, size = 108, normalized size = 1.1

$$\frac{(b^4x^4 + 4b^3x^3a + 6a^2b^2x^2 + 4b^3x^3 + 4a^3bx + 12ab^2x^2 + a^4 + 12a^2bx + 12b^2x^2 + 4a^3 + 24abx + 12a^2 + 24bx + 24a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(-b\*x-a)\*(b\*x+a)^4,x)

[Out] -(b^4\*x^4+4\*a\*b^3\*x^3+6\*a^2\*b^2\*x^2+4\*b^3\*x^3+4\*a^3\*b\*x+12\*a\*b^2\*x^2+a^4+12\*a^2\*b\*x+12\*b^2\*x^2+4\*a^3+24\*a\*b\*x+12\*a^2+24\*b\*x+24\*a+24)\*exp(-b\*x-a)/b

**Maxima [A]** time = 1.12265, size = 201, normalized size = 1.97

$$\frac{4(bx+1)a^3e^{-bx-a}}{b} - \frac{a^4e^{-bx-a}}{b} - \frac{6(b^2x^2+2bx+2)a^2e^{-bx-a}}{b} - \frac{4(b^3x^3+3b^2x^2+6bx+6)ae^{-bx-a}}{b} - \frac{(b^4x^4+4b^3x^3+12b^2x^2+24bx+24)e^{-bx-a}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(-b\*x-a)\*(b\*x+a)^4,x, algorithm="maxima")

[Out]  $-4*(b*x + 1)*a^3*e^{(-b*x - a)}/b - a^4*e^{(-b*x - a)}/b - 6*(b^2*x^2 + 2*b*x + 2)*a^2*e^{(-b*x - a)}/b - 4*(b^3*x^3 + 3*b^2*x^2 + 6*b*x + 6)*a*e^{(-b*x - a)}/b - (b^4*x^4 + 4*b^3*x^3 + 12*b^2*x^2 + 24*b*x + 24)*e^{(-b*x - a)}/b$

**Fricas [A]** time = 1.42894, size = 192, normalized size = 1.88

$$\frac{(b^4x^4 + 4(a+1)b^3x^3 + 6(a^2 + 2a + 2)b^2x^2 + a^4 + 4a^3 + 4(a^3 + 3a^2 + 6a + 6)bx + 12a^2 + 24a + 24)e^{-bx-a}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(-b\*x-a)\*(b\*x+a)^4,x, algorithm="fricas")

[Out]  $-(b^4*x^4 + 4*(a + 1)*b^3*x^3 + 6*(a^2 + 2*a + 2)*b^2*x^2 + a^4 + 4*a^3 + 4*(a^3 + 3*a^2 + 6*a + 6)*b*x + 12*a^2 + 24*a + 24)*e^{(-b*x - a)}/b$

**Sympy [A]** time = 0.164435, size = 158, normalized size = 1.55

$$\begin{cases} \frac{(-a^4 - 4a^3bx - 4a^3 - 6a^2b^2x^2 - 12a^2bx - 12a^2 - 4ab^3x^3 - 12ab^2x^2 - 24abx - 24a - b^4x^4 - 4b^3x^3 - 12b^2x^2 - 24bx - 24)e^{-a-bx}}{b} & \text{for } b \neq 0 \\ a^4x + 2a^3bx^2 + 2a^2b^2x^3 + ab^3x^4 + \frac{b^4x^5}{5} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(-b\*x-a)\*(b\*x+a)\*\*4,x)

[Out] Piecewise((((-a\*\*4 - 4\*a\*\*3\*b\*x - 4\*a\*\*3 - 6\*a\*\*2\*b\*\*2\*x\*\*2 - 12\*a\*\*2\*b\*x - 12\*a\*\*2 - 4\*a\*b\*\*3\*x\*\*3 - 12\*a\*b\*\*2\*x\*\*2 - 24\*a\*b\*x - 24\*a - b\*\*4\*x\*\*4 - 4\*b\*\*3\*x\*\*3 - 12\*b\*\*2\*x\*\*2 - 24\*b\*x - 24)\*exp(-a - b\*x)/b, Ne(b, 0)), (a\*\*4\*x

```
+ 2*a**3*b*x**2 + 2*a**2*b**2*x**3 + a*b**3*x**4 + b**4*x**5/5, True))
```

**Giac [A]** time = 1.20679, size = 178, normalized size = 1.75

$$\frac{(b^8x^4 + 4ab^7x^3 + 6a^2b^6x^2 + 4b^7x^3 + 4a^3b^5x + 12ab^6x^2 + a^4b^4 + 12a^2b^5x + 12b^6x^2 + 4a^3b^4 + 24ab^5x + 12a^2b^4 + 24a^3b^3 + 12a^4b^2 + 12a^5b + 12a^6)x^4 + (4ab^7x^3 + 6a^2b^6x^2 + 4b^7x^3 + 4a^3b^5x + 12ab^6x^2 + a^4b^4 + 12a^2b^5x + 12b^6x^2 + 4a^3b^4 + 24ab^5x + 12a^2b^4 + 24a^3b^3 + 12a^4b^2 + 12a^5b + 12a^6)x^3 + (6a^2b^6x^2 + 4b^7x^3 + 4a^3b^5x + 12ab^6x^2 + a^4b^4 + 12a^2b^5x + 12b^6x^2 + 4a^3b^4 + 24ab^5x + 12a^2b^4 + 24a^3b^3 + 12a^4b^2 + 12a^5b + 12a^6)x^2 + (4b^7x^3 + 4a^3b^5x + 12ab^6x^2 + a^4b^4 + 12a^2b^5x + 12b^6x^2 + 4a^3b^4 + 24ab^5x + 12a^2b^4 + 24a^3b^3 + 12a^4b^2 + 12a^5b + 12a^6)x + (a^4b^4 + 12a^2b^5x + 12b^6x^2 + 4a^3b^4 + 24ab^5x + 12a^2b^4 + 24a^3b^3 + 12a^4b^2 + 12a^5b + 12a^6)}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(-b*x-a)*(b*x+a)^4,x, algorithm="giac")
```

```
[Out] -(b^8*x^4 + 4*a*b^7*x^3 + 6*a^2*b^6*x^2 + 4*b^7*x^3 + 4*a^3*b^5*x + 12*a*b^6*x^2 + a^4*b^4 + 12*a^2*b^5*x + 12*b^6*x^2 + 4*a^3*b^4 + 24*a*b^5*x + 12*a^2*b^4 + 24*b^5*x + 24*a*b^4 + 24*b^4)*e^(-b*x - a)/b^5
```

$$3.78 \quad \int \frac{e^{-a-bx}(a+bx)^4}{c+dx} dx$$

**Optimal.** Leaf size=277

$$\frac{e^{\frac{bc}{d}-a}(bc-ad)^4 \operatorname{Ei}\left(-\frac{b(c+dx)}{d}\right)}{d^5} + \frac{e^{-a-bx}(bc-ad)^3}{d^4} - \frac{e^{-a-bx}(bc-ad)^2}{d^3} - \frac{e^{-a-bx}(a+bx)(bc-ad)^2}{d^3} + \frac{2e^{-a-bx}(bc-ad)}{d^2} + \frac{e^{-a-bx}}{d}$$

[Out]  $(-6 * E^{-a - b * x}) / d + (2 * (b * c - a * d) * E^{-a - b * x}) / d^2 - ((b * c - a * d)^2 * E^{-a - b * x}) / d^3 + ((b * c - a * d)^3 * E^{-a - b * x}) / d^4 - (6 * E^{-a - b * x} * (a + b * x)) / d + (2 * (b * c - a * d) * E^{-a - b * x} * (a + b * x)) / d^2 - ((b * c - a * d)^2 * E^{-a - b * x} * (a + b * x)) / d^3 - (3 * E^{-a - b * x} * (a + b * x)^2) / d + ((b * c - a * d) * E^{-a - b * x} * (a + b * x)^2) / d^2 - (E^{-a - b * x} * (a + b * x)^3) / d + ((b * c - a * d)^4 * E^{-a - b * x} * (a + b * x) / d) * \operatorname{ExpIntegralEi}[-(b * (c + d * x) / d)] / d^5$

**Rubi [A]** time = 0.338364, antiderivative size = 277, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 4, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.16$ , Rules used = {2199, 2194, 2176, 2178}

$$\frac{e^{\frac{bc}{d}-a}(bc-ad)^4 \operatorname{Ei}\left(-\frac{b(c+dx)}{d}\right)}{d^5} + \frac{e^{-a-bx}(bc-ad)^3}{d^4} - \frac{e^{-a-bx}(bc-ad)^2}{d^3} - \frac{e^{-a-bx}(a+bx)(bc-ad)^2}{d^3} + \frac{2e^{-a-bx}(bc-ad)}{d^2} + \frac{e^{-a-bx}}{d}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(E^{-a - b * x} * (a + b * x)^4) / (c + d * x), x]$

[Out]  $(-6 * E^{-a - b * x}) / d + (2 * (b * c - a * d) * E^{-a - b * x}) / d^2 - ((b * c - a * d)^2 * E^{-a - b * x}) / d^3 + ((b * c - a * d)^3 * E^{-a - b * x}) / d^4 - (6 * E^{-a - b * x} * (a + b * x)) / d + (2 * (b * c - a * d) * E^{-a - b * x} * (a + b * x)) / d^2 - ((b * c - a * d)^2 * E^{-a - b * x} * (a + b * x)) / d^3 - (3 * E^{-a - b * x} * (a + b * x)^2) / d + ((b * c - a * d) * E^{-a - b * x} * (a + b * x)^2) / d^2 - (E^{-a - b * x} * (a + b * x)^3) / d + ((b * c - a * d)^4 * E^{-a - b * x} * (a + b * x) / d) * \operatorname{ExpIntegralEi}[-(b * (c + d * x) / d)] / d^5$

### Rule 2199

$\operatorname{Int}[(F\_)^{((c\_)*(v\_))*(u\_)^{(m\_)}*(w\_)}, x\_Symbol] \rightarrow \operatorname{Int}[\operatorname{ExpandIntegrand}[F^{(c * \operatorname{ExpandToSum}[v, x])}, w * \operatorname{NormalizePowerOfLinear}[u, x]^m, x] /; \operatorname{FreeQ}\{F, c\}, x] \&\& \operatorname{PolynomialQ}[w, x] \&\& \operatorname{LinearQ}[v, x] \&\& \operatorname{PowerOfLinearQ}[u, x] \&\& \operatorname{IntegerQ}[m] \&\& !\$UseGamma == True$

### Rule 2194

```
Int[((F_)^((c_)*(a_) + (b_)*(x_)))^(n_), x_Symbol] := Simp[(F^(c*(a +
b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

### Rule 2176

```
Int[((b_)*(F_)^((g_)*((e_) + (f_)*(x_))))^(n_)*((c_) + (d_)*(x_))^(m
_), x_Symbol] := Simp[((c + d*x)^m*(b*F^(g*(e + f*x)))^n)/(f*g*n*Log[F]),
x] - Dist[(d*m)/(f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m]
] && !$UseGamma == True
```

### Rule 2178

```
Int[(F_)^((g_)*((e_) + (f_)*(x_)))/((c_) + (d_)*(x_)), x_Symbol] := Si
mp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; F
reeQ[{F, c, d, e, f, g}, x] && !$UseGamma == True
```

### Rubi steps

$$\begin{aligned}
\int \frac{e^{-a-bx}(a+bx)^4}{c+dx} dx &= \int \left( -\frac{b(bc-ad)^3 e^{-a-bx}}{d^4} + \frac{b(bc-ad)^2 e^{-a-bx}(a+bx)}{d^3} - \frac{b(bc-ad)e^{-a-bx}(a+bx)^2}{d^2} + \frac{be^{-a-bx}(a+bx)^3}{d} \right. \\
&= \frac{b \int e^{-a-bx}(a+bx)^3 dx}{d} - \frac{(b(bc-ad)) \int e^{-a-bx}(a+bx)^2 dx}{d^2} + \frac{(b(bc-ad)^2) \int e^{-a-bx}(a+bx) dx}{d^3} - \frac{b \int e^{-a-bx} dx}{d^4} \\
&= \frac{(bc-ad)^3 e^{-a-bx}}{d^4} - \frac{(bc-ad)^2 e^{-a-bx}(a+bx)}{d^3} + \frac{(bc-ad)e^{-a-bx}(a+bx)^2}{d^2} - \frac{e^{-a-bx}(a+bx)^3}{d} + \frac{(bc-ad)^4 e^{-a-bx}}{d^4} \\
&= -\frac{(bc-ad)^2 e^{-a-bx}}{d^3} + \frac{(bc-ad)^3 e^{-a-bx}}{d^4} + \frac{2(bc-ad)e^{-a-bx}(a+bx)}{d^2} - \frac{(bc-ad)^2 e^{-a-bx}(a+bx)}{d^3} - \frac{3e^{-a-bx}(a+bx)^3}{d} \\
&= \frac{2(bc-ad)e^{-a-bx}}{d^2} - \frac{(bc-ad)^2 e^{-a-bx}}{d^3} + \frac{(bc-ad)^3 e^{-a-bx}}{d^4} - \frac{6e^{-a-bx}(a+bx)}{d} + \frac{2(bc-ad)e^{-a-bx}(a+bx)^2}{d^2} \\
&= -\frac{6e^{-a-bx}}{d} + \frac{2(bc-ad)e^{-a-bx}}{d^2} - \frac{(bc-ad)^2 e^{-a-bx}}{d^3} + \frac{(bc-ad)^3 e^{-a-bx}}{d^4} - \frac{6e^{-a-bx}(a+bx)}{d} + \frac{2(bc-ad)e^{-a-bx}(a+bx)^2}{d^2}
\end{aligned}$$

**Mathematica [A]** time = 0.298496, size = 175, normalized size = 0.63

$$\frac{e^{-a-bx} \left( (bc-ad)^4 e^{b\left(\frac{c}{d}+x\right)} \text{Ei}\left(-\frac{b(c+dx)}{d}\right) - d(2bd^2((3a^2+4a+3)dx - (3a^2+2a+1)c) + 2(2a^3+3a^2+4a+3)d^3 + b^2d^4) \right)}{d^5}$$

Antiderivative was successfully verified.

[In] Integrate[(E^(-a - b\*x)\*(a + b\*x)^4)/(c + d\*x), x]

[Out] (E^(-a - b\*x)\*(-(d\*(2\*(3 + 4\*a + 3\*a^2 + 2\*a^3)\*d^3 + 2\*b\*d^2\*(-((1 + 2\*a + 3\*a^2)\*c) + (3 + 4\*a + 3\*a^2)\*d\*x) + b^2\*d\*((1 + 4\*a)\*c^2 - 2\*(1 + 2\*a)\*c\*d\*x + (3 + 4\*a)\*d^2\*x^2) + b^3\*(-c^3 + c^2\*d\*x - c\*d^2\*x^2 + d^3\*x^3))) + (b\*c - a\*d)^4\*E^(b\*(c/d + x))\*ExpIntegralEi[-((b\*(c + d\*x))/d)])/d^5

**Maple [A]** time = 0.012, size = 489, normalized size = 1.8

$$\frac{1}{b} \left( -\frac{b((-bx-a)^3 e^{-bx-a} - 3(-bx-a)^2 e^{-bx-a} + 6(-bx-a) e^{-bx-a} - 6e^{-bx-a})}{d} + \frac{ab((-bx-a)^2 e^{-bx-a} - 2(-bx-a)e^{-bx-a})}{d} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(-b\*x-a)\*(b\*x+a)^4/(d\*x+c), x)

[Out] -1/b\*(-b/d\*((-b\*x-a)^3\*exp(-b\*x-a)-3\*(-b\*x-a)^2\*exp(-b\*x-a)+6\*(-b\*x-a)\*exp(-b\*x-a)-6\*exp(-b\*x-a))+b/d\*a\*((-b\*x-a)^2\*exp(-b\*x-a)-2\*(-b\*x-a)\*exp(-b\*x-a)+2\*exp(-b\*x-a))-b^2/d^2\*c\*((-b\*x-a)^2\*exp(-b\*x-a)-2\*(-b\*x-a)\*exp(-b\*x-a)+2\*exp(-b\*x-a))-b/d\*a^2\*((-b\*x-a)\*exp(-b\*x-a)-exp(-b\*x-a))+2\*b^2/d^2\*a\*c\*((-b\*x-a)\*exp(-b\*x-a)-exp(-b\*x-a))-b^3/d^3\*c^2\*((-b\*x-a)\*exp(-b\*x-a)-exp(-b\*x-a))+b/d\*a^3\*exp(-b\*x-a)-3\*b^2/d^2\*a^2\*c\*exp(-b\*x-a)+3\*b^3/d^3\*a\*c^2\*exp(-b\*x-a)-b^4/d^4\*c^3\*exp(-b\*x-a)+(a^4\*d^4-4\*a^3\*b\*c\*d^3+6\*a^2\*b^2\*c^2\*d^2-4\*a\*b^3\*c^3\*d+b^4\*c^4)\*b/d^5\*exp(-(a\*d-b\*c)/d)\*Ei(1, b\*x+a-(a\*d-b\*c)/d))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{a^4 e^{\left(-a + \frac{bc}{d}\right)} E_1\left(\frac{(dx+c)b}{d}\right)}{d} - \frac{(b^3 d^2 x^4 + (4 a b^2 d^2 + 3 b^2 d^2) x^3 + (6 a^2 b d^2 + b^2 c d + 8 a b d^2 + 6 b d^2) x^2 + (4 a^3 d^2 - b^2 c^2 + 6 a^2 c d) x + a^4 d^2 - b^2 c^2)}{d^3 x e^a + c d^2 e^a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(-b\*x-a)\*(b\*x+a)^4/(d\*x+c), x, algorithm="maxima")

[Out] -a^4\*e^(-a + b\*c/d)\*exp\_integral\_e(1, (d\*x + c)\*b/d)/d - (b^3\*d^2\*x^4 + (4\*a\*b^2\*d^2 + 3\*b^2\*d^2)\*x^3 + (6\*a^2\*b\*d^2 + b^2\*c\*d + 8\*a\*b\*d^2 + 6\*b\*d^2)\*x^2 + (4\*a^3\*d^2 - b^2\*c^2 + 6\*a^2\*d^2 + 4\*b\*c\*d + 4\*(b\*c\*d + 2\*d^2)\*a + 6\*d^2)\*x)\*e^(-b\*x)/(d^3\*x\*e^a + c\*d^2\*e^a) + integrate((4\*a^3\*c\*d^2 - b^2\*c^3

$$+ 6a^2c^2d^2 + 4b^2c^2d + 6c^2d^2 + 4(b^2c^2d + 2c^2d^2)a + (b^3c^3 + 6a^2b^2c^2d^2 - 2b^2c^2d + 6b^2c^2d^2 - 4(b^2c^2d - 2b^2c^2d^2)a)x) * e^{(-bx)/(d^4x^2e^a + 2c^2d^3xe^a + c^2d^2e^a)}, x$$

**Fricas [A]** time = 1.52102, size = 479, normalized size = 1.73

$$(b^4c^4 - 4ab^3c^3d + 6a^2b^2c^2d^2 - 4a^3bcd^3 + a^4d^4) \operatorname{Ei}\left(-\frac{bdx+bc}{d}\right) e^{\left(\frac{bc-ad}{d}\right)} - (b^3d^4x^3 - b^3c^3d + (4a+1)b^2c^2d^2 - 2(3a^2 + 2a +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(-b\*x-a)\*(b\*x+a)^4/(d\*x+c),x, algorithm="fricas")

[Out] ((b^4\*c^4 - 4\*a\*b^3\*c^3\*d + 6\*a^2\*b^2\*c^2\*d^2 - 4\*a^3\*b\*c\*d^3 + a^4\*d^4)\*Ei(-(b\*d\*x + b\*c)/d)\*e^((b\*c - a\*d)/d) - (b^3\*d^4\*x^3 - b^3\*c^3\*d + (4\*a + 1)\*b^2\*c^2\*d^2 - 2\*(3\*a^2 + 2\*a + 1)\*b\*c\*d^3 + 2\*(2\*a^3 + 3\*a^2 + 4\*a + 3)\*d^4 - (b^3\*c\*d^3 - (4\*a + 3)\*b^2\*d^4)\*x^2 + (b^3\*c^2\*d^2 - 2\*(2\*a + 1)\*b^2\*c\*d^3 + 2\*(3\*a^2 + 4\*a + 3)\*b\*d^4)\*x)\*e^(-b\*x - a))/d^5

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\left(\int \frac{a^4}{ce^{bx} + dxe^{bx}} dx + \int \frac{b^4x^4}{ce^{bx} + dxe^{bx}} dx + \int \frac{4ab^3x^3}{ce^{bx} + dxe^{bx}} dx + \int \frac{6a^2b^2x^2}{ce^{bx} + dxe^{bx}} dx + \int \frac{4a^3bx}{ce^{bx} + dxe^{bx}} dx\right) e^{-a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(-b\*x-a)\*(b\*x+a)\*\*4/(d\*x+c),x)

[Out] (Integral(a\*\*4/(c\*exp(b\*x) + d\*x\*exp(b\*x)), x) + Integral(b\*\*4\*x\*\*4/(c\*exp(b\*x) + d\*x\*exp(b\*x)), x) + Integral(4\*a\*b\*\*3\*x\*\*3/(c\*exp(b\*x) + d\*x\*exp(b\*x)), x) + Integral(6\*a\*\*2\*b\*\*2\*x\*\*2/(c\*exp(b\*x) + d\*x\*exp(b\*x)), x) + Integral(4\*a\*\*3\*b\*x/(c\*exp(b\*x) + d\*x\*exp(b\*x)), x))\*exp(-a)

**Giac [A]** time = 1.18869, size = 240, normalized size = 0.87

$$b^4c^4 \operatorname{Ei}\left(-\frac{bdx+bc}{d}\right) e^{\left(-a+\frac{bc}{d}\right)} - 4ab^3c^3d \operatorname{Ei}\left(-\frac{bdx+bc}{d}\right) e^{\left(-a+\frac{bc}{d}\right)} + 6a^2b^2c^2d^2 \operatorname{Ei}\left(-\frac{bdx+bc}{d}\right) e^{\left(-a+\frac{bc}{d}\right)} - 4a^3bcd^3 \operatorname{Ei}\left(-\frac{bdx+bc}{d}\right) e^{\left(-a+\frac{bc}{d}\right)}$$

$d^5$



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(-b*x-a)*(b*x+a)^4/(d*x+c),x, algorithm="giac")
```

```
[Out] (b^4*c^4*Ei(-(b*d*x + b*c)/d)*e^(-a + b*c/d) - 4*a*b^3*c^3*d*Ei(-(b*d*x + b*c)/d)*e^(-a + b*c/d) + 6*a^2*b^2*c^2*d^2*Ei(-(b*d*x + b*c)/d)*e^(-a + b*c/d) - 4*a^3*b*c*d^3*Ei(-(b*d*x + b*c)/d)*e^(-a + b*c/d) + a^4*d^4*Ei(-(b*d*x + b*c)/d)*e^(-a + b*c/d))/d^5
```

$$3.79 \quad \int \frac{e^{-a-bx}(a+bx)^4}{(c+dx)^2} dx$$

**Optimal.** Leaf size=258

$$\frac{4b^2e^{-a-bx}(c+dx)(bc-ad)}{d^4} - \frac{b^3e^{-a-bx}(c+dx)^2}{d^4} - \frac{2b^2e^{-a-bx}(c+dx)}{d^3} - \frac{be^{\frac{bc}{d}-a}(bc-ad)^4\text{Ei}\left(-\frac{b(c+dx)}{d}\right)}{d^6} - \frac{4be^{\frac{bc}{d}-a}(bc-ad)^3E}{d^5}$$

[Out]  $(-2*b*E^{-a-b*x})/d^2 + (4*b*(b*c-a*d)*E^{-a-b*x})/d^3 - (6*b*(b*c-a*d)^2*E^{-a-b*x})/d^4 - ((b*c-a*d)^4*E^{-a-b*x})/(d^5*(c+d*x)) - (2*b^2*E^{-a-b*x}*(c+d*x))/d^3 + (4*b^2*(b*c-a*d)*E^{-a-b*x}*(c+d*x))/d^4 - (b^3*E^{-a-b*x}*(c+d*x)^2)/d^4 - (4*b*(b*c-a*d)^3*E^{-a+(b*c)/d}*\text{ExpIntegralEi}[-((b*(c+d*x))/d)])/d^5 - (b*(b*c-a*d)^4*E^{-a+(b*c)/d}*\text{ExpIntegralEi}[-((b*(c+d*x))/d)])/d^6$

**Rubi [A]** time = 0.378844, antiderivative size = 258, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 5, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$ , Rules used = {2199, 2194, 2177, 2178, 2176}

$$\frac{4b^2e^{-a-bx}(c+dx)(bc-ad)}{d^4} - \frac{b^3e^{-a-bx}(c+dx)^2}{d^4} - \frac{2b^2e^{-a-bx}(c+dx)}{d^3} - \frac{be^{\frac{bc}{d}-a}(bc-ad)^4\text{Ei}\left(-\frac{b(c+dx)}{d}\right)}{d^6} - \frac{4be^{\frac{bc}{d}-a}(bc-ad)^3E}{d^5}$$

Antiderivative was successfully verified.

[In] Int[(E^{-a-b\*x}\*(a+b\*x)^4)/(c+d\*x)^2,x]

[Out]  $(-2*b*E^{-a-b*x})/d^2 + (4*b*(b*c-a*d)*E^{-a-b*x})/d^3 - (6*b*(b*c-a*d)^2*E^{-a-b*x})/d^4 - ((b*c-a*d)^4*E^{-a-b*x})/(d^5*(c+d*x)) - (2*b^2*E^{-a-b*x}*(c+d*x))/d^3 + (4*b^2*(b*c-a*d)*E^{-a-b*x}*(c+d*x))/d^4 - (b^3*E^{-a-b*x}*(c+d*x)^2)/d^4 - (4*b*(b*c-a*d)^3*E^{-a+(b*c)/d}*\text{ExpIntegralEi}[-((b*(c+d*x))/d)])/d^5 - (b*(b*c-a*d)^4*E^{-a+(b*c)/d}*\text{ExpIntegralEi}[-((b*(c+d*x))/d)])/d^6$

### Rule 2199

Int[(F\_)^((c\_.)\*(v\_))\*(u\_)^(m\_.)\*(w\_), x\_Symbol] := Int[ExpandIntegrand[F^(c\*ExpandToSum[v, x]), w\*NormalizePowerOfLinear[u, x]^m, x], x] /; FreeQ[{F, c}, x] && PolynomialQ[w, x] && LinearQ[v, x] && PowerOfLinearQ[u, x] && IntegerQ[m] && !\$UseGamma === True

Rule 2194

Int[((F\_)^((c\_)\*(a\_) + (b\_)\*(x\_)))^(n\_), x\_Symbol] := Simp[(F^(c\*(a + b\*x)))^n/(b\*c\*n\*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]

Rule 2177

Int[((b\_)\*(F\_)^((g\_)\*((e\_) + (f\_)\*(x\_))))^(n\_)\*((c\_) + (d\_)\*(x\_))^(m\_), x\_Symbol] := Simp[((c + d\*x)^(m + 1)\*(b\*F^(g\*(e + f\*x)))^n)/(d\*(m + 1)), x] - Dist[(f\*g\*n\*Log[F])/(d\*(m + 1)), Int[(c + d\*x)^(m + 1)\*(b\*F^(g\*(e + f\*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && LtQ[m, -1] && IntegerQ[2\*m] && !\$UseGamma == True

Rule 2178

Int[(F\_)^((g\_)\*((e\_) + (f\_)\*(x\_)))/((c\_) + (d\_)\*(x\_)), x\_Symbol] := Simp[(F^(g\*(e - (c\*f)/d))\*ExpIntegralEi[(f\*g\*(c + d\*x)\*Log[F])/d])/d, x] /; FreeQ[{F, c, d, e, f, g}, x] && !\$UseGamma == True

Rule 2176

Int[((b\_)\*(F\_)^((g\_)\*((e\_) + (f\_)\*(x\_))))^(n\_)\*((c\_) + (d\_)\*(x\_))^(m\_), x\_Symbol] := Simp[((c + d\*x)^m\*(b\*F^(g\*(e + f\*x)))^n/(f\*g\*n\*Log[F]), x] - Dist[(d\*m)/(f\*g\*n\*Log[F]), Int[(c + d\*x)^(m - 1)\*(b\*F^(g\*(e + f\*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2\*m] && !\$UseGamma == True

Rubi steps

$$\begin{aligned}
 \int \frac{e^{-a-bx}(a+bx)^4}{(c+dx)^2} dx &= \int \left( \frac{6b^2(bc-ad)^2 e^{-a-bx}}{d^4} + \frac{(-bc+ad)^4 e^{-a-bx}}{d^4(c+dx)^2} - \frac{4b(bc-ad)^3 e^{-a-bx}}{d^4(c+dx)} - \frac{4b^3(bc-ad)e^{-a-bx}(c+dx)}{d^4} \right) dx \\
 &= \frac{b^4 \int e^{-a-bx}(c+dx)^2 dx}{d^4} - \frac{(4b^3(bc-ad)) \int e^{-a-bx}(c+dx) dx}{d^4} + \frac{(6b^2(bc-ad)^2) \int e^{-a-bx} dx}{d^4} - \frac{4b^3 \int e^{-a-bx}(c+dx) dx}{d^4} \\
 &= -\frac{6b(bc-ad)^2 e^{-a-bx}}{d^4} - \frac{(bc-ad)^4 e^{-a-bx}}{d^5(c+dx)} + \frac{4b^2(bc-ad)e^{-a-bx}(c+dx)}{d^4} - \frac{b^3 e^{-a-bx}(c+dx)^2}{d^4} - \frac{4b^3 \int e^{-a-bx}(c+dx) dx}{d^4} \\
 &= \frac{4b(bc-ad)e^{-a-bx}}{d^3} - \frac{6b(bc-ad)^2 e^{-a-bx}}{d^4} - \frac{(bc-ad)^4 e^{-a-bx}}{d^5(c+dx)} - \frac{2b^2 e^{-a-bx}(c+dx)}{d^3} + \frac{4b^2(bc-ad)e^{-a-bx}(c+dx)}{d^4} \\
 &= -\frac{2be^{-a-bx}}{d^2} + \frac{4b(bc-ad)e^{-a-bx}}{d^3} - \frac{6b(bc-ad)^2 e^{-a-bx}}{d^4} - \frac{(bc-ad)^4 e^{-a-bx}}{d^5(c+dx)} - \frac{2b^2 e^{-a-bx}(c+dx)}{d^3} + \frac{4b^2(bc-ad)e^{-a-bx}(c+dx)}{d^4}
 \end{aligned}$$

**Mathematica [A]** time = 0.437347, size = 163, normalized size = 0.63

$$e^{-a} \left( -\frac{de^{-bx}(bd(c+dx)(2(3a^2+2a+1)d^2-2(4a+1)bcd+3b^2c^2)-2b^2d^2x(c+dx)(bc-(2a+1)d)+(bc-ad)^4+b^3d^3x^2(c+dx))}{c+dx} - be^{\frac{bc}{d}}(bc-(a-4)d)(bc-ad)^3 \text{Ei} \right) / d^6$$

Antiderivative was successfully verified.

[In] Integrate[(E^(-a - b\*x)\*(a + b\*x)^4)/(c + d\*x)^2,x]

[Out] 
$$\frac{-((d*((b*c - a*d)^4 + b*d*(3*b^2*c^2 - 2*(1 + 4*a)*b*c*d + 2*(1 + 2*a + 3*a^2)*d^2)*(c + d*x) - 2*b^2*d^2*(b*c - (1 + 2*a)*d)*x*(c + d*x) + b^3*d^3*x^2*(c + d*x)))/(E^(b*x)*(c + d*x)) - b*(b*c - (-4 + a)*d)*(b*c - a*d)^3*E^((b*c)/d)*ExpIntegralEi[-((b*(c + d*x))/d)]}{(d^6*E^a)}$$

**Maple [A]** time = 0.015, size = 406, normalized size = 1.6

$$\frac{1}{b} \left( \frac{b^2((-bx-a)^2 e^{-bx-a} - 2(-bx-a) e^{-bx-a} + 2 e^{-bx-a})}{d^2} - 2 \frac{ab^2((-bx-a) e^{-bx-a} - e^{-bx-a})}{d^2} + 2 \frac{b^3 c((-bx-a) e^{-bx-a} - e^{-bx-a})}{d^3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(-b\*x-a)\*(b\*x+a)^4/(d\*x+c)^2,x)

[Out] 
$$\begin{aligned} & -1/b*(b^2/d^2*((-b*x-a)^2*exp(-b*x-a)-2*(-b*x-a)*exp(-b*x-a)+2*exp(-b*x-a)) \\ & -2*b^2/d^2*a*((-b*x-a)*exp(-b*x-a)-exp(-b*x-a))+2*b^3/d^3*c*((-b*x-a)*exp(-b*x-a)-exp(-b*x-a))+3*b^2/d^2*a^2*exp(-b*x-a)-6*b^3/d^3*a*c*exp(-b*x-a)+3*b^4/d^4*c^2*exp(-b*x-a)+4/d^5*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)* \\ & b^2*exp(-(a*d-b*c)/d)*Ei(1,b*x+a-(a*d-b*c)/d)+(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)*b^2/d^6*(-exp(-b*x-a)/(-b*x-a+(a*d-b*c)/d)-exp(-(a*d-b*c)/d)*Ei(1,b*x+a-(a*d-b*c)/d)) \end{aligned}$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{a^4 e^{\left(-a + \frac{bc}{d}\right)} E_2\left(\frac{(dx+c)b}{d}\right)}{(dx+c)d} - \frac{(b^3 d^2 x^4 + 2(2 a b^2 d^2 + b^2 d^2) x^3 + 2(3 a^2 b d^2 + b^2 c d + 2 a b d^2 + b d^2) x^2 + 2(2 a^3 d^2 - b^2 c^2 + 4 a b c d - a^2 c^2) x + a^4 d^2 - b^3 c^2)}{d^4 x^2 e^a + 2 c d^3 x e^a + c^2 d^2 e^a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(-b\*x-a)\*(b\*x+a)^4/(d\*x+c)^2,x, algorithm="maxima")

[Out]  $-a^4 e^{(-a + b*c/d)} \exp\_integral\_e(2, (d*x + c)*b/d)/((d*x + c)*d) - (b^3*d^2*x^4 + 2*(2*a*b^2*d^2 + b^2*d^2)*x^3 + 2*(3*a^2*b*d^2 + b^2*c*d + 2*a*b*d^2 + b*d^2)*x^2 + 2*(2*a^3*d^2 - b^2*c^2 + 4*a*b*c*d + 2*b*c*d)*x)*e^{(-b*x)}/(d^4*x^2*e^a + 2*c*d^3*x*e^a + c^2*d^2*e^a) - \int (-2*(2*a^3*c*d^2 - b^2*c^3 + 4*a*b*c^2*d + 2*b*c^2*d + (b^3*c^3 - 4*a*b^2*c^2*d + 6*a^2*b*c*d^2 - 2*a^3*d^3 + b^2*c^2*d)*x)*e^{(-b*x)}/(d^5*x^3*e^a + 3*c*d^4*x^2*e^a + 3*c^2*d^3*x*e^a + c^3*d^2*e^a), x)$

**Fricas [A]** time = 1.53417, size = 729, normalized size = 2.83

$$(b^5c^5 - 4(a-1)b^4c^4d + 6(a^2 - 2a)b^3c^3d^2 - 4(a^3 - 3a^2)b^2c^2d^3 + (a^4 - 4a^3)bcd^4 + (b^5c^4d - 4(a-1)b^4c^3d^2 + 6(a^2 -$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(-b\*x-a)\*(b\*x+a)^4/(d\*x+c)^2,x, algorithm="fricas")

[Out]  $-((b^5*c^5 - 4*(a - 1)*b^4*c^4*d + 6*(a^2 - 2*a)*b^3*c^3*d^2 - 4*(a^3 - 3*a^2)*b^2*c^2*d^3 + (a^4 - 4*a^3)*b*c*d^4 + (b^5*c^4*d - 4*(a - 1)*b^4*c^3*d^2 + 6*(a^2 - 2*a)*b^3*c^2*d^3 - 4*(a^3 - 3*a^2)*b^2*c*d^4 + (a^4 - 4*a^3)*b*d^5)*x)*Ei(-(b*d*x + b*c)/d)*e^{((b*c - a*d)/d)} + (b^3*d^5*x^3 + b^4*c^4*d - (4*a - 3)*b^3*c^3*d^2 + a^4*d^5 + 2*(3*a^2 - 4*a - 1)*b^2*c^2*d^3 - 2*(2*a^3 - 3*a^2 - 2*a - 1)*b*c*d^4 - (b^3*c*d^4 - 2*(2*a + 1)*b^2*d^5)*x^2 + (b^3*c^2*d^3 - 4*a*b^2*c*d^4 + 2*(3*a^2 + 2*a + 1)*b*d^5)*x)*e^{(-b*x - a)}/(d^7*x + c*d^6)$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(-b\*x-a)\*(b\*x+a)\*\*4/(d\*x+c)\*\*2,x)

[Out] Timed out

---

**Giac [B]** time = 1.74218, size = 1037, normalized size = 4.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(-b\*x-a)\*(b\*x+a)^4/(d\*x+c)^2,x, algorithm="giac")

[Out] 
$$\begin{aligned} & -(b^5c^4d^2x^2\text{Ei}(-(bdx + bc)/d)e^{(-a + bc/d)} - 4ab^4c^3d^2x\text{Ei}(-(bdx + bc)/d)e^{(-a + bc/d)} + 6a^2b^3c^2d^3x\text{Ei}(-(bdx + bc)/d)e^{(-a + bc/d)} - 4a^3b^2c^2d^4x\text{Ei}(-(bdx + bc)/d)e^{(-a + bc/d)} + a^4b^2c^3d^5x\text{Ei}(-(bdx + bc)/d)e^{(-a + bc/d)} + b^5c^5\text{Ei}(-(bdx + bc)/d)e^{(-a + bc/d)} - 4ab^4c^4d\text{Ei}(-(bdx + bc)/d)e^{(-a + bc/d)} + 6a^2b^3c^3d^2\text{Ei}(-(bdx + bc)/d)e^{(-a + bc/d)} - 4a^3b^2c^2d^3\text{Ei}(-(bdx + bc)/d)e^{(-a + bc/d)} + a^4b^2c^4d\text{Ei}(-(bdx + bc)/d)e^{(-a + bc/d)} + 4b^4c^3d^2x\text{Ei}(-(bdx + bc)/d)e^{(-a + bc/d)} - 12ab^3c^2d^3x\text{Ei}(-(bdx + bc)/d)e^{(-a + bc/d)} + 12a^2b^2c^2d^4x\text{Ei}(-(bdx + bc)/d)e^{(-a + bc/d)} - 4a^3b^2d^5x\text{Ei}(-(bdx + bc)/d)e^{(-a + bc/d)} + 4b^4c^4d\text{Ei}(-(bdx + bc)/d)e^{(-a + bc/d)} - 12ab^3c^3d^2\text{Ei}(-(bdx + bc)/d)e^{(-a + bc/d)} + 12a^2b^2c^2d^3\text{Ei}(-(bdx + bc)/d)e^{(-a + bc/d)} - 4a^3b^2c^2d^4\text{Ei}(-(bdx + bc)/d)e^{(-a + bc/d)} + b^4c^4d\text{Ei}(-bx - a) - 4ab^3c^3d^2e^{(-bx - a)} + 6a^2b^2c^2d^3e^{(-bx - a)} - 4a^3b^2c^2d^4e^{(-bx - a)} + a^4d^5e^{(-bx - a)})/(d^7x + cd^6) \end{aligned}$$

$$3.80 \quad \int \frac{e^{-a-bx}(a+bx)^4}{(c+dx)^3} dx$$

**Optimal.** Leaf size=294

$$\frac{b^2 e^{\frac{bc}{d}-a} (bc-ad)^4 \operatorname{Ei}\left(-\frac{b(c+dx)}{d}\right)}{2d^7} + \frac{4b^2 e^{\frac{bc}{d}-a} (bc-ad)^3 \operatorname{Ei}\left(-\frac{b(c+dx)}{d}\right)}{d^6} + \frac{6b^2 e^{\frac{bc}{d}-a} (bc-ad)^2 \operatorname{Ei}\left(-\frac{b(c+dx)}{d}\right)}{d^5} + \frac{b^2 e^{-a-bx} (3bc-4ad)}{d^4}$$

[Out]  $-\left(\frac{b^2 E^{-a-bx}}{d^3}\right) + \frac{b^2(3bc-4ad)E^{-a-bx}}{d^4} - \frac{b^3 E^{-a-bx} x}{d^3} - \frac{(b^3 c - a^2 d) E^{-a-bx}}{(2d^5(c+dx)^2)} + \frac{4b^2(b^3 c - a^2 d) E^{-a-bx}}{(d^5(c+dx))} + \frac{b^2(b^3 c - a^2 d) E^{-a-bx}}{(2d^6(c+dx))} + \frac{6b^2(b^3 c - a^2 d)^2 E^{-a-bx}}{d^5} \operatorname{ExpIntegralEi}\left[-\frac{b(c+dx)}{d}\right] + \frac{4b^2(b^3 c - a^2 d)^3 E^{-a-bx}}{d^6} \operatorname{ExpIntegralEi}\left[-\frac{b(c+dx)}{d}\right] + \frac{b^2(b^3 c - a^2 d)^4 E^{-a-bx}}{(2d^7)} \operatorname{ExpIntegralEi}\left[-\frac{b(c+dx)}{d}\right]$

**Rubi [A]** time = 0.407777, antiderivative size = 294, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 5, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$ , Rules used = {2199, 2194, 2176, 2177, 2178}

$$\frac{b^2 e^{\frac{bc}{d}-a} (bc-ad)^4 \operatorname{Ei}\left(-\frac{b(c+dx)}{d}\right)}{2d^7} + \frac{4b^2 e^{\frac{bc}{d}-a} (bc-ad)^3 \operatorname{Ei}\left(-\frac{b(c+dx)}{d}\right)}{d^6} + \frac{6b^2 e^{\frac{bc}{d}-a} (bc-ad)^2 \operatorname{Ei}\left(-\frac{b(c+dx)}{d}\right)}{d^5} + \frac{b^2 e^{-a-bx} (3bc-4ad)}{d^4}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}\left[\frac{E^{-a-bx}(a+bx)^4}{(c+dx)^3}, x\right]$

[Out]  $-\left(\frac{b^2 E^{-a-bx}}{d^3}\right) + \frac{b^2(3bc-4ad)E^{-a-bx}}{d^4} - \frac{b^3 E^{-a-bx} x}{d^3} - \frac{(b^3 c - a^2 d) E^{-a-bx}}{(2d^5(c+dx)^2)} + \frac{4b^2(b^3 c - a^2 d) E^{-a-bx}}{(d^5(c+dx))} + \frac{b^2(b^3 c - a^2 d) E^{-a-bx}}{(2d^6(c+dx))} + \frac{6b^2(b^3 c - a^2 d)^2 E^{-a-bx}}{d^5} \operatorname{ExpIntegralEi}\left[-\frac{b(c+dx)}{d}\right] + \frac{4b^2(b^3 c - a^2 d)^3 E^{-a-bx}}{d^6} \operatorname{ExpIntegralEi}\left[-\frac{b(c+dx)}{d}\right] + \frac{b^2(b^3 c - a^2 d)^4 E^{-a-bx}}{(2d^7)} \operatorname{ExpIntegralEi}\left[-\frac{b(c+dx)}{d}\right]$

### Rule 2199

$\operatorname{Int}\left[(F_{-})^{\left((c_{-}) \cdot (v_{-})\right)} \cdot (u_{-})^{\left(m_{-}\right)} \cdot (w_{-}), x_{\text{Symbol}}\right] \rightarrow \operatorname{Int}\left[\operatorname{ExpandIntegrand}\left[F^{\left(c \cdot \operatorname{ExpandToSum}[v, x]\right)} \cdot w \cdot \operatorname{NormalizePowerOfLinear}[u, x]^m, x\right] /; \operatorname{FreeQ}\left[\{F, c\}, x\right] \&\& \operatorname{PolynomialQ}[w, x] \&\& \operatorname{LinearQ}[v, x] \&\& \operatorname{PowerOfLinearQ}[u, x] \&\& \operatorname{In}\right]$

tegerQ[m] && !\$UseGamma === True

### Rule 2194

Int[((F\_)^((c\_)\*(a\_) + (b\_)\*(x\_)))^(n\_), x\_Symbol] := Simp[(F^(c\*(a + b\*x)))^n/(b\*c\*n\*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]

### Rule 2176

Int[((b\_)\*(F\_)^((g\_)\*(e\_) + (f\_)\*(x\_)))^(n\_)\*((c\_) + (d\_)\*(x\_))^(m\_), x\_Symbol] := Simp[((c + d\*x)^m\*(b\*F^(g\*(e + f\*x)))^n)/(f\*g\*n\*Log[F]), x] - Dist[(d\*m)/(f\*g\*n\*Log[F]), Int[(c + d\*x)^(m - 1)\*(b\*F^(g\*(e + f\*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2\*m] && !\$UseGamma === True

### Rule 2177

Int[((b\_)\*(F\_)^((g\_)\*(e\_) + (f\_)\*(x\_)))^(n\_)\*((c\_) + (d\_)\*(x\_))^(m\_), x\_Symbol] := Simp[((c + d\*x)^(m + 1)\*(b\*F^(g\*(e + f\*x)))^n)/(d\*(m + 1)), x] - Dist[(f\*g\*n\*Log[F])/(d\*(m + 1)), Int[(c + d\*x)^(m + 1)\*(b\*F^(g\*(e + f\*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && LtQ[m, -1] && IntegerQ[2\*m] && !\$UseGamma === True

### Rule 2178

Int[(F\_)^((g\_)\*(e\_) + (f\_)\*(x\_))/((c\_) + (d\_)\*(x\_)), x\_Symbol] := Simp[(F^(g\*(e - (c\*f)/d))\*ExpIntegralEi[(f\*g\*(c + d\*x)\*Log[F])/d])/d, x] /; FreeQ[{F, c, d, e, f, g}, x] && !\$UseGamma === True

### Rubi steps



$$\begin{aligned}
\int \frac{e^{-a-bx}(a+bx)^4}{(c+dx)^3} dx &= \int \left( -\frac{b^3(3bc-4ad)e^{-a-bx}}{d^4} + \frac{b^4e^{-a-bx}x}{d^3} + \frac{(-bc+ad)^4e^{-a-bx}}{d^4(c+dx)^3} - \frac{4b(bc-ad)^3e^{-a-bx}}{d^4(c+dx)^2} + \frac{6b^2(bc-ad)^2e^{-a-bx}}{d^4(c+dx)} \right) dx \\
&= \frac{b^4 \int e^{-a-bx}x dx}{d^3} - \frac{(b^3(3bc-4ad)) \int e^{-a-bx} dx}{d^4} + \frac{(6b^2(bc-ad)^2) \int \frac{e^{-a-bx}}{c+dx} dx}{d^4} - \frac{(4b(bc-ad)^3) \int \frac{e^{-a-bx}}{c+dx} dx}{d^4} + \frac{6b^2(bc-ad)^2 \int \frac{e^{-a-bx}}{c+dx} dx}{d^4} \\
&= \frac{b^2(3bc-4ad)e^{-a-bx}}{d^4} - \frac{b^3e^{-a-bx}x}{d^3} - \frac{(bc-ad)^4e^{-a-bx}}{2d^5(c+dx)^2} + \frac{4b(bc-ad)^3e^{-a-bx}}{d^5(c+dx)} + \frac{6b^2(bc-ad)^2e^{-a-bx}}{d^5} \\
&= -\frac{b^2e^{-a-bx}}{d^3} + \frac{b^2(3bc-4ad)e^{-a-bx}}{d^4} - \frac{b^3e^{-a-bx}x}{d^3} - \frac{(bc-ad)^4e^{-a-bx}}{2d^5(c+dx)^2} + \frac{4b(bc-ad)^3e^{-a-bx}}{d^5(c+dx)} + \frac{b(bc-ad)^2e^{-a-bx}}{2d^6} \\
&= -\frac{b^2e^{-a-bx}}{d^3} + \frac{b^2(3bc-4ad)e^{-a-bx}}{d^4} - \frac{b^3e^{-a-bx}x}{d^3} - \frac{(bc-ad)^4e^{-a-bx}}{2d^5(c+dx)^2} + \frac{4b(bc-ad)^3e^{-a-bx}}{d^5(c+dx)} + \frac{b(bc-ad)^2e^{-a-bx}}{2d^6}
\end{aligned}$$

**Mathematica [A]** time = 0.652042, size = 267, normalized size = 0.91

$$e^{-a} \left( b^2 e^{\frac{bc}{d}} \left( (a^2 - 8a + 12) d^2 - 2(a-4)bcd + b^2c^2 \right) (bc-ad)^2 \text{Ei} \left( -\frac{b(c+dx)}{d} \right) + \frac{de^{-bx}(2b^3d^2((3a^2-12a+5)c^2dx+(3a^2-10a+3)c^3+cd^2x^2-d^3x^3))}{2d^7} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(E^(-a - b\*x)\*(a + b\*x)^4)/(c + d\*x)^3,x]

[Out] ((d\*(-(a^4\*d^5) + b^5\*c^4\*(c + d\*x) + a^3\*b\*d^4\*(-4 + a)\*c + (-8 + a)\*d\*x) + b^4\*c^3\*d\*((7 - 4\*a)\*c - 4\*(-2 + a)\*d\*x) - 2\*b^2\*d^3\*((1 + 4\*a - 9\*a^2 + 2\*a^3)\*c^2 + 2\*(1 + 4\*a - 6\*a^2 + a^3)\*c\*d\*x + (1 + 4\*a)\*d^2\*x^2) + 2\*b^3\*d^2\*((3 - 10\*a + 3\*a^2)\*c^3 + (5 - 12\*a + 3\*a^2)\*c^2\*d\*x + c\*d^2\*x^2 - d^3\*x^3))/(E^(b\*x)\*(c + d\*x)^2) + b^2\*(b\*c - a\*d)^2\*(b^2\*c^2 - 2\*(-4 + a)\*b\*c\*d + (12 - 8\*a + a^2)\*d^2)\*E^((b\*c)/d)\*ExpIntegralEi[-((b\*(c + d\*x))/d)]/(2\*d^7\*E^a)

**Maple [A]** time = 0.015, size = 418, normalized size = 1.4

$$-\frac{1}{b} \left( -\frac{b^3((-bx-a)e^{-bx-a} - e^{-bx-a})}{d^3} + 3\frac{ab^3e^{-bx-a}}{d^3} - 3\frac{b^4ce^{-bx-a}}{d^4} - \frac{(a^4d^4 - 4a^3bcd^3 + 6a^2b^2c^2d^2 - 4ab^3c^3d + b^4c^4)b^3}{d^7} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(exp(-b*x-a)*(b*x+a)^4/(d*x+c)^3,x)`

[Out] 
$$-1/b*(-b^3/d^3*((-b*x-a)*\exp(-b*x-a)-\exp(-b*x-a))+3*b^3/d^3*a*\exp(-b*x-a)-3*b^4/d^4*c*\exp(-b*x-a)-(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)*b^3/d^7*(-1/2*\exp(-b*x-a)/(-b*x-a+(a*d-b*c)/d)^2-1/2*\exp(-b*x-a)/(-b*x-a+(a*d-b*c)/d)-1/2*\exp(-a*d-b*c)/d)*\text{Ei}(1,b*x+a-(a*d-b*c)/d))+6/d^5*(a^2*d^2-2*a*b*c*d+b^2*c^2)*b^3*\exp(-a*d-b*c)/d)*\text{Ei}(1,b*x+a-(a*d-b*c)/d)+4/d^6*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)*b^3*(-\exp(-b*x-a)/(-b*x-a+(a*d-b*c)/d)-\exp(-a*d-b*c)/d)*\text{Ei}(1,b*x+a-(a*d-b*c)/d))$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{a^4 e^{\left(-a + \frac{bc}{d}\right)} E_3\left(\frac{(dx+c)b}{d}\right)}{(dx+c)^2 d} - \frac{(b^3 d^2 x^4 + (4 ab^2 d^2 + b^2 d^2) x^3 + 3(2 a^2 b d^2 + b^2 c d) x^2 + (4 a^3 d^2 - 3 b^2 c^2 + 12 abcd - 6 a^2 d^2) x) e^{bc/d}}{d^5 x^3 e^a + 3 cd^4 x^2 e^a + 3 c^2 d^3 x e^a + c^3 d^2 e^a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(-b*x-a)*(b*x+a)^4/(d*x+c)^3,x, algorithm="maxima")`

[Out] 
$$-a^4 * e^{(-a + b*c/d)} * \exp\_integral\_e(3, (d*x + c)*b/d)/((d*x + c)^2*d) - (b^3*d^2*x^4 + (4*a*b^2*d^2 + b^2*d^2)*x^3 + 3*(2*a^2*b*d^2 + b^2*c*d)*x^2 + (4*a^3*d^2 - 3*b^2*c^2 + 12*a*b*c*d - 6*a^2*d^2)*x) * e^{-b*x}/(d^5*x^3*e^a + 3*c*d^4*x^2*e^a + 3*c^2*d^3*x*e^a + c^3*d^2*e^a) - \text{integrate}(- (4*a^3*c*d^2 - 3*b^2*c^3 + 12*a*b*c^2*d - 6*a^2*c*d^2 + (3*b^3*c^3 - 8*a^3*d^3 + 12*b^2*c^2*d + 6*(3*b*c*d^2 + 2*d^3))*a^2 - 12*(b^2*c^2*d + 2*b*c*d^2)*a) * x) * e^{-b*x}/(d^6*x^4*e^a + 4*c*d^5*x^3*e^a + 6*c^2*d^4*x^2*e^a + 4*c^3*d^3*x*e^a + c^4*d^2*e^a), x)$$

**Fricas [A]** time = 1.56864, size = 1158, normalized size = 3.94

$$(b^6 c^6 - 4(a-2)b^5 c^5 d + 6(a^2 - 4a + 2)b^4 c^4 d^2 - 4(a^3 - 6a^2 + 6a)b^3 c^3 d^3 + (a^4 - 8a^3 + 12a^2)b^2 c^2 d^4 + (b^6 c^4 d^2 - 4(a-2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(-b*x-a)*(b*x+a)^4/(d*x+c)^3,x, algorithm="fricas")`

```
[Out] 1/2*((b^6*c^6 - 4*(a - 2)*b^5*c^5*d + 6*(a^2 - 4*a + 2)*b^4*c^4*d^2 - 4*(a^3 - 6*a^2 + 6*a)*b^3*c^3*d^3 + (a^4 - 8*a^3 + 12*a^2)*b^2*c^2*d^4 + (b^6*c^4*d^2 - 4*(a - 2)*b^5*c^3*d^3 + 6*(a^2 - 4*a + 2)*b^4*c^2*d^4 - 4*(a^3 - 6*a^2 + 6*a)*b^3*c*d^5 + (a^4 - 8*a^3 + 12*a^2)*b^2*d^6)*x^2 + 2*(b^6*c^5*d - 4*(a - 2)*b^5*c^4*d^2 + 6*(a^2 - 4*a + 2)*b^4*c^3*d^3 - 4*(a^3 - 6*a^2 + 6*a)*b^3*c^2*d^4 + (a^4 - 8*a^3 + 12*a^2)*b^2*c*d^5)*x)*Ei(-(b*d*x + b*c)/d) * e^((b*c - a*d)/d) - (2*b^3*d^6*x^3 - b^5*c^5*d + (4*a - 7)*b^4*c^4*d^2 - 2*(3*a^2 - 10*a + 3)*b^3*c^3*d^3 + a^4*d^6 + 2*(2*a^3 - 9*a^2 + 4*a + 1)*b^2*c^2*d^4 - (a^4 - 4*a^3)*b*c*d^5 - 2*(b^3*c*d^5 - (4*a + 1)*b^2*d^6)*x^2 - (b^5*c^4*d^2 - 4*(a - 2)*b^4*c^3*d^3 + 2*(3*a^2 - 12*a + 5)*b^3*c^2*d^4 - 4*(a^3 - 6*a^2 + 4*a + 1)*b^2*c*d^5 + (a^4 - 8*a^3)*b*d^6)*x)*e^(-b*x - a))/(d^9*x^2 + 2*c*d^8*x + c^2*d^7)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(-b*x-a)*(b*x+a)**4/(d*x+c)**3,x)
```

```
[Out] Timed out
```

**Giac [B]** time = 1.312, size = 2419, normalized size = 8.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(-b*x-a)*(b*x+a)^4/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/2*(b^6*c^4*d^2*x^2*Ei(-(b*d*x + b*c)/d)*e^(-a + b*c/d) - 4*a*b^5*c^3*d^3*x^2*Ei(-(b*d*x + b*c)/d)*e^(-a + b*c/d) + 6*a^2*b^4*c^2*d^4*x^2*Ei(-(b*d*x + b*c)/d)*e^(-a + b*c/d) - 4*a^3*b^3*c*d^5*x^2*Ei(-(b*d*x + b*c)/d)*e^(-a + b*c/d) + a^4*b^2*d^6*x^2*Ei(-(b*d*x + b*c)/d)*e^(-a + b*c/d) + 2*b^6*c^5*d*x*Ei(-(b*d*x + b*c)/d)*e^(-a + b*c/d) - 8*a*b^5*c^4*d^2*x*Ei(-(b*d*x + b*c)/d)*e^(-a + b*c/d) + 12*a^2*b^4*c^3*d^3*x*Ei(-(b*d*x + b*c)/d)*e^(-a + b*c/d) - 8*a^3*b^3*c^2*d^4*x*Ei(-(b*d*x + b*c)/d)*e^(-a + b*c/d) + 2*a^4*b^2*c*d^5*x*Ei(-(b*d*x + b*c)/d)*e^(-a + b*c/d) + 8*b^5*c^3*d^3*x^2*Ei(-(b*d*x +
```

$$\begin{aligned}
& b*c)/d)*e^{(-a + b*c/d)} - 24*a*b^4*c^2*d^4*x^2*Ei(-(b*d*x + b*c)/d)*e^{(-a + b*c/d)} + 24*a^2*b^3*c*d^5*x^2*Ei(-(b*d*x + b*c)/d)*e^{(-a + b*c/d)} - 8*a^3*b^2*d^6*x^2*Ei(-(b*d*x + b*c)/d)*e^{(-a + b*c/d)} + b^6*c^6*Ei(-(b*d*x + b*c)/d)*e^{(-a + b*c/d)} - 4*a*b^5*c^5*d*Ei(-(b*d*x + b*c)/d)*e^{(-a + b*c/d)} + 6*a^2*b^4*c^4*d^2*Ei(-(b*d*x + b*c)/d)*e^{(-a + b*c/d)} - 4*a^3*b^3*c^3*d^3*Ei(-(b*d*x + b*c)/d)*e^{(-a + b*c/d)} + a^4*b^2*c^2*d^4*Ei(-(b*d*x + b*c)/d)*e^{(-a + b*c/d)} + 16*b^5*c^4*d^2*x*Ei(-(b*d*x + b*c)/d)*e^{(-a + b*c/d)} - 48*a*b^4*c^3*d^3*x*Ei(-(b*d*x + b*c)/d)*e^{(-a + b*c/d)} + 48*a^2*b^3*c^2*d^4*x*Ei(-(b*d*x + b*c)/d)*e^{(-a + b*c/d)} - 16*a^3*b^2*c*d^5*x*Ei(-(b*d*x + b*c)/d)*e^{(-a + b*c/d)} + 12*b^4*c^2*d^4*x^2*Ei(-(b*d*x + b*c)/d)*e^{(-a + b*c/d)} - 24*a*b^3*c*d^5*x^2*Ei(-(b*d*x + b*c)/d)*e^{(-a + b*c/d)} + 12*a^2*b^2*d^6*x^2*Ei(-(b*d*x + b*c)/d)*e^{(-a + b*c/d)} + b^5*c^4*d^2*x*e^{(-b*x - a)} - 4*a*b^4*c^3*d^3*x*e^{(-b*x - a)} + 6*a^2*b^3*c^2*d^4*x*e^{(-b*x - a)} - 4*a^3*b^2*c*d^5*x*e^{(-b*x - a)} + a^4*b*d^6*x*e^{(-b*x - a)} + 8*b^5*c^5*d*Ei(-(b*d*x + b*c)/d)*e^{(-a + b*c/d)} - 24*a*b^4*c^4*d^2*Ei(-(b*d*x + b*c)/d)*e^{(-a + b*c/d)} + 24*a^2*b^3*c^3*d^3*Ei(-(b*d*x + b*c)/d)*e^{(-a + b*c/d)} - 8*a^3*b^2*c^2*d^4*Ei(-(b*d*x + b*c)/d)*e^{(-a + b*c/d)} + 24*b^4*c^3*d^3*x*Ei(-(b*d*x + b*c)/d)*e^{(-a + b*c/d)} - 48*a*b^3*c^2*d^4*x*Ei(-(b*d*x + b*c)/d)*e^{(-a + b*c/d)} + 24*a^2*b^2*c*d^5*x*Ei(-(b*d*x + b*c)/d)*e^{(-a + b*c/d)} + b^5*c^5*d*e^{(-b*x - a)} - 4*a*b^4*c^4*d^2*e^{(-b*x - a)} + 6*a^2*b^3*c^3*d^3*e^{(-b*x - a)} - 4*a^3*b^2*c^2*d^4*e^{(-b*x - a)} + a^4*b*c*d^5*e^{(-b*x - a)} + 8*b^4*c^3*d^3*x*e^{(-b*x - a)} - 24*a*b^3*c^2*d^4*x*e^{(-b*x - a)} + 24*a^2*b^2*c*d^5*x*e^{(-b*x - a)} - 8*a^3*b*d^6*x*e^{(-b*x - a)} + 12*b^4*c^4*d^2*Ei(-(b*d*x + b*c)/d)*e^{(-a + b*c/d)} - 24*a*b^3*c^3*d^3*Ei(-(b*d*x + b*c)/d)*e^{(-a + b*c/d)} + 12*a^2*b^2*c^2*d^4*Ei(-(b*d*x + b*c)/d)*e^{(-a + b*c/d)} + 7*b^4*c^4*d^2*e^{(-b*x - a)} - 20*a*b^3*c^3*d^3*e^{(-b*x - a)} + 18*a^2*b^2*c^2*d^4*e^{(-b*x - a)} - 4*a^3*b*c*d^5*e^{(-b*x - a)} - a^4*d^6*e^{(-b*x - a)})/(d^9*x^2 + 2*c*d^8*x + c^2*d^7)
\end{aligned}$$

$$3.81 \quad \int \frac{e^{-a-bx}(a+bx)^4}{(c+dx)^4} dx$$

**Optimal.** Leaf size=396

$$\frac{b^3 e^{\frac{bc}{d}-a} (bc-ad)^4 \operatorname{Ei}\left(-\frac{b(c+dx)}{d}\right)}{6d^8} - \frac{2b^3 e^{\frac{bc}{d}-a} (bc-ad)^3 \operatorname{Ei}\left(-\frac{b(c+dx)}{d}\right)}{d^7} - \frac{6b^3 e^{\frac{bc}{d}-a} (bc-ad)^2 \operatorname{Ei}\left(-\frac{b(c+dx)}{d}\right)}{d^6} - \frac{4b^3 e^{\frac{bc}{d}-a} (bc-ad)}{d^5}$$

[Out]  $-\left(\frac{b^3 E^{-a-bx}}{d^4}\right) - \left(\frac{(b^3 c - a^3 d) E^{-a-bx}}{(3d^5(c+dx)^3)}\right) + \left(\frac{2b^2(b^2 c - a^2 d) E^{-a-bx}}{(d^5(c+dx)^2)}\right) + \left(\frac{b(b^2 c - a^2 d) E^{-a-bx}}{(6d^6(c+dx)^2)}\right) - \left(\frac{6b^2(b^2 c - a^2 d) E^{-a-bx}}{(d^5(c+dx))}\right) - \left(\frac{2b^2(b^2 c - a^2 d) E^{-a-bx}}{(d^6(c+dx))}\right) - \left(\frac{b^2(b^2 c - a^2 d) E^{-a-bx}}{(6d^7(c+dx))}\right) - \left(\frac{4b^3(b^2 c - a^2 d) E^{-a+(b^2 c)/d}}{(d) \operatorname{ExpIntegralEi}\left[-\frac{b(c+dx)}{d}\right]}{d^5}\right) - \left(\frac{6b^3(b^2 c - a^2 d) E^{-a+(b^2 c)/d}}{(d) \operatorname{ExpIntegralEi}\left[-\frac{b(c+dx)}{d}\right]}{d^6}\right) - \left(\frac{2b^3(b^2 c - a^2 d) E^{-a+(b^2 c)/d}}{(d) \operatorname{ExpIntegralEi}\left[-\frac{b(c+dx)}{d}\right]}{d^7}\right) - \left(\frac{b^3(b^2 c - a^2 d) E^{-a+(b^2 c)/d}}{(d) \operatorname{ExpIntegralEi}\left[-\frac{b(c+dx)}{d}\right]}{d^8}\right)$

**Rubi [A]** time = 0.520858, antiderivative size = 396, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 4, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.16$ , Rules used = {2199, 2194, 2177, 2178}

$$\frac{b^3 e^{\frac{bc}{d}-a} (bc-ad)^4 \operatorname{Ei}\left(-\frac{b(c+dx)}{d}\right)}{6d^8} - \frac{2b^3 e^{\frac{bc}{d}-a} (bc-ad)^3 \operatorname{Ei}\left(-\frac{b(c+dx)}{d}\right)}{d^7} - \frac{6b^3 e^{\frac{bc}{d}-a} (bc-ad)^2 \operatorname{Ei}\left(-\frac{b(c+dx)}{d}\right)}{d^6} - \frac{4b^3 e^{\frac{bc}{d}-a} (bc-ad)}{d^5}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}\left[\frac{E^{-a-bx}(a+bx)^4}{(c+dx)^4}, x\right]$

[Out]  $-\left(\frac{b^3 E^{-a-bx}}{d^4}\right) - \left(\frac{(b^3 c - a^3 d) E^{-a-bx}}{(3d^5(c+dx)^3)}\right) + \left(\frac{2b^2(b^2 c - a^2 d) E^{-a-bx}}{(d^5(c+dx)^2)}\right) + \left(\frac{b(b^2 c - a^2 d) E^{-a-bx}}{(6d^6(c+dx)^2)}\right) - \left(\frac{6b^2(b^2 c - a^2 d) E^{-a-bx}}{(d^5(c+dx))}\right) - \left(\frac{2b^2(b^2 c - a^2 d) E^{-a-bx}}{(d^6(c+dx))}\right) - \left(\frac{b^2(b^2 c - a^2 d) E^{-a-bx}}{(6d^7(c+dx))}\right) - \left(\frac{4b^3(b^2 c - a^2 d) E^{-a+(b^2 c)/d}}{(d) \operatorname{ExpIntegralEi}\left[-\frac{b(c+dx)}{d}\right]}{d^5}\right) - \left(\frac{6b^3(b^2 c - a^2 d) E^{-a+(b^2 c)/d}}{(d) \operatorname{ExpIntegralEi}\left[-\frac{b(c+dx)}{d}\right]}{d^6}\right) - \left(\frac{2b^3(b^2 c - a^2 d) E^{-a+(b^2 c)/d}}{(d) \operatorname{ExpIntegralEi}\left[-\frac{b(c+dx)}{d}\right]}{d^7}\right) - \left(\frac{b^3(b^2 c - a^2 d) E^{-a+(b^2 c)/d}}{(d) \operatorname{ExpIntegralEi}\left[-\frac{b(c+dx)}{d}\right]}{d^8}\right)$

Rule 2199

```
Int[(F_)^((c_)*(v_))*(u_)^(m_)*(w_), x_Symbol] := Int[ExpandIntegrand[F^(
c*ExpandToSum[v, x]), w*NormalizePowerOfLinear[u, x]^m, x], x] /; FreeQ[{F,
c}, x] && PolynomialQ[w, x] && LinearQ[v, x] && PowerOfLinearQ[u, x] && In
tegerQ[m] && !$UseGamma === True
```

### Rule 2194

```
Int[((F_)^((c_)*((a_) + (b_)*(x_))))^(n_), x_Symbol] := Simp[(F^(c*(a +
b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

### Rule 2177

```
Int[((b_)*(F_)^((g_)*((e_) + (f_)*(x_))))^(n_)*((c_) + (d_)*(x_))^(m
_), x_Symbol] := Simp[((c + d*x)^(m + 1)*(b*F^(g*(e + f*x)))^n)/(d*(m + 1))
, x] - Dist[(f*g*n*Log[F])/(d*(m + 1)), Int[(c + d*x)^(m + 1)*(b*F^(g*(e +
f*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && LtQ[m, -1] && Int
egerQ[2*m] && !$UseGamma === True
```

### Rule 2178

```
Int[(F_)^((g_)*((e_) + (f_)*(x_)))/((c_) + (d_)*(x_)), x_Symbol] := Si
mp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; F
reeQ[{F, c, d, e, f, g}, x] && !$UseGamma === True
```

### Rubi steps

$$\begin{aligned}
\int \frac{e^{-a-bx}(a+bx)^4}{(c+dx)^4} dx &= \int \left( \frac{b^4 e^{-a-bx}}{d^4} + \frac{(-bc+ad)^4 e^{-a-bx}}{d^4(c+dx)^4} - \frac{4b(bc-ad)^3 e^{-a-bx}}{d^4(c+dx)^3} + \frac{6b^2(bc-ad)^2 e^{-a-bx}}{d^4(c+dx)^2} - \frac{4b^3(bc-ad) e^{-a-bx}}{d^4(c+dx)} \right) dx \\
&= \frac{b^4 \int e^{-a-bx} dx}{d^4} - \frac{(4b^3(bc-ad)) \int \frac{e^{-a-bx}}{c+dx} dx}{d^4} + \frac{(6b^2(bc-ad)^2) \int \frac{e^{-a-bx}}{(c+dx)^2} dx}{d^4} - \frac{(4b(bc-ad)^3) \int \frac{e^{-a-bx}}{(c+dx)^3} dx}{d^4} + \frac{4b^3(bc-ad) \int \frac{e^{-a-bx}}{(c+dx)^4} dx}{d^4} \\
&= -\frac{b^3 e^{-a-bx}}{d^4} - \frac{(bc-ad)^4 e^{-a-bx}}{3d^5(c+dx)^3} + \frac{2b(bc-ad)^3 e^{-a-bx}}{d^5(c+dx)^2} - \frac{6b^2(bc-ad)^2 e^{-a-bx}}{d^5(c+dx)} - \frac{4b^3(bc-ad) e^{-a-bx}}{d^5} \\
&= -\frac{b^3 e^{-a-bx}}{d^4} - \frac{(bc-ad)^4 e^{-a-bx}}{3d^5(c+dx)^3} + \frac{2b(bc-ad)^3 e^{-a-bx}}{d^5(c+dx)^2} + \frac{b(bc-ad)^4 e^{-a-bx}}{6d^6(c+dx)^2} - \frac{6b^2(bc-ad)^2 e^{-a-bx}}{d^5(c+dx)} \\
&= -\frac{b^3 e^{-a-bx}}{d^4} - \frac{(bc-ad)^4 e^{-a-bx}}{3d^5(c+dx)^3} + \frac{2b(bc-ad)^3 e^{-a-bx}}{d^5(c+dx)^2} + \frac{b(bc-ad)^4 e^{-a-bx}}{6d^6(c+dx)^2} - \frac{6b^2(bc-ad)^2 e^{-a-bx}}{d^5(c+dx)} \\
&= -\frac{b^3 e^{-a-bx}}{d^4} - \frac{(bc-ad)^4 e^{-a-bx}}{3d^5(c+dx)^3} + \frac{2b(bc-ad)^3 e^{-a-bx}}{d^5(c+dx)^2} + \frac{b(bc-ad)^4 e^{-a-bx}}{6d^6(c+dx)^2} - \frac{6b^2(bc-ad)^2 e^{-a-bx}}{d^5(c+dx)}
\end{aligned}$$

**Mathematica [A]** time = 0.807385, size = 389, normalized size = 0.98

$$e^{-a} \left( b^3 e^{\frac{bc}{d}} \left( - \left( 6(a^2 - 6a + 6) b^2 c^2 d^2 - 4(a^3 - 9a^2 + 18a - 6) bcd^3 + a(a^3 - 12a^2 + 36a - 24) d^4 - 4(a-3)b^3 c^3 d + b^4 c^4 \right) \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(E^(-a - b\*x)\*(a + b\*x)^4)/(c + d\*x)^4, x]

[Out] 
$$\frac{-((d*(2*a^4*d^6 + b^6*c^4*(c + d*x)^2 - a^3*b*d^5*((-4 + a)*c + (-12 + a)*d*x) - b^5*c^3*d*(c + d*x)*((-11 + 4*a)*c + 4*(-3 + a)*d*x) + a^2*b^2*d^4*((12 - 8*a + a^2)*c^2 + 2*(18 - 10*a + a^2)*c*d*x + (-6 + a)^2*d^2*x^2) + 2*b^4*c^2*d^2*((13 - 16*a + 3*a^2)*c^2 + 2*(15 - 17*a + 3*a^2)*c*d*x + 3*(6 - 6*a + a^2)*d^2*x^2) + 2*b^3*d^3*((3 - 22*a + 15*a^2 - 2*a^3)*c^3 + (9 - 54*a + 33*a^2 - 4*a^3)*c^2*d*x + (9 - 36*a + 18*a^2 - 2*a^3)*c*d^2*x^2 + 3*d^3*x^3)))/(E^(b*x)*(c + d*x)^3) - b^3*(b^4*c^4 - 4*(-3 + a)*b^3*c^3*d + 6*(6 - 6*a + a^2)*b^2*c^2*d^2 - 4*(-6 + 18*a - 9*a^2 + a^3)*b*c*d^3 + a*(-24 + 36*a - 12*a^2 + a^3)*d^4)*E^((b*c)/d)*ExpIntegralEi[-((b*(c + d*x))/d)]/(6*d^8*E^a)$$

**Maple [A]** time = 0.013, size = 511, normalized size = 1.3

$$-\frac{1}{b} \left( \frac{b^4 e^{-bx-a}}{d^4} - 4 \frac{(a^3 d^3 - 3 a^2 b c d^2 + 3 a b^2 c^2 d - b^3 c^3) b^4}{d^7} \left( -\frac{1}{2} e^{-bx-a} \left( -bx - a + \frac{ad - bc}{d} \right)^{-2} - \frac{1}{2} e^{-bx-a} \left( -bx - a + \frac{ad - bc}{d} \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(-b\*x-a)\*(b\*x+a)^4/(d\*x+c)^4,x)

[Out]  $-1/b*(b^4/d^4*\exp(-b*x-a)-4/d^7*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)*b^4*(-1/2*\exp(-b*x-a)/(-b*x-a+(a*d-b*c)/d)^2-1/2*\exp(-b*x-a)/(-b*x-a+(a*d-b*c)/d)-1/2*\exp(-(a*d-b*c)/d)*\text{Ei}(1,b*x+a-(a*d-b*c)/d))+4/d^5*(a*d-b*c)*b^4*\exp(-(a*d-b*c)/d)*\text{Ei}(1,b*x+a-(a*d-b*c)/d)+(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)*b^4/d^8*(-1/3*\exp(-b*x-a)/(-b*x-a+(a*d-b*c)/d)^3-1/6*\exp(-b*x-a)/(-b*x-a+(a*d-b*c)/d)^2-1/6*\exp(-b*x-a)/(-b*x-a+(a*d-b*c)/d)-1/6*\exp(-(a*d-b*c)/d)*\text{Ei}(1,b*x+a-(a*d-b*c)/d))+6/d^6*(a^2*d^2-2*a*b*c*d+b^2*c^2)*b^4*(-\exp(-b*x-a)/(-b*x-a+(a*d-b*c)/d)-\exp(-(a*d-b*c)/d)*\text{Ei}(1,b*x+a-(a*d-b*c)/d))$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{a^4 e^{\left(-a + \frac{bc}{d}\right)} E_4\left(\frac{(dx+c)b}{d}\right)}{(dx+c)^3 d} - \frac{(b^3 d^2 x^4 + 4 a b^2 d^2 x^3 + 2(3 a^2 b d^2 + 2 b^2 c d - 2 a b d^2) x^2 + 4(a^3 d^2 - b^2 c^2 - 3 a^2 d^2 - 2 b c d + 2(2 b^2 c^2 d - 2 a b c d^2 + 3 a^2 b^2 c^2 d - b^3 c^3)) x + (a^4 d^3 - 3 a^3 b c d^2 + 3 a^2 b^2 c^2 d - b^3 c^3)) e^{\left(-a + \frac{bc}{d}\right)}}{d^6 x^4 e^a + 4 c d^5 x^3 e^a + 6 c^2 d^4 x^2 e^a + 4 c^3 d^3 x e^a + c^4 d^2 e^a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(-b\*x-a)\*(b\*x+a)^4/(d\*x+c)^4,x, algorithm="maxima")

[Out]  $-a^4 * e^{(-a + b*c/d)} * \text{exp\_integral\_e}(4, (d*x + c)*b/d)/((d*x + c)^3*d) - (b^3*d^2*x^4 + 4*a*b^2*d^2*x^3 + 2*(3*a^2*b*d^2 + 2*b^2*c*d - 2*a*b*d^2)*x^2 + 4*(a^3*d^2 - b^2*c^2 - 3*a^2*d^2 - 2*b*c*d + 2*(2*b*c*d + d^2)*a)*x)*e^{(-b*x)}/(d^6*x^4*e^a + 4*c*d^5*x^3*e^a + 6*c^2*d^4*x^2*e^a + 4*c^3*d^3*x*e^a + c^4*d^2*e^a) - \text{integrate}(-4*(a^3*c*d^2 - b^2*c^3 - 3*a^2*c*d^2 - 2*b*c^2*d + 2*(2*b*c^2*d + c*d^2)*a + (b^3*c^3 - 3*a^3*d^3 + 7*b^2*c^2*d + 6*b*c*d^2 + 3*(2*b*c*d^2 + 3*d^3)*a^2 - 2*(2*b^2*c^2*d + 8*b*c*d^2 + 3*d^3)*a)*x)*e^{(-b*x)}/(d^7*x^5*e^a + 5*c*d^6*x^4*e^a + 10*c^2*d^5*x^3*e^a + 10*c^3*d^4*x^2*e^a + 5*c^4*d^3*x*e^a + c^5*d^2*e^a), x)$



**Fricas [B]** time = 1.59546, size = 1733, normalized size = 4.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(-b\*x-a)\*(b\*x+a)^4/(d\*x+c)^4,x, algorithm="fricas")

[Out] 
$$-1/6*((b^7*c^7 - 4*(a - 3)*b^6*c^6*d + 6*(a^2 - 6*a + 6)*b^5*c^5*d^2 - 4*(a^3 - 9*a^2 + 18*a - 6)*b^4*c^4*d^3 + (a^4 - 12*a^3 + 36*a^2 - 24*a)*b^3*c^3*d^4 + (b^7*c^4*d^3 - 4*(a - 3)*b^6*c^3*d^4 + 6*(a^2 - 6*a + 6)*b^5*c^2*d^5 - 4*(a^3 - 9*a^2 + 18*a - 6)*b^4*c*d^6 + (a^4 - 12*a^3 + 36*a^2 - 24*a)*b^3*d^7)*x^3 + 3*(b^7*c^5*d^2 - 4*(a - 3)*b^6*c^4*d^3 + 6*(a^2 - 6*a + 6)*b^5*c^3*d^4 - 4*(a^3 - 9*a^2 + 18*a - 6)*b^4*c^2*d^5 + (a^4 - 12*a^3 + 36*a^2 - 24*a)*b^3*c*d^6)*x^2 + 3*(b^7*c^6*d - 4*(a - 3)*b^6*c^5*d^2 + 6*(a^2 - 6*a + 6)*b^5*c^4*d^3 - 4*(a^3 - 9*a^2 + 18*a - 6)*b^4*c^3*d^4 + (a^4 - 12*a^3 + 36*a^2 - 24*a)*b^3*c^2*d^5)*x)*Ei(-(b*d*x + b*c)/d)*e^((b*c - a*d)/d) + (b^6*c^6*d - (4*a - 11)*b^5*c^5*d^2 + 6*b^3*d^7*x^3 + 2*(3*a^2 - 16*a + 13)*b^4*c^4*d^3 - 2*(2*a^3 - 15*a^2 + 22*a - 3)*b^3*c^3*d^4 + 2*a^4*d^7 + (a^4 - 8*a^3 + 12*a^2)*b^2*c^2*d^5 - (a^4 - 4*a^3)*b*c*d^6 + (b^6*c^4*d^3 - 4*(a - 3)*b^5*c^3*d^4 + 6*(a^2 - 6*a + 6)*b^4*c^2*d^5 - 2*(2*a^3 - 18*a^2 + 36*a - 9)*b^3*c*d^6 + (a^4 - 12*a^3 + 36*a^2)*b^2*d^7)*x^2 + (2*b^6*c^5*d^2 - (8*a - 23)*b^5*c^4*d^3 + 4*(3*a^2 - 17*a + 15)*b^4*c^3*d^4 - 2*(4*a^3 - 33*a^2 + 54*a - 9)*b^3*c^2*d^5 + 2*(a^4 - 10*a^3 + 18*a^2)*b^2*c*d^6 - (a^4 - 12*a^3)*b*d^7)*x)*e^(-b*x - a))/(d^11*x^3 + 3*c*d^10*x^2 + 3*c^2*d^9*x + c^3*d^8)$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(-b\*x-a)\*(b\*x+a)\*\*4/(d\*x+c)\*\*4,x)

[Out] Timed out

**Giac [B]** time = 1.29438, size = 4180, normalized size = 10.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(-b\*x-a)\*(b\*x+a)^4/(d\*x+c)^4,x, algorithm="giac")

[Out] 
$$\begin{aligned} & -1/6*(b^7*c^4*d^3*x^3*Ei(-(b*d*x + b*c)/d)*e^{(-a + b*c/d)} - 4*a*b^6*c^3*d^4 \\ & *x^3*Ei(-(b*d*x + b*c)/d)*e^{(-a + b*c/d)} + 6*a^2*b^5*c^2*d^5*x^3*Ei(-(b*d*x \\ & + b*c)/d)*e^{(-a + b*c/d)} - 4*a^3*b^4*c*d^6*x^3*Ei(-(b*d*x + b*c)/d)*e^{(-a \\ & + b*c/d)} + a^4*b^3*d^7*x^3*Ei(-(b*d*x + b*c)/d)*e^{(-a + b*c/d)} + 3*b^7*c^5* \\ & d^2*x^2*Ei(-(b*d*x + b*c)/d)*e^{(-a + b*c/d)} - 12*a*b^6*c^4*d^3*x^2*Ei(-(b*d \\ & *x + b*c)/d)*e^{(-a + b*c/d)} + 18*a^2*b^5*c^3*d^4*x^2*Ei(-(b*d*x + b*c)/d)*e \\ & ^{(-a + b*c/d)} - 12*a^3*b^4*c^2*d^5*x^2*Ei(-(b*d*x + b*c)/d)*e^{(-a + b*c/d)} \\ & + 3*a^4*b^3*c*d^6*x^2*Ei(-(b*d*x + b*c)/d)*e^{(-a + b*c/d)} + 12*b^6*c^3*d^4* \\ & x^3*Ei(-(b*d*x + b*c)/d)*e^{(-a + b*c/d)} - 36*a*b^5*c^2*d^5*x^3*Ei(-(b*d*x + \\ & b*c)/d)*e^{(-a + b*c/d)} + 36*a^2*b^4*c*d^6*x^3*Ei(-(b*d*x + b*c)/d)*e^{(-a + \\ & b*c/d)} - 12*a^3*b^3*d^7*x^3*Ei(-(b*d*x + b*c)/d)*e^{(-a + b*c/d)} + 3*b^7*c^ \\ & 6*d*x*Ei(-(b*d*x + b*c)/d)*e^{(-a + b*c/d)} - 12*a*b^6*c^5*d^2*x*Ei(-(b*d*x + \\ & b*c)/d)*e^{(-a + b*c/d)} + 18*a^2*b^5*c^4*d^3*x*Ei(-(b*d*x + b*c)/d)*e^{(-a + \\ & b*c/d)} - 12*a^3*b^4*c^3*d^4*x*Ei(-(b*d*x + b*c)/d)*e^{(-a + b*c/d)} + 3*a^4* \\ & b^3*c^2*d^5*x*Ei(-(b*d*x + b*c)/d)*e^{(-a + b*c/d)} + 36*b^6*c^4*d^3*x^2*Ei(- \\ & (b*d*x + b*c)/d)*e^{(-a + b*c/d)} - 108*a*b^5*c^3*d^4*x^2*Ei(-(b*d*x + b*c)/d \\ & )*e^{(-a + b*c/d)} + 108*a^2*b^4*c^2*d^5*x^2*Ei(-(b*d*x + b*c)/d)*e^{(-a + b*c \\ & /d)} - 36*a^3*b^3*c*d^6*x^2*Ei(-(b*d*x + b*c)/d)*e^{(-a + b*c/d)} + 36*b^5*c^2 \\ & *d^5*x^3*Ei(-(b*d*x + b*c)/d)*e^{(-a + b*c/d)} - 72*a*b^4*c*d^6*x^3*Ei(-(b*d* \\ & x + b*c)/d)*e^{(-a + b*c/d)} + 36*a^2*b^3*d^7*x^3*Ei(-(b*d*x + b*c)/d)*e^{(-a \\ & + b*c/d)} + b^6*c^4*d^3*x^2*e^{(-b*x - a)} - 4*a*b^5*c^3*d^4*x^2*e^{(-b*x - a)} \\ & + 6*a^2*b^4*c^2*d^5*x^2*e^{(-b*x - a)} - 4*a^3*b^3*c*d^6*x^2*e^{(-b*x - a)} + a \\ & ^4*b^2*d^7*x^2*e^{(-b*x - a)} + b^7*c^7*Ei(-(b*d*x + b*c)/d)*e^{(-a + b*c/d)} - \\ & 4*a*b^6*c^6*d*Ei(-(b*d*x + b*c)/d)*e^{(-a + b*c/d)} + 6*a^2*b^5*c^5*d^2*Ei(- \\ & (b*d*x + b*c)/d)*e^{(-a + b*c/d)} - 4*a^3*b^4*c^4*d^3*Ei(-(b*d*x + b*c)/d)*e^{ \\ & (-a + b*c/d)} + a^4*b^3*c^3*d^4*Ei(-(b*d*x + b*c)/d)*e^{(-a + b*c/d)} + 36*b^6 \\ & *c^5*d^2*x*Ei(-(b*d*x + b*c)/d)*e^{(-a + b*c/d)} - 108*a*b^5*c^4*d^3*x*Ei(-(b \\ & *d*x + b*c)/d)*e^{(-a + b*c/d)} + 108*a^2*b^4*c^3*d^4*x*Ei(-(b*d*x + b*c)/d)* \\ & e^{(-a + b*c/d)} - 36*a^3*b^3*c^2*d^5*x*Ei(-(b*d*x + b*c)/d)*e^{(-a + b*c/d)} + \\ & 108*b^5*c^3*d^4*x^2*Ei(-(b*d*x + b*c)/d)*e^{(-a + b*c/d)} - 216*a*b^4*c^2*d^ \\ & 5*x^2*Ei(-(b*d*x + b*c)/d)*e^{(-a + b*c/d)} + 108*a^2*b^3*c*d^6*x^2*Ei(-(b*d* \\ & x + b*c)/d)*e^{(-a + b*c/d)} + 24*b^4*c*d^6*x^3*Ei(-(b*d*x + b*c)/d)*e^{(-a + \\ & b*c/d)} - 24*a*b^3*d^7*x^3*Ei(-(b*d*x + b*c)/d)*e^{(-a + b*c/d)} + 2*b^6*c^5*d \\ & ^2*x*e^{(-b*x - a)} - 8*a*b^5*c^4*d^3*x*e^{(-b*x - a)} + 12*a^2*b^4*c^3*d^4*x*e \\ & ^{(-b*x - a)} - 8*a^3*b^3*c^2*d^5*x*e^{(-b*x - a)} + 2*a^4*b^2*c*d^6*x*e^{(-b*x \\ & - a)} + 12*b^5*c^3*d^4*x^2*e^{(-b*x - a)} - 36*a*b^4*c^2*d^5*x^2*e^{(-b*x - a)} \\ & + 36*a^2*b^3*c*d^6*x^2*e^{(-b*x - a)} - 12*a^3*b^2*d^7*x^2*e^{(-b*x - a)} + 12* \\ & b^6*c^6*d*Ei(-(b*d*x + b*c)/d)*e^{(-a + b*c/d)} - 36*a*b^5*c^5*d^2*Ei(-(b*d*x \\ & + b*c)/d)*e^{(-a + b*c/d)} + 36*a^2*b^4*c^4*d^3*Ei(-(b*d*x + b*c)/d)*e^{(-a + \\ & b*c/d)} - 12*a^3*b^3*c^3*d^4*Ei(-(b*d*x + b*c)/d)*e^{(-a + b*c/d)} + 108*b^5*c \end{aligned}$$

$$\begin{aligned}
& c^4 d^3 x \operatorname{Ei}(-b d x + b c) / d * e^{-a + b c / d} - 216 a b^4 c^3 d^4 x \operatorname{Ei}(-b d x + b c) / d * e^{-a + b c / d} + 108 a^2 b^3 c^2 d^5 x \operatorname{Ei}(-b d x + b c) / d * e^{-a + b c / d} \\
& + 72 b^4 c^2 d^5 x^2 \operatorname{Ei}(-b d x + b c) / d * e^{-a + b c / d} - 72 a b^3 c d^6 x^2 \operatorname{Ei}(-b d x + b c) / d * e^{-a + b c / d} + b^6 c^6 d e^{-b x - a} \\
& - 4 a b^5 c^5 d^2 e^{-b x - a} + 6 a^2 b^4 c^4 d^3 e^{-b x - a} - 4 a^3 b^3 c^3 d^4 e^{-b x - a} + a^4 b^2 c^2 d^5 e^{-b x - a} + 23 b^5 c^4 d^3 x e^{-b x - a} \\
& - 68 a b^4 c^3 d^4 x e^{-b x - a} + 66 a^2 b^3 c^2 d^5 x e^{-b x - a} - 20 a^3 b^2 c d^6 x e^{-b x - a} - a^4 b d^7 x e^{-b x - a} + 36 b^4 c^2 d^5 x^2 e^{-b x - a} \\
& - 72 a b^3 c d^6 x^2 e^{-b x - a} + 36 a^2 b^2 d^7 x^2 e^{-b x - a} + 36 b^5 c^5 d^2 \operatorname{Ei}(-b d x + b c) / d * e^{-a + b c / d} - 72 a b^4 c^4 d^3 \operatorname{Ei}(-b d x + b c) / d * e^{-a + b c / d} \\
& + 36 a^2 b^3 c^3 d^4 \operatorname{Ei}(-b d x + b c) / d * e^{-a + b c / d} + 72 b^4 c^3 d^4 x \operatorname{Ei}(-b d x + b c) / d * e^{-a + b c / d} - 72 a b^3 c^2 d^5 x \operatorname{Ei}(-b d x + b c) / d * e^{-a + b c / d} \\
& + 11 b^5 c^5 d^2 e^{-b x - a} - 32 a b^4 c^4 d^3 e^{-b x - a} + 30 a^2 b^3 c^3 d^4 e^{-b x - a} - 8 a^3 b^2 c^2 d^5 e^{-b x - a} - a^4 b c d^6 e^{-b x - a} \\
& + 60 b^4 c^3 d^4 x e^{-b x - a} - 108 a b^3 c^2 d^5 x e^{-b x - a} + 36 a^2 b^2 c d^6 x e^{-b x - a} + 12 a^3 b d^7 x e^{-b x - a} + 24 b^4 c^4 d^3 \operatorname{Ei}(-b d x + b c) / d * e^{-a + b c / d} \\
& - 24 a b^3 c^3 d^4 \operatorname{Ei}(-b d x + b c) / d * e^{-a + b c / d} + 26 b^4 c^4 d^3 e^{-b x - a} - 44 a b^3 c^3 d^4 e^{-b x - a} + 12 a^2 b^2 c^2 d^5 e^{-b x - a} + 4 a^3 b c d^6 e^{-b x - a} + 2 a^4 d^7 e^{-b x - a} \\
& \left. \right) / (d^{11} x^3 + 3 c d^{10} x^2 + 3 c^2 d^9 x + c^3 d^8)
\end{aligned}$$

$$3.82 \quad \int \frac{e^{-a-bx}(a+bx)^4}{(c+dx)^5} dx$$

**Optimal.** Leaf size=557

$$\frac{b^4(bc-ad)^4 e^{\frac{bc}{d}-a} \operatorname{Ei}\left(-\frac{b(c+dx)}{d}\right)}{24d^9} + \frac{2b^4(bc-ad)^3 e^{\frac{bc}{d}-a} \operatorname{Ei}\left(-\frac{b(c+dx)}{d}\right)}{3d^8} + \frac{3b^4(bc-ad)^2 e^{\frac{bc}{d}-a} \operatorname{Ei}\left(-\frac{b(c+dx)}{d}\right)}{d^7} + \frac{4b^4(bc-ad) e^{\frac{bc}{d}-a} \operatorname{Ei}\left(-\frac{b(c+dx)}{d}\right)}{d^6}$$

[Out]  $-\left(\frac{(b^2c - a^2d)^4 E^{-a - bx}}{(4d^5(c + dx))^4} + \frac{4b(b^2c - a^2d)^3 E^{-a - bx}}{(3d^5(c + dx))^3} + \frac{b(b^2c - a^2d)^4 E^{-a - bx}}{(12d^6(c + dx))^3} - \frac{3b^2(b^2c - a^2d)^2 E^{-a - bx}}{(d^5(c + dx))^2} - \frac{2b^2(b^2c - a^2d)^3 E^{-a - bx}}{(3d^6(c + dx))^2} - \frac{b^2(b^2c - a^2d)^4 E^{-a - bx}}{(24d^7(c + dx))^2} + \frac{4b^3(b^2c - a^2d) E^{-a - bx}}{(d^5(c + dx))} + \frac{3b^3(b^2c - a^2d)^2 E^{-a - bx}}{(d^6(c + dx))} + \frac{2b^3(b^2c - a^2d)^3 E^{-a - bx}}{(3d^7(c + dx))} + \frac{b^3(b^2c - a^2d)^4 E^{-a - bx}}{(24d^8(c + dx))} + \frac{b^4 E^{-a + (bc)/d} \operatorname{ExpIntegralEi}\left[-\frac{b(c + dx)}{d}\right]}{d^5} + \frac{4b^4(b^2c - a^2d) E^{-a + (bc)/d} \operatorname{ExpIntegralEi}\left[-\frac{b(c + dx)}{d}\right]}{d^6} + \frac{3b^4(b^2c - a^2d)^2 E^{-a + (bc)/d} \operatorname{ExpIntegralEi}\left[-\frac{b(c + dx)}{d}\right]}{d^7} + \frac{2b^4(b^2c - a^2d)^3 E^{-a + (bc)/d} \operatorname{ExpIntegralEi}\left[-\frac{b(c + dx)}{d}\right]}{(3d^8)} + \frac{b^4(b^2c - a^2d)^4 E^{-a + (bc)/d} \operatorname{ExpIntegralEi}\left[-\frac{b(c + dx)}{d}\right]}{(24d^9)}\right)$

**Rubi [A]** time = 0.684236, antiderivative size = 557, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 3, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.12$ , Rules used = {2199, 2177, 2178}

$$\frac{b^4(bc-ad)^4 e^{\frac{bc}{d}-a} \operatorname{Ei}\left(-\frac{b(c+dx)}{d}\right)}{24d^9} + \frac{2b^4(bc-ad)^3 e^{\frac{bc}{d}-a} \operatorname{Ei}\left(-\frac{b(c+dx)}{d}\right)}{3d^8} + \frac{3b^4(bc-ad)^2 e^{\frac{bc}{d}-a} \operatorname{Ei}\left(-\frac{b(c+dx)}{d}\right)}{d^7} + \frac{4b^4(bc-ad) e^{\frac{bc}{d}-a} \operatorname{Ei}\left(-\frac{b(c+dx)}{d}\right)}{d^6}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}\left[\frac{E^{-a - bx}(a + bx)^4}{(c + dx)^5}, x\right]$

[Out]  $-\left(\frac{(b^2c - a^2d)^4 E^{-a - bx}}{(4d^5(c + dx))^4} + \frac{4b(b^2c - a^2d)^3 E^{-a - bx}}{(3d^5(c + dx))^3} + \frac{b(b^2c - a^2d)^4 E^{-a - bx}}{(12d^6(c + dx))^3} - \frac{3b^2(b^2c - a^2d)^2 E^{-a - bx}}{(d^5(c + dx))^2} - \frac{2b^2(b^2c - a^2d)^3 E^{-a - bx}}{(3d^6(c + dx))^2} - \frac{b^2(b^2c - a^2d)^4 E^{-a - bx}}{(24d^7(c + dx))^2} + \frac{4b^3(b^2c - a^2d) E^{-a - bx}}{(d^5(c + dx))} + \frac{3b^3(b^2c - a^2d)^2 E^{-a - bx}}{(d^6(c + dx))} + \frac{2b^3(b^2c - a^2d)^3 E^{-a - bx}}{(3d^7(c + dx))} + \frac{b^3(b^2c - a^2d)^4 E^{-a - bx}}{(24d^8(c + dx))} + \frac{b^4 E^{-a + (bc)/d} \operatorname{ExpIntegralEi}\left[-\frac{b(c + dx)}{d}\right]}{d^5} + \frac{4b^4(b^2c - a^2d) E^{-a + (bc)/d} \operatorname{ExpIntegralEi}\left[-\frac{b(c + dx)}{d}\right]}{d^6} + \frac{3b^4(b^2c - a^2d)^2 E^{-a + (bc)/d} \operatorname{ExpIntegralEi}\left[-\frac{b(c + dx)}{d}\right]}{d^7} + \frac{2b^4(b^2c - a^2d)^3 E^{-a + (bc)/d} \operatorname{ExpIntegralEi}\left[-\frac{b(c + dx)}{d}\right]}{(3d^8)} + \frac{b^4(b^2c - a^2d)^4 E^{-a + (bc)/d} \operatorname{ExpIntegralEi}\left[-\frac{b(c + dx)}{d}\right]}{(24d^9)}\right)$

$$\begin{aligned} &^5 + (4*b^4*(b*c - a*d)*E^{-a + (b*c)/d}*ExpIntegralEi[-((b*(c + d*x))/d)]) \\ &/d^6 + (3*b^4*(b*c - a*d)^2*E^{-a + (b*c)/d}*ExpIntegralEi[-((b*(c + d*x))/ \\ &d)])/d^7 + (2*b^4*(b*c - a*d)^3*E^{-a + (b*c)/d}*ExpIntegralEi[-((b*(c + d* \\ &x))/d)])/(3*d^8) + (b^4*(b*c - a*d)^4*E^{-a + (b*c)/d}*ExpIntegralEi[-((b*( \\ &c + d*x))/d)])/(24*d^9) \end{aligned}$$

### Rule 2199

```
Int[(F_)^((c_)*(v_))*(u_)^(m_)*(w_), x_Symbol] := Int[ExpandIntegrand[F^(
c*ExpandToSum[v, x]), w*NormalizePowerOfLinear[u, x]^m, x] /; FreeQ[{F,
c}, x] && PolynomialQ[w, x] && LinearQ[v, x] && PowerOfLinearQ[u, x] && In
tegerQ[m] && !$UseGamma === True
```

### Rule 2177

```
Int[((b_)*(F_)^((g_)*((e_) + (f_)*(x_))))^(n_)*((c_) + (d_)*(x_))^(m
_), x_Symbol] := Simp[((c + d*x)^(m + 1)*(b*F^(g*(e + f*x)))^n)/(d*(m + 1))
, x] - Dist[(f*g*n*Log[F])/(d*(m + 1)), Int[(c + d*x)^(m + 1)*(b*F^(g*(e +
f*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && LtQ[m, -1] && Int
egerQ[2*m] && !$UseGamma === True
```

### Rule 2178

```
Int[(F_)^((g_)*((e_) + (f_)*(x_)))/((c_) + (d_)*(x_)), x_Symbol] := Si
mp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; F
reeQ[{F, c, d, e, f, g}, x] && !$UseGamma === True
```

### Rubi steps

$$\begin{aligned}
\int \frac{e^{-a-bx}(a+bx)^4}{(c+dx)^5} dx &= \int \left( \frac{(-bc+ad)^4 e^{-a-bx}}{d^4(c+dx)^5} - \frac{4b(bc-ad)^3 e^{-a-bx}}{d^4(c+dx)^4} + \frac{6b^2(bc-ad)^2 e^{-a-bx}}{d^4(c+dx)^3} - \frac{4b^3(bc-ad) e^{-a-bx}}{d^4(c+dx)^2} + \frac{b^4 e^{-a-bx}}{d^4(c+dx)} \right) dx \\
&= \frac{b^4 \int \frac{e^{-a-bx}}{c+dx} dx}{d^4} - \frac{(4b^3(bc-ad)) \int \frac{e^{-a-bx}}{(c+dx)^2} dx}{d^4} + \frac{(6b^2(bc-ad)^2) \int \frac{e^{-a-bx}}{(c+dx)^3} dx}{d^4} - \frac{(4b(bc-ad)^3) \int \frac{e^{-a-bx}}{(c+dx)^4} dx}{d^4} + \frac{b^4 \int \frac{e^{-a-bx}}{c+dx} dx}{d^4} \\
&= -\frac{(bc-ad)^4 e^{-a-bx}}{4d^5(c+dx)^4} + \frac{4b(bc-ad)^3 e^{-a-bx}}{3d^5(c+dx)^3} - \frac{3b^2(bc-ad)^2 e^{-a-bx}}{d^5(c+dx)^2} + \frac{4b^3(bc-ad) e^{-a-bx}}{d^5(c+dx)} + \frac{b^4 e^{-a-bx}}{d^4} \operatorname{Ei}\left(-\frac{b(c+dx)}{d}\right) \\
&= -\frac{(bc-ad)^4 e^{-a-bx}}{4d^5(c+dx)^4} + \frac{4b(bc-ad)^3 e^{-a-bx}}{3d^5(c+dx)^3} + \frac{b(bc-ad)^4 e^{-a-bx}}{12d^6(c+dx)^3} - \frac{3b^2(bc-ad)^2 e^{-a-bx}}{d^5(c+dx)^2} - \frac{2b^2(bc-ad)^3 e^{-a-bx}}{3d^6(c+dx)^2} \\
&= -\frac{(bc-ad)^4 e^{-a-bx}}{4d^5(c+dx)^4} + \frac{4b(bc-ad)^3 e^{-a-bx}}{3d^5(c+dx)^3} + \frac{b(bc-ad)^4 e^{-a-bx}}{12d^6(c+dx)^3} - \frac{3b^2(bc-ad)^2 e^{-a-bx}}{d^5(c+dx)^2} - \frac{2b^2(bc-ad)^3 e^{-a-bx}}{3d^6(c+dx)^2} \\
&= -\frac{(bc-ad)^4 e^{-a-bx}}{4d^5(c+dx)^4} + \frac{4b(bc-ad)^3 e^{-a-bx}}{3d^5(c+dx)^3} + \frac{b(bc-ad)^4 e^{-a-bx}}{12d^6(c+dx)^3} - \frac{3b^2(bc-ad)^2 e^{-a-bx}}{d^5(c+dx)^2} - \frac{2b^2(bc-ad)^3 e^{-a-bx}}{3d^6(c+dx)^2} \\
&= -\frac{(bc-ad)^4 e^{-a-bx}}{4d^5(c+dx)^4} + \frac{4b(bc-ad)^3 e^{-a-bx}}{3d^5(c+dx)^3} + \frac{b(bc-ad)^4 e^{-a-bx}}{12d^6(c+dx)^3} - \frac{3b^2(bc-ad)^2 e^{-a-bx}}{d^5(c+dx)^2} - \frac{2b^2(bc-ad)^3 e^{-a-bx}}{3d^6(c+dx)^2}
\end{aligned}$$

**Mathematica [A]** time = 0.725518, size = 669, normalized size = 1.2

$$e^{-a} \left( 6a^2 b^6 c^2 d^2 e^{\frac{bc}{d}} \operatorname{Ei}\left(-\frac{b(c+dx)}{d}\right) + \frac{de^{-bx}(-b^2 d(c+dx)^2((a^2-16a+72)d^2-2(a-8)bcd+b^2c^2)(bc-ad)^2+b^3(c+dx)^3(6(a^2-8a+12)b^2c^2d^2-4(a^3-12a^2+36a-24)bc^2d^2-4(a^3-12a^2+36a-24)bc^2d^2-4(a^3-12a^2+36a-24)bc^2d^2-4(a^3-12a^2+36a-24)bc^2d^2)}{(c+dx)^4} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(E^(-a - b\*x)\*(a + b\*x)^4)/(c + d\*x)^5,x]

[Out] ((d\*(-6\*d^3\*(b\*c - a\*d)^4 + 2\*b\*d^2\*(b\*c - (-16 + a)\*d)\*(b\*c - a\*d)^3\*(c + d\*x) - b^2\*d\*(b\*c - a\*d)^2\*(b^2\*c^2 - 2\*(-8 + a)\*b\*c\*d + (72 - 16\*a + a^2)\*d^2)\*(c + d\*x)^2 + b^3\*(b^4\*c^4 - 4\*(-4 + a)\*b^3\*c^3\*d + 6\*(12 - 8\*a + a^2)\*b^2\*c^2\*d^2 - 4\*(-24 + 36\*a - 12\*a^2 + a^3)\*b\*c\*d^3 + a\*(-96 + 72\*a - 16\*a^2 + a^3)\*d^4)\*(c + d\*x)^3)/(E^(b\*x)\*(c + d\*x)^4) + b^8\*c^4\*E^((b\*c)/d)\*ExpIntegralEi[-((b\*(c + d\*x))/d)] + 16\*b^7\*c^3\*d\*E^((b\*c)/d)\*ExpIntegralEi[-((b\*(c + d\*x))/d)] - 4\*a\*b^7\*c^3\*d\*E^((b\*c)/d)\*ExpIntegralEi[-((b\*(c + d\*x))/d)] + 72\*b^6\*c^2\*d^2\*E^((b\*c)/d)\*ExpIntegralEi[-((b\*(c + d\*x))/d)] - 48\*a\*b^6\*c^2\*d^2\*E^((b\*c)/d)\*ExpIntegralEi[-((b\*(c + d\*x))/d)] + 6\*a^2\*b^6\*c^2\*d^2\*E^((b\*c)/d)\*ExpIntegralEi[-((b\*(c + d\*x))/d)] + 96\*b^5\*c\*d^3\*E^((b\*c)/d)\*ExpIntegralEi[-((b\*(c + d\*x))/d)] - 144\*a\*b^5\*c\*d^3\*E^((b\*c)/d)\*ExpIntegralEi[-((b\*(c + d\*x))/d)]

$$\frac{1}{b} \left( -6 \frac{(ad-bc)^2 b^5}{d^7} \left( -\frac{1}{2} e^{-bx-a} \left( -bx-a + \frac{ad-bc}{d} \right)^{-2} - \frac{1}{2} e^{-bx-a} \left( -bx-a + \frac{ad-bc}{d} \right)^{-1} - \frac{1}{2} e^{-\frac{ad-bc}{d}} \text{Ei} \left( 1, bx+a - \frac{(b(c+dx))}{d} \right) + 48a^2 b^5 c d^3 E^{\left( \frac{b(c+dx)}{d} \right)} \text{ExpIntegralEi} \left[ -\left( \frac{b(c+dx)}{d} \right) \right] - 4a^3 b^5 c d^3 E^{\left( \frac{b(c+dx)}{d} \right)} \text{ExpIntegralEi} \left[ -\left( \frac{b(c+dx)}{d} \right) \right] + 24b^4 d^4 E^{\left( \frac{b(c+dx)}{d} \right)} \text{ExpIntegralEi} \left[ -\left( \frac{b(c+dx)}{d} \right) \right] - 96a^2 b^4 d^4 E^{\left( \frac{b(c+dx)}{d} \right)} \text{ExpIntegralEi} \left[ -\left( \frac{b(c+dx)}{d} \right) \right] + 72a^2 b^4 d^4 E^{\left( \frac{b(c+dx)}{d} \right)} \text{ExpIntegralEi} \left[ -\left( \frac{b(c+dx)}{d} \right) \right] - 16a^3 b^4 d^4 E^{\left( \frac{b(c+dx)}{d} \right)} \text{ExpIntegralEi} \left[ -\left( \frac{b(c+dx)}{d} \right) \right] + a^4 b^4 d^4 E^{\left( \frac{b(c+dx)}{d} \right)} \text{ExpIntegralEi} \left[ -\left( \frac{b(c+dx)}{d} \right) \right] \right) / (24d^9 E^a)$$

**Maple [A]** time = 0.013, size = 596, normalized size = 1.1

$$-\frac{1}{b} \left( -6 \frac{(ad-bc)^2 b^5}{d^7} \left( -\frac{1}{2} e^{-bx-a} \left( -bx-a + \frac{ad-bc}{d} \right)^{-2} - \frac{1}{2} e^{-bx-a} \left( -bx-a + \frac{ad-bc}{d} \right)^{-1} - \frac{1}{2} e^{-\frac{ad-bc}{d}} \text{Ei} \left( 1, bx+a - \frac{(b(c+dx))}{d} \right) + 48a^2 b^5 c d^3 E^{\left( \frac{b(c+dx)}{d} \right)} \text{ExpIntegralEi} \left[ -\left( \frac{b(c+dx)}{d} \right) \right] - 4a^3 b^5 c d^3 E^{\left( \frac{b(c+dx)}{d} \right)} \text{ExpIntegralEi} \left[ -\left( \frac{b(c+dx)}{d} \right) \right] + 24b^4 d^4 E^{\left( \frac{b(c+dx)}{d} \right)} \text{ExpIntegralEi} \left[ -\left( \frac{b(c+dx)}{d} \right) \right] - 96a^2 b^4 d^4 E^{\left( \frac{b(c+dx)}{d} \right)} \text{ExpIntegralEi} \left[ -\left( \frac{b(c+dx)}{d} \right) \right] + 72a^2 b^4 d^4 E^{\left( \frac{b(c+dx)}{d} \right)} \text{ExpIntegralEi} \left[ -\left( \frac{b(c+dx)}{d} \right) \right] - 16a^3 b^4 d^4 E^{\left( \frac{b(c+dx)}{d} \right)} \text{ExpIntegralEi} \left[ -\left( \frac{b(c+dx)}{d} \right) \right] + a^4 b^4 d^4 E^{\left( \frac{b(c+dx)}{d} \right)} \text{ExpIntegralEi} \left[ -\left( \frac{b(c+dx)}{d} \right) \right] \right) / (24d^9 E^a)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(-b\*x-a)\*(b\*x+a)^4/(d\*x+c)^5,x)

[Out] 
$$-1/b * (-6 * (a*d-b*c)^2 / d^7 * b^5 * (-1/2 * \exp(-b*x-a) / (-b*x-a + (a*d-b*c)/d)^2 - 1/2 * \exp(-b*x-a) / (-b*x-a + (a*d-b*c)/d) - 1/2 * \exp(-\frac{a*d-b*c}{d}) * \text{Ei}(1, b*x+a - \frac{a*d-b*c}{d})) + b^5 / d^5 * \exp(-\frac{a*d-b*c}{d}) * \text{Ei}(1, b*x+a - \frac{a*d-b*c}{d}) - (a*d-b*c)^4 / d^9 * b^5 * (-1/4 * \exp(-b*x-a) / (-b*x-a + (a*d-b*c)/d)^4 - 1/12 * \exp(-b*x-a) / (-b*x-a + (a*d-b*c)/d)^3 - 1/24 * \exp(-b*x-a) / (-b*x-a + (a*d-b*c)/d)^2 - 1/24 * \exp(-b*x-a) / (-b*x-a + (a*d-b*c)/d) - 1/24 * \exp(-\frac{a*d-b*c}{d}) * \text{Ei}(1, b*x+a - \frac{a*d-b*c}{d})) + 4 * (a*d-b*c)^3 / d^8 * b^5 * (-1/3 * \exp(-b*x-a) / (-b*x-a + (a*d-b*c)/d)^3 - 1/6 * \exp(-b*x-a) / (-b*x-a + (a*d-b*c)/d)^2 - 1/6 * \exp(-b*x-a) / (-b*x-a + (a*d-b*c)/d) - 1/6 * \exp(-\frac{a*d-b*c}{d}) * \text{Ei}(1, b*x+a - \frac{a*d-b*c}{d})) + 4 * (a*d-b*c) / d^6 * b^5 * (-\exp(-b*x-a) / (-b*x-a + (a*d-b*c)/d) - \exp(-\frac{a*d-b*c}{d}) * \text{Ei}(1, b*x+a - \frac{a*d-b*c}{d}))$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{(b^3 d^2 x^4 + (4 a b^2 d^2 - b^2 d^2) x^3 + (6 a^2 b d^2 + 5 b^2 c d - 8 a b d^2 + 2 b d^2) x^2 + (4 a^3 d^2 - 5 b^2 c^2 - 18 a^2 d^2 - 20 b c d + 4 (5 b c d - 4 a^2 d^2)) x + (4 a^3 d^2 - 5 b^2 c^2 - 18 a^2 d^2 - 20 b c d + 4 (5 b c d - 4 a^2 d^2)))}{d^7 x^5 e^a + 5 c d^6 x^4 e^a + 10 c^2 d^5 x^3 e^a + 10 c^3 d^4 x^2 e^a + 5 c^4 d^3 x e^a + c^5 d^2 e^a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(-b\*x-a)\*(b\*x+a)^4/(d\*x+c)^5,x, algorithm="maxima")

[Out] 
$$-(b^3 d^2 x^4 + (4 a b^2 d^2 - b^2 d^2) x^3 + (6 a^2 b d^2 + 5 b^2 c d - 8 a b d^2 + 2 b d^2) x^2 + (4 a^3 d^2 - 5 b^2 c^2 - 18 a^2 d^2 - 20 b c d + 4 (5 b c d - 4 a^2 d^2))) / (d^7 x^5 e^a + 5 c d^6 x^4 e^a + 10 c^2 d^5 x^3 e^a + 10 c^3 d^4 x^2 e^a + 5 c^4 d^3 x e^a + c^5 d^2 e^a)$$

$$\begin{aligned} &*(5*b*c*d + 6*d^2)*a - 6*d^2)*x)*e^{(-b*x)/(d^7*x^5*e^a + 5*c*d^6*x^4*e^a + \\ &10*c^2*d^5*x^3*e^a + 10*c^3*d^4*x^2*e^a + 5*c^4*d^3*x*e^a + c^5*d^2*e^a) - \\ &a^4*e^{(-a + b*c/d)*\exp\_integral\_e(5, (d*x + c)*b/d)/((d*x + c)^4*d) - \text{integ} \\ &\text{rate}(-(4*a^3*c*d^2 - 5*b^2*c^3 - 18*a^2*c*d^2 - 20*b*c^2*d - 6*c*d^2 + 4*(5 \\ &*b*c^2*d + 6*c*d^2)*a + (5*b^3*c^3 - 16*a^3*d^3 + 50*b^2*c^2*d + 90*b*c*d^2 \\ &+ 6*(5*b*c*d^2 + 12*d^3)*a^2 + 24*d^3 - 4*(5*b^2*c^2*d + 30*b*c*d^2 + 24*d \\ &^3)*a)*x)*e^{(-b*x)/(d^8*x^6*e^a + 6*c*d^7*x^5*e^a + 15*c^2*d^6*x^4*e^a + 20 \\ &*c^3*d^5*x^3*e^a + 15*c^4*d^4*x^2*e^a + 6*c^5*d^3*x*e^a + c^6*d^2*e^a), x) \end{aligned}$$

**Fricas [B]** time = 1.64307, size = 2442, normalized size = 4.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(-b\*x-a)\*(b\*x+a)^4/(d\*x+c)^5,x, algorithm="fricas")

[Out]  $\frac{1}{24}*((b^8*c^8 - 4*(a - 4)*b^7*c^7*d + 6*(a^2 - 8*a + 12)*b^6*c^6*d^2 - 4*(a^3 - 12*a^2 + 36*a - 24)*b^5*c^5*d^3 + (a^4 - 16*a^3 + 72*a^2 - 96*a + 24)*b^4*c^4*d^4 + (b^8*c^4*d^4 - 4*(a - 4)*b^7*c^3*d^5 + 6*(a^2 - 8*a + 12)*b^6*c^2*d^6 - 4*(a^3 - 12*a^2 + 36*a - 24)*b^5*c*d^7 + (a^4 - 16*a^3 + 72*a^2 - 96*a + 24)*b^4*d^8)*x^4 + 4*(b^8*c^5*d^3 - 4*(a - 4)*b^7*c^4*d^4 + 6*(a^2 - 8*a + 12)*b^6*c^3*d^5 - 4*(a^3 - 12*a^2 + 36*a - 24)*b^5*c^2*d^6 + (a^4 - 16*a^3 + 72*a^2 - 96*a + 24)*b^4*c*d^7)*x^3 + 6*(b^8*c^6*d^2 - 4*(a - 4)*b^7*c^5*d^3 + 6*(a^2 - 8*a + 12)*b^6*c^4*d^4 - 4*(a^3 - 12*a^2 + 36*a - 24)*b^5*c^3*d^5 + (a^4 - 16*a^3 + 72*a^2 - 96*a + 24)*b^4*c^2*d^6)*x^2 + 4*(b^8*c^7*d - 4*(a - 4)*b^7*c^6*d^2 + 6*(a^2 - 8*a + 12)*b^6*c^5*d^3 - 4*(a^3 - 12*a^2 + 36*a - 24)*b^5*c^4*d^4 + (a^4 - 16*a^3 + 72*a^2 - 96*a + 24)*b^4*c^3*d^5)*x)*\text{Ei}(-(b*d*x + b*c)/d)*e^{((b*c - a*d)/d) + (b^7*c^7*d - (4*a - 15)*b^6*c^6*d^2 + 2*(3*a^2 - 22*a + 29)*b^5*c^5*d^3 - 2*(2*a^3 - 21*a^2 + 52*a - 25)*b^4*c^4*d^4 + (a^4 - 12*a^3 + 36*a^2 - 24*a)*b^3*c^3*d^5 - 6*a^4*d^8 - (a^4 - 8*a^3 + 12*a^2)*b^2*c^2*d^6 + 2*(a^4 - 4*a^3)*b*c*d^7 + (b^7*c^4*d^4 - 4*(a - 4)*b^6*c^3*d^5 + 6*(a^2 - 8*a + 12)*b^5*c^2*d^6 - 4*(a^3 - 12*a^2 + 36*a - 24)*b^4*c*d^7 + (a^4 - 16*a^3 + 72*a^2 - 96*a)*b^3*d^8)*x^3 + (3*b^7*c^5*d^3 - (12*a - 47)*b^6*c^4*d^4 + 2*(9*a^2 - 70*a + 100)*b^5*c^3*d^5 - 6*(2*a^3 - 23*a^2 + 64*a - 36)*b^4*c^2*d^6 + (3*a^4 - 44*a^3 + 168*a^2 - 144*a)*b^3*c*d^7 - (a^4 - 16*a^3 + 72*a^2)*b^2*d^8)*x^2 + (3*b^7*c^6*d^2 - 2*(6*a - 23)*b^6*c^5*d^3 + 2*(9*a^2 - 68*a + 93)*b^5*c^4*d^4 - 4*(3*a^3 - 33*a^2 + 86*a - 44)*b^4*c^3*d^5 + (3*a^4 - 40*a^3 + 132*a^2 - 96*a)*b^3*c^2*d^6 - 2*(a^4 - 12*a^3 + 24*a^2)*b^2*c*d^7 + 2*(a^4 - 16*a^3)*b*d^8)*x)*e^{(-b*x - a)}/(d^13*x^4 + 4*c*d^12*x^3 + 6*c^2*d^11*x^2 + 4*c^3*d^10*x + c^4*d^9)$



---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(-b\*x-a)\*(b\*x+a)\*\*4/(d\*x+c)\*\*5,x)

[Out] Timed out

---

**Giac [B]** time = 1.51575, size = 6118, normalized size = 10.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(-b\*x-a)\*(b\*x+a)^4/(d\*x+c)^5,x, algorithm="giac")

[Out] 
$$\begin{aligned} & 1/24*(b^8*c^4*d^4*x^4*Ei(-(b*d*x + b*c)/d)*e^{-a + b*c/d} - 4*a*b^7*c^3*d^5 \\ & *x^4*Ei(-(b*d*x + b*c)/d)*e^{-a + b*c/d} + 6*a^2*b^6*c^2*d^6*x^4*Ei(-(b*d*x \\ & + b*c)/d)*e^{-a + b*c/d} - 4*a^3*b^5*c*d^7*x^4*Ei(-(b*d*x + b*c)/d)*e^{-a \\ & + b*c/d} + a^4*b^4*d^8*x^4*Ei(-(b*d*x + b*c)/d)*e^{-a + b*c/d} + 4*b^8*c^5 \\ & d^3*x^3*Ei(-(b*d*x + b*c)/d)*e^{-a + b*c/d} - 16*a*b^7*c^4*d^4*x^3*Ei(-(b*d \\ & *x + b*c)/d)*e^{-a + b*c/d} + 24*a^2*b^6*c^3*d^5*x^3*Ei(-(b*d*x + b*c)/d)*e \\ & ^{-a + b*c/d} - 16*a^3*b^5*c^2*d^6*x^3*Ei(-(b*d*x + b*c)/d)*e^{-a + b*c/d} \\ & + 4*a^4*b^4*c*d^7*x^3*Ei(-(b*d*x + b*c)/d)*e^{-a + b*c/d} + 16*b^7*c^3*d^5* \\ & x^4*Ei(-(b*d*x + b*c)/d)*e^{-a + b*c/d} - 48*a*b^6*c^2*d^6*x^4*Ei(-(b*d*x + \\ & b*c)/d)*e^{-a + b*c/d} + 48*a^2*b^5*c*d^7*x^4*Ei(-(b*d*x + b*c)/d)*e^{-a + \\ & b*c/d} - 16*a^3*b^4*d^8*x^4*Ei(-(b*d*x + b*c)/d)*e^{-a + b*c/d} + 6*b^8*c^ \\ & 6*d^2*x^2*Ei(-(b*d*x + b*c)/d)*e^{-a + b*c/d} - 24*a*b^7*c^5*d^3*x^2*Ei(-(b \\ & *d*x + b*c)/d)*e^{-a + b*c/d} + 36*a^2*b^6*c^4*d^4*x^2*Ei(-(b*d*x + b*c)/d) \\ & *e^{-a + b*c/d} - 24*a^3*b^5*c^3*d^5*x^2*Ei(-(b*d*x + b*c)/d)*e^{-a + b*c/d} \\ & ) + 6*a^4*b^4*c^2*d^6*x^2*Ei(-(b*d*x + b*c)/d)*e^{-a + b*c/d} + 64*b^7*c^4* \\ & d^4*x^3*Ei(-(b*d*x + b*c)/d)*e^{-a + b*c/d} - 192*a*b^6*c^3*d^5*x^3*Ei(-(b* \\ & d*x + b*c)/d)*e^{-a + b*c/d} + 192*a^2*b^5*c^2*d^6*x^3*Ei(-(b*d*x + b*c)/d) \\ & *e^{-a + b*c/d} - 64*a^3*b^4*c*d^7*x^3*Ei(-(b*d*x + b*c)/d)*e^{-a + b*c/d} \\ & + 72*b^6*c^2*d^6*x^4*Ei(-(b*d*x + b*c)/d)*e^{-a + b*c/d} - 144*a*b^5*c*d^7* \\ & x^4*Ei(-(b*d*x + b*c)/d)*e^{-a + b*c/d} + 72*a^2*b^4*d^8*x^4*Ei(-(b*d*x + b \\ & *c)/d)*e^{-a + b*c/d} + b^7*c^4*d^4*x^3*e^{-b*x - a} - 4*a*b^6*c^3*d^5*x^3* \\ & e^{-b*x - a} + 6*a^2*b^5*c^2*d^6*x^3*e^{-b*x - a} - 4*a^3*b^4*c*d^7*x^3*e^{-} \end{aligned}$$

$$\begin{aligned}
& -b*x - a) + a^4*b^3*d^8*x^3*e^{(-b*x - a)} + 4*b^8*c^7*d*x*Ei(-(b*d*x + b*c)/d)*e^{(-a + b*c/d)} - 16*a*b^7*c^6*d^2*x*Ei(-(b*d*x + b*c)/d)*e^{(-a + b*c/d)} \\
& + 24*a^2*b^6*c^5*d^3*x*Ei(-(b*d*x + b*c)/d)*e^{(-a + b*c/d)} - 16*a^3*b^5*c^4*d^4*x*Ei(-(b*d*x + b*c)/d)*e^{(-a + b*c/d)} + 4*a^4*b^4*c^3*d^5*x*Ei(-(b*d*x + b*c)/d)*e^{(-a + b*c/d)} + 96*b^7*c^5*d^3*x^2*Ei(-(b*d*x + b*c)/d)*e^{(-a + b*c/d)} - 288*a*b^6*c^4*d^4*x^2*Ei(-(b*d*x + b*c)/d)*e^{(-a + b*c/d)} + 288*a^2*b^5*c^3*d^5*x^2*Ei(-(b*d*x + b*c)/d)*e^{(-a + b*c/d)} - 96*a^3*b^4*c^2*d^6*x^2*Ei(-(b*d*x + b*c)/d)*e^{(-a + b*c/d)} + 288*b^6*c^3*d^5*x^3*Ei(-(b*d*x + b*c)/d)*e^{(-a + b*c/d)} - 576*a*b^5*c^2*d^6*x^3*Ei(-(b*d*x + b*c)/d)*e^{(-a + b*c/d)} + 288*a^2*b^4*c*d^7*x^3*Ei(-(b*d*x + b*c)/d)*e^{(-a + b*c/d)} + 96*b^5*c*d^7*x^4*Ei(-(b*d*x + b*c)/d)*e^{(-a + b*c/d)} - 96*a*b^4*d^8*x^4*Ei(-(b*d*x + b*c)/d)*e^{(-a + b*c/d)} + 3*b^7*c^5*d^3*x^2*e^{(-b*x - a)} - 12*a*b^6*c^4*d^4*x^2*e^{(-b*x - a)} + 18*a^2*b^5*c^3*d^5*x^2*e^{(-b*x - a)} - 12*a^3*b^4*c^2*d^6*x^2*e^{(-b*x - a)} + 3*a^4*b^3*c*d^7*x^2*e^{(-b*x - a)} + 16*b^6*c^3*d^5*x^3*e^{(-b*x - a)} - 48*a*b^5*c^2*d^6*x^3*e^{(-b*x - a)} + 48*a^2*b^4*c*d^7*x^3*e^{(-b*x - a)} - 16*a^3*b^3*d^8*x^3*e^{(-b*x - a)} + b^8*c^8*Ei(-(b*d*x + b*c)/d)*e^{(-a + b*c/d)} - 4*a*b^7*c^7*d*Ei(-(b*d*x + b*c)/d)*e^{(-a + b*c/d)} + 6*a^2*b^6*c^6*d^2*Ei(-(b*d*x + b*c)/d)*e^{(-a + b*c/d)} - 4*a^3*b^5*c^5*d^3*Ei(-(b*d*x + b*c)/d)*e^{(-a + b*c/d)} + a^4*b^4*c^4*d^4*Ei(-(b*d*x + b*c)/d)*e^{(-a + b*c/d)} + 64*b^7*c^6*d^2*x*Ei(-(b*d*x + b*c)/d)*e^{(-a + b*c/d)} - 192*a*b^6*c^5*d^3*x*Ei(-(b*d*x + b*c)/d)*e^{(-a + b*c/d)} + 192*a^2*b^5*c^4*d^4*x*Ei(-(b*d*x + b*c)/d)*e^{(-a + b*c/d)} - 64*a^3*b^4*c^3*d^5*x*Ei(-(b*d*x + b*c)/d)*e^{(-a + b*c/d)} + 432*b^6*c^4*d^4*x^2*Ei(-(b*d*x + b*c)/d)*e^{(-a + b*c/d)} - 864*a*b^5*c^3*d^5*x^2*Ei(-(b*d*x + b*c)/d)*e^{(-a + b*c/d)} + 432*a^2*b^4*c^2*d^6*x^2*Ei(-(b*d*x + b*c)/d)*e^{(-a + b*c/d)} + 384*b^5*c^2*d^6*x^3*Ei(-(b*d*x + b*c)/d)*e^{(-a + b*c/d)} - 384*a*b^4*c*d^7*x^3*Ei(-(b*d*x + b*c)/d)*e^{(-a + b*c/d)} + 24*b^4*d^8*x^4*Ei(-(b*d*x + b*c)/d)*e^{(-a + b*c/d)} + 3*b^7*c^6*d^2*x*e^{(-b*x - a)} - 12*a*b^6*c^5*d^3*x*e^{(-b*x - a)} + 18*a^2*b^5*c^4*d^4*x*e^{(-b*x - a)} - 12*a^3*b^4*c^3*d^5*x*e^{(-b*x - a)} + 3*a^4*b^3*c^2*d^6*x*e^{(-b*x - a)} + 47*b^6*c^4*d^4*x^2*e^{(-b*x - a)} - 140*a*b^5*c^3*d^5*x^2*e^{(-b*x - a)} + 138*a^2*b^4*c^2*d^6*x^2*e^{(-b*x - a)} - 44*a^3*b^3*c*d^7*x^2*e^{(-b*x - a)} - a^4*b^2*d^8*x^2*e^{(-b*x - a)} + 72*b^5*c^2*d^6*x^3*e^{(-b*x - a)} - 144*a*b^4*c*d^7*x^3*e^{(-b*x - a)} + 72*a^2*b^3*d^8*x^3*e^{(-b*x - a)} + 16*b^7*c^7*d*Ei(-(b*d*x + b*c)/d)*e^{(-a + b*c/d)} - 48*a*b^6*c^6*d^2*Ei(-(b*d*x + b*c)/d)*e^{(-a + b*c/d)} + 48*a^2*b^5*c^5*d^3*Ei(-(b*d*x + b*c)/d)*e^{(-a + b*c/d)} - 16*a^3*b^4*c^4*d^4*Ei(-(b*d*x + b*c)/d)*e^{(-a + b*c/d)} + 288*b^6*c^5*d^3*x*Ei(-(b*d*x + b*c)/d)*e^{(-a + b*c/d)} - 576*a*b^5*c^4*d^4*x*Ei(-(b*d*x + b*c)/d)*e^{(-a + b*c/d)} + 288*a^2*b^4*c^3*d^5*x*Ei(-(b*d*x + b*c)/d)*e^{(-a + b*c/d)} + 576*b^5*c^3*d^5*x^2*Ei(-(b*d*x + b*c)/d)*e^{(-a + b*c/d)} - 576*a*b^4*c^2*d^6*x^2*Ei(-(b*d*x + b*c)/d)*e^{(-a + b*c/d)} + 96*b^4*c*d^7*x^3*Ei(-(b*d*x + b*c)/d)*e^{(-a + b*c/d)} + b^7*c^7*d*e^{(-b*x - a)} - 4*a*b^6*c^6*d^2*e^{(-b*x - a)} + 6*a^2*b^5*c^5*d^3*e^{(-b*x - a)} - 4*a^3*b^4*c^4*d^4*e^{(-b*x - a)} + a^4*b^3*c^3*d^5*e^{(-b*x - a)} + 46*b^6*c^5*d^3*x*e^{(-b*x - a)} - 136*a*b^5*c^4*d^4*x*e^{(-b*x - a)} + 132*a^2*b^4*c^3*d^5*x*e^{(-b*x - a)} - 40*a^3*b^3*c^2*d^6*x*e^{(-b*x - a)} - 2*a^4*b^2*c*d^7*x*e^{(-b*x - a)} + 200*b^5*c
\end{aligned}$$

$$\begin{aligned}
&^3d^5x^2e^{(-bx - a)} - 384a^2b^4c^2d^6x^2e^{(-bx - a)} + 168a^2b^3c^2d^7x^2e^{(-bx - a)} + 16a^3b^2d^8x^2e^{(-bx - a)} + 96b^4c^2d^7x^3 \\
&e^{(-bx - a)} - 96a^2b^3d^8x^3e^{(-bx - a)} + 72b^6c^6d^2\text{Ei}(-(b^2dx + bc)/d)e^{(-a + bc/d)} - 144a^2b^5c^5d^3\text{Ei}(-(b^2dx + bc)/d)e^{(-a + bc/d)} \\
&+ 72a^2b^4c^4d^4\text{Ei}(-(b^2dx + bc)/d)e^{(-a + bc/d)} + 384b^5c^4d^4x\text{Ei}(-(b^2dx + bc)/d)e^{(-a + bc/d)} - 384a^2b^4c^3d^5x\text{Ei}(-(b^2dx + bc)/d)e^{(-a + bc/d)} \\
&+ 144b^4c^2d^6x^2\text{Ei}(-(b^2dx + bc)/d)e^{(-a + bc/d)} + 15b^6c^6d^2e^{(-bx - a)} - 44a^2b^5c^5d^3e^{(-bx - a)} + 42a^2b^4c^4d^4e^{(-bx - a)} - 12a^3b^3c^3d^5e^{(-bx - a)} - a^4b^2c^2d^6e^{(-bx - a)} \\
&+ 186b^5c^4d^4x\text{Ei}(-(b^2dx + bc)/d)e^{(-a + bc/d)} - 344a^2b^4c^3d^5x\text{Ei}(-(b^2dx + bc)/d)e^{(-a + bc/d)} + 132a^2b^3c^2d^6x\text{Ei}(-(b^2dx + bc)/d)e^{(-a + bc/d)} \\
&+ 24a^3b^2c^2d^7x\text{Ei}(-(b^2dx + bc)/d)e^{(-a + bc/d)} + 2a^4b^2d^8x\text{Ei}(-(b^2dx + bc)/d)e^{(-a + bc/d)} + 216b^4c^2d^6x^2e^{(-bx - a)} - 144a^2b^3c^2d^7x^2e^{(-bx - a)} \\
&- 72a^2b^2d^8x^2e^{(-bx - a)} + 96b^5c^5d^3\text{Ei}(-(b^2dx + bc)/d)e^{(-a + bc/d)} - 96a^2b^4c^4d^4\text{Ei}(-(b^2dx + bc)/d)e^{(-a + bc/d)} + 96b^4c^3d^5x\text{Ei}(-(b^2dx + bc)/d)e^{(-a + bc/d)} \\
&+ 58b^5c^5d^3e^{(-bx - a)} - 104a^2b^4c^4d^4e^{(-bx - a)} + 36a^2b^3c^3d^5e^{(-bx - a)} + 8a^3b^2c^2d^6e^{(-bx - a)} + 2a^4b^2c^2d^7e^{(-bx - a)} + 176b^4c^3d^5x\text{Ei}(-(b^2dx + bc)/d)e^{(-a + bc/d)} \\
&- 96a^2b^3c^2d^6x\text{Ei}(-(b^2dx + bc)/d)e^{(-a + bc/d)} - 48a^2b^2c^2d^7x\text{Ei}(-(b^2dx + bc)/d)e^{(-a + bc/d)} - 32a^3b^2d^8x\text{Ei}(-(b^2dx + bc)/d)e^{(-a + bc/d)} + 24b^4c^4d^4\text{Ei}(-(b^2dx + bc)/d)e^{(-a + bc/d)} \\
&+ 50b^4c^4d^4e^{(-bx - a)} - 24a^2b^3c^3d^5e^{(-bx - a)} - 12a^2b^2c^2d^6e^{(-bx - a)} - 8a^3b^2c^2d^7e^{(-bx - a)} - 6a^4d^8e^{(-bx - a)} \\
&)/(d^{13}x^4 + 4c^2d^{12}x^3 + 6c^2d^{11}x^2 + 4c^3d^{10}x + c^4d^9)
\end{aligned}$$

$$3.83 \quad \int F^{c(a+bx)} x^m \log^n(dx) (e + en + e(1+m+bcx \log(F)) \log(dx)) dx$$

**Optimal.** Leaf size=24

$$e x^{m+1} \log^{n+1}(dx) F^{c(a+bx)}$$

[Out]  $e F^{c(a + b x)} x^{1 + m} \text{Log}[d x]^{1 + n}$

**Rubi [A]** time = 0.145645, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 39,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.026$ , Rules used = {2202}

$$e x^{m+1} \log^{n+1}(dx) F^{c(a+bx)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[F^{c(a + b x)} x^m \text{Log}[d x]^n (e + e n + e(1 + m + b c x \text{Log}[F])) \text{Log}[d x], x]$

[Out]  $e F^{c(a + b x)} x^{1 + m} \text{Log}[d x]^{1 + n}$

**Rule 2202**

$\text{Int}[\text{Log}[(d_.)(x_.)]^{(n_.)}(F_.)^{((c_.)((a_.) + (b_.)(x_.)))(x_.)^{(m_.)}((e_.) + \text{Log}[(d_.)(x_.)](h_.)((f_.) + (g_.)(x_.))), x\_Symbol] :> \text{Simp}[(e x^{m+1} F^{c(a+bx)} \text{Log}[d x]^{n+1}) / (n+1), x] /; \text{FreeQ}\{F, a, b, c, d, e, f, g, h, m, n\}, x] \&\& \text{EqQ}[e(m+1) - f h(n+1), 0] \&\& \text{EqQ}[g h(n+1) - b c e \text{Log}[F], 0] \&\& \text{NeQ}[n, -1]$

**Rubi steps**

$$\int F^{c(a+bx)} x^m \log^n(dx) (e + en + e(1+m+bcx \log(F)) \log(dx)) dx = e F^{c(a+bx)} x^{1+m} \log^{1+n}(dx)$$

**Mathematica [A]** time = 0.366007, size = 24, normalized size = 1.

$$e x^{m+1} \log^{n+1}(dx) F^{c(a+bx)}$$

Antiderivative was successfully verified.

[In] Integrate[F^(c\*(a + b\*x))\*x^m\*Log[d\*x]^n\*(e + e\*n + e\*(1 + m + b\*c\*x\*Log[F])  
)\*Log[d\*x]], x]

[Out] e\*F^(c\*(a + b\*x))\*x^(1 + m)\*Log[d\*x]^(1 + n)

**Maple [C]** time = 0.2, size = 192, normalized size = 8.

$$\frac{(2exF^{c(bx+a)} \ln(x) - ixF^{c(bx+a)} e\pi \operatorname{csgn}(id) \operatorname{csgn}(ix) \operatorname{csgn}(idx) + ixF^{c(bx+a)} e\pi \operatorname{csgn}(id) (\operatorname{csgn}(idx))^2 + ixF^{c(bx+a)} e\pi \operatorname{csgn}(id) \operatorname{csgn}(ix) \operatorname{csgn}(idx))}{\dots}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(c\*(b\*x+a))\*x^m\*ln(d\*x)^n\*(e+e\*n+e\*(1+m+b\*c\*x\*ln(F))\*ln(d\*x)), x)

[Out]  $\frac{1}{2} * (2 * e * x * F^{c(b*x+a)} * \ln(x) - I * x * F^{c(b*x+a)} * e * \operatorname{Pi} * \operatorname{csgn}(I*d) * \operatorname{csgn}(I*x) * \operatorname{csgn}(I*d*x) + I * x * F^{c(b*x+a)} * e * \operatorname{Pi} * \operatorname{csgn}(I*d) * \operatorname{csgn}(I*d*x)^2 + I * x * F^{c(b*x+a)} * e * \operatorname{Pi} * \operatorname{csgn}(I*x) * \operatorname{csgn}(I*d*x)^2 + 2 * x * F^{c(b*x+a)} * e * \ln(d) - I * x * F^{c(b*x+a)} * e * \operatorname{Pi} * \operatorname{csgn}(I*d*x)^3) * x^m * (\ln(d) + \ln(x) - 1/2 * I * \operatorname{Pi} * \operatorname{csgn}(I*d*x) * (-\operatorname{csgn}(I*d*x) + \operatorname{csgn}(I*d)) * (-\operatorname{csgn}(I*d*x) + \operatorname{csgn}(I*x)))^n$

**Maxima [A]** time = 1.44309, size = 57, normalized size = 2.38

$$(F^{ac} ex \log(d) + F^{ac} ex \log(x)) e^{(bcx \log(F) + m \log(x) + n \log(\log(d) + \log(x)))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(c\*(b\*x+a))\*x^m\*log(d\*x)^n\*(e+e\*n+e\*(1+m+b\*c\*x\*log(F))\*log(d\*x)), x, algorithm="maxima")

[Out]  $(F^{a*c} * e * x * \log(d) + F^{a*c} * e * x * \log(x)) * e^{(b*c*x*\log(F) + m*\log(x) + n*\log(\log(d) + \log(x)))}$

**Fricas [A]** time = 1.55793, size = 90, normalized size = 3.75

$$(ex \log(d) + ex \log(x)) F^{bcx+ac} x^m (\log(d) + \log(x))^n$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*x^m*log(d*x)^n*(e+e*n+e*(1+m+b*c*x*log(F))*log(d*x)
),x, algorithm="fricas")
```

```
[Out] (e*x*log(d) + e*x*log(x))*F^(b*c*x + a*c)*x^m*(log(d) + log(x))^n
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(c*(b*x+a))*x**m*ln(d*x)**n*(e+e*n+e*(1+m+b*c*x*ln(F))*ln(d*x)
),x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int ((bcx \log(F) + m + 1)e \log(dx) + en + e)F^{(bx+a)c}x^m \log(dx)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*x^m*log(d*x)^n*(e+e*n+e*(1+m+b*c*x*log(F))*log(d*x)
),x, algorithm="giac")
```

```
[Out] integrate(((b*c*x*log(F) + m + 1)*e*log(d*x) + e*n + e)*F^((b*x + a)*c)*x^m
*log(d*x)^n, x)
```

$$3.84 \quad \int F^{c(a+bx)} x^2 \log^n(dx) (e + en + e(3 + bcx \log(F)) \log(dx)) dx$$

Optimal. Leaf size=22

$$ex^3 \log^{n+1}(dx) F^{c(a+bx)}$$

[Out]  $e F^{c(a + b x)} x^3 \text{Log}[d x]^{(1 + n)}$

**Rubi [A]** time = 0.130544, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 38,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.026$ , Rules used = {2202}

$$ex^3 \log^{n+1}(dx) F^{c(a+bx)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[F^{c(a + b x)} x^2 \text{Log}[d x]^n (e + e n + e(3 + b c x \text{Log}[F]) \text{Log}[d x]), x]$

[Out]  $e F^{c(a + b x)} x^3 \text{Log}[d x]^{(1 + n)}$

Rule 2202

$\text{Int}[\text{Log}[(d \cdot) (x \cdot)]^{(n \cdot)} (F \cdot)^{((c \cdot) ((a \cdot) + (b \cdot) (x \cdot)))} (x \cdot)^{(m \cdot)} ((e \cdot) + \text{Log}[(d \cdot) (x \cdot)] (h \cdot) ((f \cdot) + (g \cdot) (x \cdot))), x\_Symbol] \rightarrow \text{Simp}[(e x^{(m + 1)} F^{c(a + b x)} \text{Log}[d x]^{(n + 1)}) / (n + 1), x] /;$  FreeQ[{F, a, b, c, d, e, f, g, h, m, n}, x] && EqQ[e\*(m + 1) - f\*h\*(n + 1), 0] && EqQ[g\*h\*(n + 1) - b\*c\*e\*Log[F], 0] && NeQ[n, -1]

Rubi steps

$$\int F^{c(a+bx)} x^2 \log^n(dx) (e + en + e(3 + bcx \log(F)) \log(dx)) dx = e F^{c(a+bx)} x^3 \log^{1+n}(dx)$$

**Mathematica [A]** time = 0.298562, size = 23, normalized size = 1.05

$$ex^3 \log^{n+1}(dx) F^{ac+bcx}$$

Antiderivative was successfully verified.

[In] Integrate[F^(c\*(a + b\*x))\*x^2\*Log[d\*x]^n\*(e + e\*n + e\*(3 + b\*c\*x\*Log[F])\*Log[d\*x]), x]

[Out] e\*F^(a\*c + b\*c\*x)\*x^3\*Log[d\*x]^(1 + n)

**Maple [C]** time = 0.098, size = 198, normalized size = 9.

$$\left(-\frac{i}{2}\pi ex^3 \operatorname{csgn}(id) \operatorname{csgn}(ix) \operatorname{csgn}(idx) F^{c(bx+a)} + \frac{i}{2}\pi ex^3 \operatorname{csgn}(id) (\operatorname{csgn}(idx))^2 F^{c(bx+a)} + \frac{i}{2}\pi ex^3 \operatorname{csgn}(ix) (\operatorname{csgn}(idx))^2 F^{c(bx+a)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(c\*(b\*x+a))\*x^2\*ln(d\*x)^n\*(e+e\*n+e\*(3+b\*c\*x\*ln(F))\*ln(d\*x)), x)

[Out] (-1/2\*I\*Pi\*e\*x^3\*csgn(I\*d)\*csgn(I\*x)\*csgn(I\*d\*x)\*F^(c\*(b\*x+a))+1/2\*I\*Pi\*e\*x^3\*csgn(I\*d)\*csgn(I\*d\*x)^2\*F^(c\*(b\*x+a))+1/2\*I\*Pi\*e\*x^3\*csgn(I\*x)\*csgn(I\*d\*x)^2\*F^(c\*(b\*x+a))-1/2\*I\*Pi\*e\*x^3\*csgn(I\*d\*x)^3\*F^(c\*(b\*x+a))+ln(d)\*e\*x^3\*F^(c\*(b\*x+a))+e\*x^3\*F^(c\*(b\*x+a))\*ln(x))\*(ln(d)+ln(x)-1/2\*I\*Pi\*csgn(I\*d\*x)\*(-csgn(I\*d\*x)+csgn(I\*d))\*(-csgn(I\*d\*x)+csgn(I\*x)))^n

**Maxima [A]** time = 1.37368, size = 57, normalized size = 2.59

$$\left(F^{ac} ex^3 \log(d) + F^{ac} ex^3 \log(x)\right) e^{(bcx \log(F) + n \log(\log(d) + \log(x)))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(c\*(b\*x+a))\*x^2\*log(d\*x)^n\*(e+e\*n+e\*(3+b\*c\*x\*log(F))\*log(d\*x)), x, algorithm="maxima")

[Out] (F^(a\*c)\*e\*x^3\*log(d) + F^(a\*c)\*e\*x^3\*log(x))\*e^(b\*c\*x\*log(F) + n\*log(log(d) + log(x)))

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.



```
[In] integrate(F^(c*(b*x+a))*x^2*log(d*x)^n*(e+e*n+e*(3+b*c*x*log(F))*log(d*x)),
x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(c*(b*x+a))*x**2*ln(d*x)**n*(e+e*n+e*(3+b*c*x*ln(F))*ln(d*x)),
x)
```

```
[Out] Timed out
```

**Giac [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*x^2*log(d*x)^n*(e+e*n+e*(3+b*c*x*log(F))*log(d*x)),
x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError
```

$$3.85 \quad \int F^{c(a+bx)} x \log^n(dx) (e + en + e(2 + bcx \log(F)) \log(dx)) dx$$

**Optimal.** Leaf size=22

$$ex^2 \log^{n+1}(dx) F^{c(a+bx)}$$

[Out]  $e F^{c(a + b x)} x^2 \text{Log}[d x]^{(1 + n)}$

**Rubi [A]** time = 0.0901636, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 36,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.028$ , Rules used = {2202}

$$ex^2 \log^{n+1}(dx) F^{c(a+bx)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[F^{c(a + b x)} x \text{Log}[d x]^n (e + e n + e(2 + b c x \text{Log}[F]) \text{Log}[d x]), x]$

[Out]  $e F^{c(a + b x)} x^2 \text{Log}[d x]^{(1 + n)}$

#### Rule 2202

$\text{Int}[\text{Log}[(d_.)(x_.)]^{(n_.)}(F_.)^{((c_.)((a_.) + (b_.)(x_.)))(x_.)^{(m_.)}((e_.) + \text{Log}[(d_.)(x_.)](h_.)((f_.) + (g_.)(x_.))), x\_Symbol] :> \text{Simp}[(e x^{(m + 1)} F^{c(a + b x)} \text{Log}[d x]^{(n + 1)}) / (n + 1), x] /; \text{FreeQ}\{F, a, b, c, d, e, f, g, h, m, n\}, x] \&\& \text{EqQ}[e^{(m + 1)} - f h^{(n + 1)}, 0] \&\& \text{EqQ}[g h^{(n + 1)} - b c e \text{Log}[F], 0] \&\& \text{NeQ}[n, -1]$

#### Rubi steps

$$\int F^{c(a+bx)} x \log^n(dx) (e + en + e(2 + bcx \log(F)) \log(dx)) dx = e F^{c(a+bx)} x^2 \log^{1+n}(dx)$$

**Mathematica [A]** time = 0.269473, size = 23, normalized size = 1.05

$$ex^2 \log^{n+1}(dx) F^{ac+bcx}$$

Antiderivative was successfully verified.

[In] Integrate[F^(c\*(a + b\*x))\*x\*Log[d\*x]^n\*(e + e\*n + e\*(2 + b\*c\*x\*Log[F])\*Log[d\*x]),x]

[Out] e\*F^(a\*c + b\*c\*x)\*x^2\*Log[d\*x]^(1 + n)

**Maple [C]** time = 0.097, size = 198, normalized size = 9.

$$\left(-\frac{i}{2}\pi ex^2 \operatorname{csgn}(id) \operatorname{csgn}(ix) \operatorname{csgn}(idx) F^{c(bx+a)} + \frac{i}{2}\pi ex^2 \operatorname{csgn}(id) (\operatorname{csgn}(idx))^2 F^{c(bx+a)} + \frac{i}{2}\pi ex^2 \operatorname{csgn}(ix) (\operatorname{csgn}(idx))^2 F^{c(bx+a)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(c\*(b\*x+a))\*x\*ln(d\*x)^n\*(e+e\*n+e\*(2+b\*c\*x\*ln(F))\*ln(d\*x)),x)

[Out]  $(-1/2*I*Pi*e*x^2*\operatorname{csgn}(I*d)*\operatorname{csgn}(I*x)*\operatorname{csgn}(I*d*x)*F^{c*(b*x+a)}+1/2*I*Pi*e*x^2*\operatorname{csgn}(I*d)*\operatorname{csgn}(I*d*x)^2*F^{c*(b*x+a)}+1/2*I*Pi*e*x^2*\operatorname{csgn}(I*x)*\operatorname{csgn}(I*d*x)^2*F^{c*(b*x+a)}-1/2*I*Pi*e*x^2*\operatorname{csgn}(I*d*x)^3*F^{c*(b*x+a)}+\ln(d)*e*x^2*F^{c*(b*x+a)}+e*x^2*F^{c*(b*x+a)}*\ln(x))*(\ln(d)+\ln(x)-1/2*I*Pi*\operatorname{csgn}(I*d*x)*(-\operatorname{csgn}(I*d*x)+\operatorname{csgn}(I*d))*(-\operatorname{csgn}(I*d*x)+\operatorname{csgn}(I*x)))^n$

**Maxima [A]** time = 1.40364, size = 57, normalized size = 2.59

$$(F^{ac}ex^2 \log(d) + F^{ac}ex^2 \log(x))e^{(bcx \log(F)+n \log(\log(d)+\log(x)))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(c\*(b\*x+a))\*x\*log(d\*x)^n\*(e+e\*n+e\*(2+b\*c\*x\*log(F))\*log(d\*x)),x, algorithm="maxima")

[Out] (F^(a\*c)\*e\*x^2\*log(d) + F^(a\*c)\*e\*x^2\*log(x))\*e^(b\*c\*x\*log(F) + n\*log(log(d) + log(x)))

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*x*log(d*x)^n*(e+e*n+e*(2+b*c*x*log(F))*log(d*x)),x,
algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(c*(b*x+a))*x*ln(d*x)**n*(e+e*n+e*(2+b*c*x*ln(F))*ln(d*x)),x)
```

```
[Out] Timed out
```

**Giac [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*x*log(d*x)^n*(e+e*n+e*(2+b*c*x*log(F))*log(d*x)),x,
algorithm="giac")
```

```
[Out] Exception raised: RuntimeError
```

$$3.86 \quad \int F^{c(a+bx)} \log^n(dx) (e + en + e(1 + bcx \log(F)) \log(dx)) dx$$

**Optimal.** Leaf size=20

$$ex \log^{n+1}(dx) F^{c(a+bx)}$$

[Out]  $e F^{c(a + b x)} x \text{Log}[d x]^{(1 + n)}$

**Rubi [A]** time = 0.0464877, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 35,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.029$ , Rules used = {2201}

$$ex \log^{n+1}(dx) F^{c(a+bx)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[F^{c(a + b x)} \text{Log}[d x]^n (e + e n + e (1 + b c x \text{Log}[F]) \text{Log}[d x]), x]$

[Out]  $e F^{c(a + b x)} x \text{Log}[d x]^{(1 + n)}$

#### Rule 2201

$\text{Int}[\text{Log}[(d \cdot) (x \cdot)]^{(n \cdot)} (F \cdot)^{((c \cdot) ((a \cdot) + (b \cdot) (x \cdot)))} ((e \cdot) + \text{Log}[(d \cdot) (x \cdot)] (h \cdot) ((f \cdot) + (g \cdot) (x \cdot))), x\_Symbol] \text{ :> } \text{Simp}[(e \cdot x \cdot F^{c(a + b x)}) \cdot \text{Log}[d x]^{(n + 1)} / (n + 1), x] \text{ /; } \text{FreeQ}\{F, a, b, c, d, e, f, g, h, n\}, x] \ \&\& \ \text{EqQ}[e - f \cdot h \cdot (n + 1), 0] \ \&\& \ \text{EqQ}[g \cdot h \cdot (n + 1) - b \cdot c \cdot e \cdot \text{Log}[F], 0] \ \&\& \ \text{NeQ}[n, -1]$

#### Rubi steps

$$\int F^{c(a+bx)} \log^n(dx) (e + en + e(1 + bcx \log(F)) \log(dx)) dx = e F^{c(a+bx)} x \log^{1+n}(dx)$$

**Mathematica [A]** time = 0.17145, size = 21, normalized size = 1.05

$$ex \log^{n+1}(dx) F^{ac+bcx}$$

Antiderivative was successfully verified.

[In] Integrate[F^(c\*(a + b\*x))\*Log[d\*x]^n\*(e + e\*n + e\*(1 + b\*c\*x\*Log[F])\*Log[d\*x]),x]

[Out] e\*F^(a\*c + b\*c\*x)\*x\*Log[d\*x]^(1 + n)

**Maple [C]** time = 0.108, size = 186, normalized size = 9.3

$$\left(-\frac{i}{2}x^{F^{c(bx+a)}}e^{\pi \operatorname{csgn}(id)} \operatorname{csgn}(ix) \operatorname{csgn}(idx) + \frac{i}{2}x^{F^{c(bx+a)}}e^{\pi \operatorname{csgn}(id)} (\operatorname{csgn}(idx))^2 + \frac{i}{2}x^{F^{c(bx+a)}}e^{\pi \operatorname{csgn}(ix)} (\operatorname{csgn}(idx))\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(c\*(b\*x+a))\*ln(d\*x)^n\*(e+e\*n+e\*(1+b\*c\*x\*ln(F))\*ln(d\*x)),x)

[Out]  $(-1/2*I*x*F^{c*(b*x+a)}*e*Pi*csgn(I*d)*csgn(I*x)*csgn(I*d*x)+1/2*I*x*F^{c*(b*x+a)}*e*Pi*csgn(I*d)*csgn(I*d*x)^2+1/2*I*x*F^{c*(b*x+a)}*e*Pi*csgn(I*x)*csgn(I*d*x)^2-1/2*I*x*F^{c*(b*x+a)}*e*Pi*csgn(I*d*x)^3+x*F^{c*(b*x+a)}*e*ln(d)+e*x*F^{c*(b*x+a)}*ln(x))*(ln(d)+ln(x)-1/2*I*Pi*csgn(I*d*x)*(-csgn(I*d*x)+csgn(I*d))*(-csgn(I*d*x)+csgn(I*x)))^n$

**Maxima [A]** time = 1.3577, size = 51, normalized size = 2.55

$$(F^{ac}ex \log(d) + F^{ac}ex \log(x))e^{(bcx \log(F) + n \log(\log(d) + \log(x)))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(c\*(b\*x+a))\*log(d\*x)^n\*(e+e\*n+e\*(1+b\*c\*x\*log(F))\*log(d\*x)),x, algorithm="maxima")

[Out]  $(F^{(a*c)}*e*x*\log(d) + F^{(a*c)}*e*x*\log(x))*e^{(b*c*x*\log(F) + n*\log(\log(d) + \log(x)))}$

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*log(d*x)^n*(e+e*n+e*(1+b*c*x*log(F))*log(d*x)),x, a
lgorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(c*(b*x+a))*ln(d*x)**n*(e+e*n+e*(1+b*c*x*ln(F))*ln(d*x)),x)
```

```
[Out] Timed out
```

**Giac [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*log(d*x)^n*(e+e*n+e*(1+b*c*x*log(F))*log(d*x)),x, a
lgorithm="giac")
```

```
[Out] Exception raised: RuntimeError
```

$$3.87 \quad \int \frac{F^{c(a+bx)} \log^n(dx) (e + en + bcex \log(F) \log(dx))}{x} dx$$

**Optimal.** Leaf size=19

$$e \log^{n+1}(dx) F^{c(a+bx)}$$

[Out]  $e F^{c(a + b x)} \text{Log}[d x]^{(1 + n)}$

**Rubi [A]** time = 0.129157, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 35,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.029$ , Rules used = {2202}

$$e \log^{n+1}(dx) F^{c(a+bx)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(F^{c(a + b x)}) \text{Log}[d x]^n (e + e n + b c e x \text{Log}[F] \text{Log}[d x])]/x, x]$

[Out]  $e F^{c(a + b x)} \text{Log}[d x]^{(1 + n)}$

### Rule 2202

$\text{Int}[\text{Log}[(d \cdot) (x)]^{(n \cdot)} (F)^{((c \cdot) ((a \cdot) + (b \cdot) (x)))} (x)^{(m \cdot)} ((e \cdot) + \text{Log}[(d \cdot) (x)] (h \cdot) ((f \cdot) + (g \cdot) (x))), x_{\text{Symbol}}] \text{:> Simp}[(e x^{(m + 1)} F^{c(a + b x)} \text{Log}[d x]^{(n + 1)}) / (n + 1), x] \text{/; FreeQ}\{F, a, b, c, d, e, f, g, h, m, n\}, x] \&\& \text{EqQ}[e(m + 1) - f h (n + 1), 0] \&\& \text{EqQ}[g h (n + 1) - b c e \text{Log}[F], 0] \&\& \text{NeQ}[n, -1]$

### Rubi steps

$$\int \frac{F^{c(a+bx)} \log^n(dx) (e + en + bcex \log(F) \log(dx))}{x} dx = e F^{c(a+bx)} \log^{1+n}(dx)$$

**Mathematica [A]** time = 0.032266, size = 19, normalized size = 1.

$$e \log^{n+1}(dx) F^{c(a+bx)}$$

Antiderivative was successfully verified.



[In] Integrate[(F^(c\*(a + b\*x))\*Log[d\*x]^n\*(e + e\*n + b\*c\*e\*x\*Log[F]\*Log[d\*x]))/x,x]

[Out] e\*F^(c\*(a + b\*x))\*Log[d\*x]^(1 + n)

**Maple [C]** time = 0.088, size = 180, normalized size = 9.5

$$\left(-\frac{i}{2}\pi \operatorname{ecsgn}(id) \operatorname{csgn}(ix) \operatorname{csgn}(idx) F^{c(bx+a)} + \frac{i}{2}\pi \operatorname{ecsgn}(id) (\operatorname{csgn}(idx))^2 F^{c(bx+a)} + \frac{i}{2}\pi \operatorname{ecsgn}(ix) (\operatorname{csgn}(idx))^2 F^{c(bx+a)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(c\*(b\*x+a))\*ln(d\*x)^n\*(e+e\*n+b\*c\*e\*x\*ln(F)\*ln(d\*x))/x,x)

[Out]  $(-1/2*I*Pi*e*csgn(I*d)*csgn(I*x)*csgn(I*d*x)*F^{c*(b*x+a)} + 1/2*I*Pi*e*csgn(I*d)*csgn(I*d*x)^2*F^{c*(b*x+a)} + 1/2*I*Pi*e*csgn(I*x)*csgn(I*d*x)^2*F^{c*(b*x+a)} - 1/2*I*Pi*e*csgn(I*d*x)^3*F^{c*(b*x+a)} + \ln(d)*e*F^{c*(b*x+a)} + e*F^{c*(b*x+a)}*\ln(x))*(\ln(d)+\ln(x)-1/2*I*Pi*csgn(I*d*x)*(-csgn(I*d*x)+csgn(I*d))*(-csgn(I*d*x)+csgn(I*x)))^n$

**Maxima [A]** time = 1.38625, size = 49, normalized size = 2.58

$$(F^{ac}e \log(d) + F^{ac}e \log(x))e^{(bcx \log(F) + n \log(\log(d) + \log(x)))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(c\*(b\*x+a))\*log(d\*x)^n\*(e+e\*n+b\*c\*e\*x\*log(F)\*log(d\*x))/x,x, algorithm="maxima")

[Out] (F^(a\*c)\*e\*log(d) + F^(a\*c)\*e\*log(x))\*e^(b\*c\*x\*log(F) + n\*log(log(d) + log(x)))

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*log(d*x)^n*(e+e*n+b*c*e*x*log(F)*log(d*x))/x,x, alg
orithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(c*(b*x+a))*ln(d*x)**n*(e+e*n+b*c*e*x*ln(F)*ln(d*x))/x,x)
```

```
[Out] Timed out
```

**Giac [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*log(d*x)^n*(e+e*n+b*c*e*x*log(F)*log(d*x))/x,x, alg
orithm="giac")
```

```
[Out] Exception raised: RuntimeError
```

$$3.88 \quad \int \frac{F^{c(a+bx)} \log^n(dx) (e + en + e(-1 + bcx \log(F)) \log(dx))}{x^2} dx$$

**Optimal.** Leaf size=22

$$\frac{e \log^{n+1}(dx) F^{c(a+bx)}}{x}$$

[Out] (e\*F^(c\*(a + b\*x))\*Log[d\*x]^(1 + n))/x

**Rubi [A]** time = 0.131073, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 38,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.026$ , Rules used = {2202}

$$\frac{e \log^{n+1}(dx) F^{c(a+bx)}}{x}$$

Antiderivative was successfully verified.

[In] Int[(F^(c\*(a + b\*x))\*Log[d\*x]^n\*(e + e\*n + e\*(-1 + b\*c\*x\*Log[F]))\*Log[d\*x])/x^2,x]

[Out] (e\*F^(c\*(a + b\*x))\*Log[d\*x]^(1 + n))/x

### Rule 2202

Int[Log[(d\_.)\*(x\_)]^(n\_.)\*(F\_)^(c\_.\*((a\_.) + (b\_.)\*(x\_)))\*(x\_)^(m\_.)\*((e\_ + Log[(d\_.)\*(x\_)]\*(h\_.)\*((f\_.) + (g\_.)\*(x\_))), x\_Symbol] :> Simp[(e\*x^(m + 1)\*F^(c\*(a + b\*x))\*Log[d\*x]^(n + 1))/(n + 1), x] /; FreeQ[{F, a, b, c, d, e, f, g, h, m, n}, x] && EqQ[e\*(m + 1) - f\*h\*(n + 1), 0] && EqQ[g\*h\*(n + 1) - b\*c\*e\*Log[F], 0] && NeQ[n, -1]

### Rubi steps

$$\int \frac{F^{c(a+bx)} \log^n(dx) (e + en + e(-1 + bcx \log(F)) \log(dx))}{x^2} dx = \frac{e F^{c(a+bx)} \log^{1+n}(dx)}{x}$$

**Mathematica [A]** time = 0.327632, size = 23, normalized size = 1.05

$$\frac{e \log^{n+1}(dx) F^{ac+bcx}}{x}$$

Antiderivative was successfully verified.

[In] Integrate[(F^(c\*(a + b\*x))\*Log[d\*x]^n\*(e + e\*n + e\*(-1 + b\*c\*x\*Log[F]))\*Log[d\*x]))/x^2,x]

[Out] (e\*F^(a\*c + b\*c\*x)\*Log[d\*x]^(1 + n))/x

**Maple [C]** time = 0.105, size = 136, normalized size = 6.2

$$\frac{F^{c(bx+a)} e \left( -i (\operatorname{csgn}(idx))^3 \pi + i (\operatorname{csgn}(idx))^2 \operatorname{csgn}(id) \pi + i (\operatorname{csgn}(idx))^2 \operatorname{csgn}(ix) \pi - i \operatorname{csgn}(idx) \operatorname{csgn}(id) \operatorname{csgn}(ix) \pi + \dots \right)}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(c\*(b\*x+a))\*ln(d\*x)^n\*(e+e\*n+e\*(-1+b\*c\*x\*ln(F))\*ln(d\*x))/x^2,x)

[Out] 1/2\*F^(c\*(b\*x+a))\*e\*(-I\*csgn(I\*d\*x)^3\*Pi+I\*csgn(I\*d\*x)^2\*csgn(I\*d)\*Pi+I\*csgn(I\*d\*x)^2\*csgn(I\*x)\*Pi-I\*csgn(I\*d\*x)\*csgn(I\*d)\*csgn(I\*x)\*Pi+2\*ln(x)+2\*ln(d))/x\*(ln(d)+ln(x)-1/2\*I\*Pi\*csgn(I\*d\*x)\*(-csgn(I\*d\*x)+csgn(I\*d))\*(-csgn(I\*d\*x)+csgn(I\*x)))^n

**Maxima [A]** time = 1.42827, size = 53, normalized size = 2.41

$$\frac{(F^{ac} e \log(d) + F^{ac} e \log(x)) e^{(bcx \log(F) + n \log(\log(d) + \log(x)))}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(c\*(b\*x+a))\*log(d\*x)^n\*(e+e\*n+e\*(-1+b\*c\*x\*log(F))\*log(d\*x))/x^2,x, algorithm="maxima")

[Out] (F^(a\*c)\*e\*log(d) + F^(a\*c)\*e\*log(x))\*e^(b\*c\*x\*log(F) + n\*log(log(d) + log(x)))/x

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*log(d*x)^n*(e+e*n+e*(-1+b*c*x*log(F))*log(d*x))/x^2
,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(c*(b*x+a))*ln(d*x)**n*(e+e*n+e*(-1+b*c*x*ln(F))*ln(d*x))/x**2
,x)
```

```
[Out] Timed out
```

---

**Giac [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*log(d*x)^n*(e+e*n+e*(-1+b*c*x*log(F))*log(d*x))/x^2
,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError
```

$$3.89 \quad \int \frac{F^{c(a+bx)} \log^n(dx) (e + en + e(-2 + bcx \log(F)) \log(dx))}{x^3} dx$$

**Optimal.** Leaf size=22

$$\frac{e \log^{n+1}(dx) F^{c(a+bx)}}{x^2}$$

[Out] (e\*F^(c\*(a + b\*x))\*Log[d\*x]^(1 + n))/x^2

**Rubi [A]** time = 0.131368, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 38,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.026$ , Rules used = {2202}

$$\frac{e \log^{n+1}(dx) F^{c(a+bx)}}{x^2}$$

Antiderivative was successfully verified.

[In] Int[(F^(c\*(a + b\*x))\*Log[d\*x]^n\*(e + e\*n + e\*(-2 + b\*c\*x\*Log[F]))\*Log[d\*x])/x^3,x]

[Out] (e\*F^(c\*(a + b\*x))\*Log[d\*x]^(1 + n))/x^2

### Rule 2202

Int[Log[(d\_.)\*(x\_)]^(n\_.)\*(F\_)^((c\_.)\*((a\_.) + (b\_.)\*(x\_)))\*(x\_)^(m\_.)\*((e\_ + Log[(d\_.)\*(x\_)]\*(h\_.)\*((f\_.) + (g\_.)\*(x\_))), x\_Symbol] :> Simp[(e\*x^(m + 1)\*F^(c\*(a + b\*x))\*Log[d\*x]^(n + 1))/(n + 1), x] /; FreeQ[{F, a, b, c, d, e, f, g, h, m, n}, x] && EqQ[e\*(m + 1) - f\*h\*(n + 1), 0] && EqQ[g\*h\*(n + 1) - b\*c\*e\*Log[F], 0] && NeQ[n, -1]

### Rubi steps

$$\int \frac{F^{c(a+bx)} \log^n(dx) (e + en + e(-2 + bcx \log(F)) \log(dx))}{x^3} dx = \frac{e F^{c(a+bx)} \log^{1+n}(dx)}{x^2}$$

**Mathematica [A]** time = 0.322739, size = 23, normalized size = 1.05

$$\frac{e \log^{n+1}(dx) F^{ac+bcx}}{x^2}$$

Antiderivative was successfully verified.

[In] Integrate[(F^(c\*(a + b\*x))\*Log[d\*x]^n\*(e + e\*n + e\*(-2 + b\*c\*x\*Log[F]))\*Log[d\*x]))/x^3,x]

[Out] (e\*F^(a\*c + b\*c\*x)\*Log[d\*x]^(1 + n))/x^2

**Maple [C]** time = 0.102, size = 136, normalized size = 6.2

$$\frac{F^{c(bx+a)} e \left( -i (\operatorname{csgn}(idx))^3 \pi + i (\operatorname{csgn}(idx))^2 \operatorname{csgn}(id) \pi + i (\operatorname{csgn}(idx))^2 \operatorname{csgn}(ix) \pi - i \operatorname{csgn}(idx) \operatorname{csgn}(id) \operatorname{csgn}(ix) \pi \right)}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(c\*(b\*x+a))\*ln(d\*x)^n\*(e+e\*n+e\*(-2+b\*c\*x\*ln(F))\*ln(d\*x))/x^3,x)

[Out] 1/2\*F^(c\*(b\*x+a))\*e\*(-I\*csgn(I\*d\*x)^3\*Pi+I\*csgn(I\*d\*x)^2\*csgn(I\*d)\*Pi+I\*csgn(I\*d\*x)^2\*csgn(I\*x)\*Pi-I\*csgn(I\*d\*x)\*csgn(I\*d)\*csgn(I\*x)\*Pi+2\*ln(x)+2\*ln(d))/x^2\*(ln(d)+ln(x)-1/2\*I\*Pi\*csgn(I\*d\*x)\*(-csgn(I\*d\*x)+csgn(I\*d))\*(-csgn(I\*d\*x)+csgn(I\*x)))^n

**Maxima [A]** time = 1.38074, size = 53, normalized size = 2.41

$$\frac{(F^{ac} e \log(d) + F^{ac} e \log(x)) e^{(bcx \log(F) + n \log(\log(d) + \log(x)))}}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(c\*(b\*x+a))\*log(d\*x)^n\*(e+e\*n+e\*(-2+b\*c\*x\*log(F))\*log(d\*x))/x^3,x,algorithm="maxima")

[Out] (F^(a\*c)\*e\*log(d) + F^(a\*c)\*e\*log(x))\*e^(b\*c\*x\*log(F) + n\*log(log(d) + log(x)))/x^2

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*log(d*x)^n*(e+e*n+e*(-2+b*c*x*log(F))*log(d*x))/x^3
,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(c*(b*x+a))*ln(d*x)**n*(e+e*n+e*(-2+b*c*x*ln(F))*ln(d*x))/x**3
,x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{((bcx \log(F) - 2)e \log(dx) + en + e)F^{(bx+a)c} \log(dx)^n}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*log(d*x)^n*(e+e*n+e*(-2+b*c*x*log(F))*log(d*x))/x^3
,x, algorithm="giac")
```

```
[Out] integrate(((b*c*x*log(F) - 2)*e*log(d*x) + e*n + e)*F^((b*x + a)*c)*log(d*x)
)^n/x^3, x)
```



### 3.90 $\int \sqrt{e^{a+bx}} x^4 dx$

**Optimal.** Leaf size=91

$$-\frac{16x^3\sqrt{e^{a+bx}}}{b^2} + \frac{96x^2\sqrt{e^{a+bx}}}{b^3} - \frac{384x\sqrt{e^{a+bx}}}{b^4} + \frac{768\sqrt{e^{a+bx}}}{b^5} + \frac{2x^4\sqrt{e^{a+bx}}}{b}$$

[Out] (768\*sqrt[E^(a + b\*x)])/b^5 - (384\*sqrt[E^(a + b\*x)]\*x)/b^4 + (96\*sqrt[E^(a + b\*x)]\*x^2)/b^3 - (16\*sqrt[E^(a + b\*x)]\*x^3)/b^2 + (2\*sqrt[E^(a + b\*x)]\*x^4)/b

**Rubi [A]** time = 0.143393, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$ , Rules used = {2176, 2194}

$$-\frac{16x^3\sqrt{e^{a+bx}}}{b^2} + \frac{96x^2\sqrt{e^{a+bx}}}{b^3} - \frac{384x\sqrt{e^{a+bx}}}{b^4} + \frac{768\sqrt{e^{a+bx}}}{b^5} + \frac{2x^4\sqrt{e^{a+bx}}}{b}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[E^(a + b\*x)]\*x^4, x]

[Out] (768\*sqrt[E^(a + b\*x)])/b^5 - (384\*sqrt[E^(a + b\*x)]\*x)/b^4 + (96\*sqrt[E^(a + b\*x)]\*x^2)/b^3 - (16\*sqrt[E^(a + b\*x)]\*x^3)/b^2 + (2\*sqrt[E^(a + b\*x)]\*x^4)/b

#### Rule 2176

Int[((b\_.)\*(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[((c + d\*x)^m\*(b\*F^(g\*(e + f\*x)))^n)/(f\*g\*n\*Log[F]), x] - Dist[(d\*m)/(f\*g\*n\*Log[F]), Int[(c + d\*x)^(m - 1)\*(b\*F^(g\*(e + f\*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2\*m] && !\$UseGamma === True

#### Rule 2194

Int[((F\_)^((c\_.)\*((a\_.) + (b\_.)\*(x\_))))^(n\_.), x\_Symbol] :> Simp[(F^(c\*(a + b\*x)))^n/(b\*c\*n\*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]

#### Rubi steps

$$\begin{aligned}
\int \sqrt{e^{a+bx}} x^4 dx &= \frac{2\sqrt{e^{a+bx}} x^4}{b} - \frac{8 \int \sqrt{e^{a+bx}} x^3 dx}{b} \\
&= -\frac{16\sqrt{e^{a+bx}} x^3}{b^2} + \frac{2\sqrt{e^{a+bx}} x^4}{b} + \frac{48 \int \sqrt{e^{a+bx}} x^2 dx}{b^2} \\
&= \frac{96\sqrt{e^{a+bx}} x^2}{b^3} - \frac{16\sqrt{e^{a+bx}} x^3}{b^2} + \frac{2\sqrt{e^{a+bx}} x^4}{b} - \frac{192 \int \sqrt{e^{a+bx}} x dx}{b^3} \\
&= -\frac{384\sqrt{e^{a+bx}} x}{b^4} + \frac{96\sqrt{e^{a+bx}} x^2}{b^3} - \frac{16\sqrt{e^{a+bx}} x^3}{b^2} + \frac{2\sqrt{e^{a+bx}} x^4}{b} + \frac{384 \int \sqrt{e^{a+bx}} dx}{b^4} \\
&= \frac{768\sqrt{e^{a+bx}}}{b^5} - \frac{384\sqrt{e^{a+bx}} x}{b^4} + \frac{96\sqrt{e^{a+bx}} x^2}{b^3} - \frac{16\sqrt{e^{a+bx}} x^3}{b^2} + \frac{2\sqrt{e^{a+bx}} x^4}{b}
\end{aligned}$$

**Mathematica [A]** time = 0.0206717, size = 45, normalized size = 0.49

$$\frac{2(b^4 x^4 - 8b^3 x^3 + 48b^2 x^2 - 192bx + 384)\sqrt{e^{a+bx}}}{b^5}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[E^(a + b\*x)]\*x^4, x]

[Out] (2\*Sqrt[E^(a + b\*x)]\*(384 - 192\*b\*x + 48\*b^2\*x^2 - 8\*b^3\*x^3 + b^4\*x^4))/b^5

**Maple [A]** time = 0.003, size = 43, normalized size = 0.5

$$2 \frac{(x^4 b^4 - 8 x^3 b^3 + 48 x^2 b^2 - 192 b x + 384) \sqrt{e^{bx+a}}}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4\*exp(b\*x+a)^(1/2), x)

[Out] 2\*(b^4\*x^4-8\*b^3\*x^3+48\*b^2\*x^2-192\*b\*x+384)\*exp(b\*x+a)^(1/2)/b^5

**Maxima [A]** time = 1.027, size = 81, normalized size = 0.89

$$\frac{2\left(b^4x^4e^{\left(\frac{1}{2}a\right)} - 8b^3x^3e^{\left(\frac{1}{2}a\right)} + 48b^2x^2e^{\left(\frac{1}{2}a\right)} - 192bx e^{\left(\frac{1}{2}a\right)} + 384e^{\left(\frac{1}{2}a\right)}\right)e^{\left(\frac{1}{2}bx\right)}}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4\*exp(b\*x+a)^(1/2),x, algorithm="maxima")

[Out] 2\*(b^4\*x^4\*e^(1/2\*a) - 8\*b^3\*x^3\*e^(1/2\*a) + 48\*b^2\*x^2\*e^(1/2\*a) - 192\*b\*x\*e^(1/2\*a) + 384\*e^(1/2\*a))\*e^(1/2\*b\*x)/b^5

**Fricas [A]** time = 1.44212, size = 105, normalized size = 1.15

$$\frac{2\left(b^4x^4 - 8b^3x^3 + 48b^2x^2 - 192bx + 384\right)e^{\left(\frac{1}{2}bx + \frac{1}{2}a\right)}}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4\*exp(b\*x+a)^(1/2),x, algorithm="fricas")

[Out] 2\*(b^4\*x^4 - 8\*b^3\*x^3 + 48\*b^2\*x^2 - 192\*b\*x + 384)\*e^(1/2\*b\*x + 1/2\*a)/b^5

**Sympy [A]** time = 0.114484, size = 51, normalized size = 0.56

$$\begin{cases} \frac{\left(2b^4x^4 - 16b^3x^3 + 96b^2x^2 - 384bx + 768\right)\sqrt{e^{a+bx}}}{b^5} & \text{for } b^5 \neq 0 \\ \frac{x^5}{5} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*4\*exp(b\*x+a)\*\*(1/2),x)

[Out] Piecewise(((2\*b\*\*4\*x\*\*4 - 16\*b\*\*3\*x\*\*3 + 96\*b\*\*2\*x\*\*2 - 384\*b\*x + 768)\*sqrt(exp(a + b\*x))/b\*\*5, Ne(b\*\*5, 0)), (x\*\*5/5, True))

---

**Giac [A]** time = 1.31474, size = 58, normalized size = 0.64

$$\frac{2(b^4x^4 - 8b^3x^3 + 48b^2x^2 - 192bx + 384)e^{\left(\frac{1}{2}bx + \frac{1}{2}a\right)}}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4\*exp(b\*x+a)^(1/2),x, algorithm="giac")

[Out] 2\*(b^4\*x^4 - 8\*b^3\*x^3 + 48\*b^2\*x^2 - 192\*b\*x + 384)\*e^(1/2\*b\*x + 1/2\*a)/b^5

### 3.91 $\int \sqrt{e^{a+bx}} x^3 dx$

**Optimal.** Leaf size=72

$$-\frac{12x^2\sqrt{e^{a+bx}}}{b^2} + \frac{48x\sqrt{e^{a+bx}}}{b^3} - \frac{96\sqrt{e^{a+bx}}}{b^4} + \frac{2x^3\sqrt{e^{a+bx}}}{b}$$

[Out]  $(-96*\text{Sqrt}[E^{(a + b*x)}])/b^4 + (48*\text{Sqrt}[E^{(a + b*x)}]*x)/b^3 - (12*\text{Sqrt}[E^{(a + b*x)}]*x^2)/b^2 + (2*\text{Sqrt}[E^{(a + b*x)}]*x^3)/b$

**Rubi [A]** time = 0.101391, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$ , Rules used = {2176, 2194}

$$-\frac{12x^2\sqrt{e^{a+bx}}}{b^2} + \frac{48x\sqrt{e^{a+bx}}}{b^3} - \frac{96\sqrt{e^{a+bx}}}{b^4} + \frac{2x^3\sqrt{e^{a+bx}}}{b}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Sqrt}[E^{(a + b*x)}]*x^3, x]$

[Out]  $(-96*\text{Sqrt}[E^{(a + b*x)}])/b^4 + (48*\text{Sqrt}[E^{(a + b*x)}]*x)/b^3 - (12*\text{Sqrt}[E^{(a + b*x)}]*x^2)/b^2 + (2*\text{Sqrt}[E^{(a + b*x)}]*x^3)/b$

#### Rule 2176

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Simp[((c + d*x)^m*(b*F^(g*(e + f*x)))^n)/(f*g*n*Log[F]), x] - Dist[(d*m)/(f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m] && !$UseGamma === True
```

#### Rule 2194

```
Int[(F)^((c_.)*((a_.) + (b_.)*(x_)))^(n_.), x_Symbol] := Simp[(F^(c*(a + b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

#### Rubi steps

$$\begin{aligned}
\int \sqrt{e^{a+bx}} x^3 dx &= \frac{2\sqrt{e^{a+bx}} x^3}{b} - \frac{6 \int \sqrt{e^{a+bx}} x^2 dx}{b} \\
&= -\frac{12\sqrt{e^{a+bx}} x^2}{b^2} + \frac{2\sqrt{e^{a+bx}} x^3}{b} + \frac{24 \int \sqrt{e^{a+bx}} x dx}{b^2} \\
&= \frac{48\sqrt{e^{a+bx}} x}{b^3} - \frac{12\sqrt{e^{a+bx}} x^2}{b^2} + \frac{2\sqrt{e^{a+bx}} x^3}{b} - \frac{48 \int \sqrt{e^{a+bx}} dx}{b^3} \\
&= -\frac{96\sqrt{e^{a+bx}}}{b^4} + \frac{48\sqrt{e^{a+bx}} x}{b^3} - \frac{12\sqrt{e^{a+bx}} x^2}{b^2} + \frac{2\sqrt{e^{a+bx}} x^3}{b}
\end{aligned}$$

**Mathematica [A]** time = 0.0134152, size = 37, normalized size = 0.51

$$\frac{2(b^3 x^3 - 6b^2 x^2 + 24bx - 48)\sqrt{e^{a+bx}}}{b^4}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[E^(a + b\*x)]\*x^3,x]

[Out] (2\*Sqrt[E^(a + b\*x)]\*(-48 + 24\*b\*x - 6\*b^2\*x^2 + b^3\*x^3))/b^4

**Maple [A]** time = 0.003, size = 35, normalized size = 0.5

$$2 \frac{(x^3 b^3 - 6 x^2 b^2 + 24 b x - 48) \sqrt{e^{bx+a}}}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*exp(b\*x+a)^(1/2),x)

[Out] 2\*(b^3\*x^3-6\*b^2\*x^2+24\*b\*x-48)\*exp(b\*x+a)^(1/2)/b^4

**Maxima [A]** time = 1.09332, size = 65, normalized size = 0.9

$$\frac{2 \left( b^3 x^3 e^{\left(\frac{1}{2} a\right)} - 6 b^2 x^2 e^{\left(\frac{1}{2} a\right)} + 24 b x e^{\left(\frac{1}{2} a\right)} - 48 e^{\left(\frac{1}{2} a\right)} \right) e^{\left(\frac{1}{2} b x\right)}}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*exp(b\*x+a)^(1/2),x, algorithm="maxima")

[Out]  $2*(b^3*x^3*e^{(1/2*a)} - 6*b^2*x^2*e^{(1/2*a)} + 24*b*x*e^{(1/2*a)} - 48*e^{(1/2*a)})*e^{(1/2*b*x)}/b^4$

**Fricas [A]** time = 1.45923, size = 85, normalized size = 1.18

$$\frac{2(b^3x^3 - 6b^2x^2 + 24bx - 48)e^{\left(\frac{1}{2}bx + \frac{1}{2}a\right)}}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*exp(b\*x+a)^(1/2),x, algorithm="fricas")

[Out]  $2*(b^3*x^3 - 6*b^2*x^2 + 24*b*x - 48)*e^{(1/2*b*x + 1/2*a)}/b^4$

**Sympy [A]** time = 0.108289, size = 42, normalized size = 0.58

$$\begin{cases} \frac{(2b^3x^3 - 12b^2x^2 + 48bx - 96)\sqrt{e^{a+bx}}}{b^4} & \text{for } b^4 \neq 0 \\ \frac{x^4}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*3\*exp(b\*x+a)\*\*(1/2),x)

[Out] Piecewise(((2\*b\*\*3\*x\*\*3 - 12\*b\*\*2\*x\*\*2 + 48\*b\*x - 96)\*sqrt(exp(a + b\*x)))/b\*\*4, Ne(b\*\*4, 0)), (x\*\*4/4, True))

**Giac [A]** time = 1.25374, size = 47, normalized size = 0.65

$$\frac{2(b^3x^3 - 6b^2x^2 + 24bx - 48)e^{\left(\frac{1}{2}bx + \frac{1}{2}a\right)}}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*exp(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2*(b^3*x^3 - 6*b^2*x^2 + 24*b*x - 48)*e^(1/2*b*x + 1/2*a)/b^4
```



### 3.92 $\int \sqrt{e^{a+bx}} x^2 dx$

**Optimal.** Leaf size=53

$$-\frac{8x\sqrt{e^{a+bx}}}{b^2} + \frac{16\sqrt{e^{a+bx}}}{b^3} + \frac{2x^2\sqrt{e^{a+bx}}}{b}$$

[Out] (16\*Sqrt[E^(a + b\*x)]/b^3 - (8\*Sqrt[E^(a + b\*x)]\*x)/b^2 + (2\*Sqrt[E^(a + b\*x)]\*x^2)/b

**Rubi [A]** time = 0.0635734, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$ , Rules used = {2176, 2194}

$$-\frac{8x\sqrt{e^{a+bx}}}{b^2} + \frac{16\sqrt{e^{a+bx}}}{b^3} + \frac{2x^2\sqrt{e^{a+bx}}}{b}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[E^(a + b\*x)]\*x^2,x]

[Out] (16\*Sqrt[E^(a + b\*x)]/b^3 - (8\*Sqrt[E^(a + b\*x)]\*x)/b^2 + (2\*Sqrt[E^(a + b\*x)]\*x^2)/b

#### Rule 2176

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Simp[((c + d*x)^m*(b*F^(g*(e + f*x)))^n)/(f*g*n*Log[F]), x] - Dist[(d*m)/(f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m] && !$UseGamma === True
```

#### Rule 2194

```
Int[(F)^((c_.)*((a_.) + (b_.)*(x_)))^(n_.), x_Symbol] := Simp[(F^(c*(a + b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

#### Rubi steps

$$\begin{aligned}
\int \sqrt{e^{a+bx}} x^2 dx &= \frac{2\sqrt{e^{a+bx}} x^2}{b} - \frac{4 \int \sqrt{e^{a+bx}} x dx}{b} \\
&= -\frac{8\sqrt{e^{a+bx}} x}{b^2} + \frac{2\sqrt{e^{a+bx}} x^2}{b} + \frac{8 \int \sqrt{e^{a+bx}} dx}{b^2} \\
&= \frac{16\sqrt{e^{a+bx}}}{b^3} - \frac{8\sqrt{e^{a+bx}} x}{b^2} + \frac{2\sqrt{e^{a+bx}} x^2}{b}
\end{aligned}$$

**Mathematica [A]** time = 0.0126865, size = 29, normalized size = 0.55

$$\frac{2(b^2 x^2 - 4bx + 8)\sqrt{e^{a+bx}}}{b^3}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[E^(a + b\*x)]\*x^2,x]

[Out] (2\*Sqrt[E^(a + b\*x)]\*(8 - 4\*b\*x + b^2\*x^2))/b^3

**Maple [A]** time = 0.001, size = 27, normalized size = 0.5

$$2 \frac{(x^2 b^2 - 4bx + 8)\sqrt{e^{bx+a}}}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*exp(b\*x+a)^(1/2),x)

[Out] 2\*(b^2\*x^2-4\*b\*x+8)\*exp(b\*x+a)^(1/2)/b^3

**Maxima [A]** time = 1.02316, size = 49, normalized size = 0.92

$$\frac{2\left(b^2 x^2 e^{\left(\frac{1}{2}a\right)} - 4bx e^{\left(\frac{1}{2}a\right)} + 8e^{\left(\frac{1}{2}a\right)}\right) e^{\left(\frac{1}{2}bx\right)}}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*exp(b\*x+a)^(1/2),x, algorithm="maxima")

[Out]  $2*(b^2*x^2*e^{(1/2*a)} - 4*b*x*e^{(1/2*a)} + 8*e^{(1/2*a)})*e^{(1/2*b*x)}/b^3$

**Fricas [A]** time = 1.47572, size = 66, normalized size = 1.25

$$\frac{2(b^2x^2 - 4bx + 8)e^{\left(\frac{1}{2}bx + \frac{1}{2}a\right)}}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*exp(b\*x+a)^(1/2),x, algorithm="fricas")

[Out]  $2*(b^2*x^2 - 4*b*x + 8)*e^{(1/2*b*x + 1/2*a)}/b^3$

**Sympy [A]** time = 0.103519, size = 34, normalized size = 0.64

$$\begin{cases} \frac{(2b^2x^2 - 8bx + 16)\sqrt{e^{a+bx}}}{b^3} & \text{for } b^3 \neq 0 \\ \frac{x^3}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2\*exp(b\*x+a)\*\*(1/2),x)

[Out] Piecewise(((2\*b\*\*2\*x\*\*2 - 8\*b\*x + 16)\*sqrt(exp(a + b\*x))/b\*\*3, Ne(b\*\*3, 0)), (x\*\*3/3, True))

**Giac [A]** time = 1.22556, size = 36, normalized size = 0.68

$$\frac{2(b^2x^2 - 4bx + 8)e^{\left(\frac{1}{2}bx + \frac{1}{2}a\right)}}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*exp(b\*x+a)^(1/2),x, algorithm="giac")

[Out]  $2*(b^2*x^2 - 4*b*x + 8)*e^{(1/2*b*x + 1/2*a)}/b^3$

### 3.93 $\int \sqrt{e^{a+bx}} x dx$

**Optimal.** Leaf size=34

$$\frac{2x\sqrt{e^{a+bx}}}{b} - \frac{4\sqrt{e^{a+bx}}}{b^2}$$

[Out]  $(-4*\text{Sqrt}[E^{(a + b*x)}])/b^2 + (2*\text{Sqrt}[E^{(a + b*x)}]*x)/b$

**Rubi [A]** time = 0.0285417, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$ , Rules used = {2176, 2194}

$$\frac{2x\sqrt{e^{a+bx}}}{b} - \frac{4\sqrt{e^{a+bx}}}{b^2}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Sqrt}[E^{(a + b*x)}]*x, x]$

[Out]  $(-4*\text{Sqrt}[E^{(a + b*x)}])/b^2 + (2*\text{Sqrt}[E^{(a + b*x)}]*x)/b$

#### Rule 2176

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_
_.), x_Symbol] :> Simp[((c + d*x)^m*(b*F^(g*(e + f*x)))^n)/(f*g*n*Log[F]),
x] - Dist[(d*m)/(f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m]
] && !$UseGamma == True
```

#### Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

#### Rubi steps

$$\begin{aligned}\int \sqrt{e^{a+bx}} x dx &= \frac{2\sqrt{e^{a+bx}} x}{b} - \frac{2 \int \sqrt{e^{a+bx}} dx}{b} \\ &= -\frac{4\sqrt{e^{a+bx}}}{b^2} + \frac{2\sqrt{e^{a+bx}} x}{b}\end{aligned}$$

**Mathematica [A]** time = 0.0101217, size = 21, normalized size = 0.62

$$\frac{2(bx - 2)\sqrt{e^{a+bx}}}{b^2}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[E^(a + b\*x)]\*x,x]

[Out] (2\*Sqrt[E^(a + b\*x)]\*(-2 + b\*x))/b^2

**Maple [A]** time = 0.001, size = 19, normalized size = 0.6

$$2 \frac{(bx - 2) \sqrt{e^{bx+a}}}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*exp(b\*x+a)^(1/2),x)

[Out] 2\*(b\*x-2)\*exp(b\*x+a)^(1/2)/b^2

**Maxima [A]** time = 1.0455, size = 32, normalized size = 0.94

$$\frac{2 \left( b x e^{\left(\frac{1}{2} a\right)} - 2 e^{\left(\frac{1}{2} a\right)} \right) e^{\left(\frac{1}{2} b x\right)}}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*exp(b\*x+a)^(1/2),x, algorithm="maxima")

[Out]  $2*(b*x*e^{(1/2*a)} - 2*e^{(1/2*a)})*e^{(1/2*b*x)}/b^2$

---

**Fricas [A]** time = 1.5013, size = 50, normalized size = 1.47

$$\frac{2(bx - 2)e^{\left(\frac{1}{2}bx + \frac{1}{2}a\right)}}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*exp(b*x+a)^(1/2),x, algorithm="fricas")`

[Out]  $2*(b*x - 2)*e^{(1/2*b*x + 1/2*a)}/b^2$

---

**Sympy [A]** time = 0.094429, size = 26, normalized size = 0.76

$$\begin{cases} \frac{(2bx-4)\sqrt{e^{a+bx}}}{b^2} & \text{for } b^2 \neq 0 \\ \frac{x^2}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*exp(b*x+a)**(1/2),x)`

[Out] `Piecewise(((2*b*x - 4)*sqrt(exp(a + b*x))/b**2, Ne(b**2, 0)), (x**2/2, True))`

---

**Giac [A]** time = 1.2432, size = 26, normalized size = 0.76

$$\frac{2(bx - 2)e^{\left(\frac{1}{2}bx + \frac{1}{2}a\right)}}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*exp(b*x+a)^(1/2),x, algorithm="giac")`

[Out]  $2*(b*x - 2)*e^{(1/2*b*x + 1/2*a)}/b^2$

### 3.94 $\int \sqrt{e^{a+bx}} dx$

**Optimal.** Leaf size=16

$$\frac{2\sqrt{e^{a+bx}}}{b}$$

[Out] (2\*Sqrt[E^(a + b\*x)])/b

**Rubi [A]** time = 0.0065371, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$ , Rules used = {2194}

$$\frac{2\sqrt{e^{a+bx}}}{b}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[E^(a + b\*x)], x]

[Out] (2\*Sqrt[E^(a + b\*x)])/b

**Rule 2194**

Int[((F\_)^((c\_.)\*((a\_.) + (b\_.)\*(x\_))))^(n\_.), x\_Symbol] := Simp[(F^(c\*(a + b\*x)))^n/(b\*c\*n\*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]

**Rubi steps**

$$\int \sqrt{e^{a+bx}} dx = \frac{2\sqrt{e^{a+bx}}}{b}$$

**Mathematica [A]** time = 0.0050288, size = 16, normalized size = 1.

$$\frac{2\sqrt{e^{a+bx}}}{b}$$

Antiderivative was successfully verified.



[In] Integrate[Sqrt[E^(a + b\*x)],x]

[Out] (2\*Sqrt[E^(a + b\*x)])/b

**Maple [A]** time = 0.001, size = 14, normalized size = 0.9

$$2 \frac{\sqrt{e^{bx+a}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(b\*x+a)^(1/2),x)

[Out] 2\*exp(b\*x+a)^(1/2)/b

**Maxima [A]** time = 1.04315, size = 19, normalized size = 1.19

$$\frac{2 e^{\left(\frac{1}{2} b x + \frac{1}{2} a\right)}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(b\*x+a)^(1/2),x, algorithm="maxima")

[Out] 2\*e^(1/2\*b\*x + 1/2\*a)/b

**Fricas [A]** time = 1.46732, size = 34, normalized size = 2.12

$$\frac{2 e^{\left(\frac{1}{2} b x + \frac{1}{2} a\right)}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(b\*x+a)^(1/2),x, algorithm="fricas")

[Out]  $2*e^{(1/2*b*x + 1/2*a)}/b$

**Sympy [A]** time = 0.082823, size = 14, normalized size = 0.88

$$\begin{cases} \frac{2\sqrt{e^{a+bx}}}{b} & \text{for } b \neq 0 \\ x & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(b*x+a)**(1/2),x)`

[Out] `Piecewise((2*sqrt(exp(a + b*x))/b, Ne(b, 0)), (x, True))`

**Giac [A]** time = 1.24524, size = 19, normalized size = 1.19

$$\frac{2e^{\left(\frac{1}{2}bx + \frac{1}{2}a\right)}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(b*x+a)^(1/2),x, algorithm="giac")`

[Out]  $2*e^{(1/2*b*x + 1/2*a)}/b$

$$3.95 \quad \int \frac{\sqrt{e^{a+bx}}}{x} dx$$

Optimal. Leaf size=27

$$e^{-\frac{bx}{2}} \sqrt{e^{a+bx}} \operatorname{Ei}\left(\frac{bx}{2}\right)$$

[Out] (Sqrt[E^(a + b\*x)]\*ExpIntegralEi[(b\*x)/2])/E^((b\*x)/2)

**Rubi [A]** time = 0.0583287, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$ , Rules used = {2182, 2178}

$$e^{-\frac{bx}{2}} \sqrt{e^{a+bx}} \operatorname{Ei}\left(\frac{bx}{2}\right)$$

Antiderivative was successfully verified.

[In] Int[Sqrt[E^(a + b\*x)]/x,x]

[Out] (Sqrt[E^(a + b\*x)]\*ExpIntegralEi[(b\*x)/2])/E^((b\*x)/2)

#### Rule 2182

Int[((b\_.)\*(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_)))^(n\_)\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] :> Dist[(b\*F^(g\*(e + f\*x)))^n/F^(g\*n\*(e + f\*x)), Int[(c + d\*x)^m\*F^(g\*n\*(e + f\*x)), x], x] /; FreeQ[{F, b, c, d, e, f, g, m, n}, x]

#### Rule 2178

Int[(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_)))/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] :> Simp[(F^(g\*(e - (c\*f)/d))\*ExpIntegralEi[(f\*g\*(c + d\*x)\*Log[F])/d])/d, x] /; FreeQ[{F, c, d, e, f, g}, x] && !\$UseGamma == True

#### Rubi steps

$$\int \frac{\sqrt{e^{a+bx}}}{x} dx = \left( e^{\frac{1}{2}(-a-bx)} \sqrt{e^{a+bx}} \right) \int \frac{e^{\frac{1}{2}(a+bx)}}{x} dx$$

$$= e^{-\frac{bx}{2}} \sqrt{e^{a+bx}} \operatorname{Ei} \left( \frac{bx}{2} \right)$$

**Mathematica [A]** time = 0.0156963, size = 27, normalized size = 1.

$$e^{-\frac{bx}{2}} \sqrt{e^{a+bx}} \operatorname{Ei} \left( \frac{bx}{2} \right)$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[E^(a + b\*x)]/x,x]

[Out] (Sqrt[E^(a + b\*x)]\*ExpIntegralEi[(b\*x)/2])/E^((b\*x)/2)

**Maple [B]** time = 0.039, size = 57, normalized size = 2.1

$$\sqrt{e^{bx+a}} e^{-\frac{bx}{2}} e^{\frac{a}{2}} \left( \ln(x) - \ln(2) + \ln\left(-be^{\frac{a}{2}}\right) - \ln\left(-\frac{bx}{2} e^{\frac{a}{2}}\right) - \operatorname{Ei}\left(1, -\frac{bx}{2} e^{\frac{a}{2}}\right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(b\*x+a)^(1/2)/x,x)

[Out] exp(b\*x+a)^(1/2)\*exp(-1/2\*b\*x\*exp(1/2\*a))\*(ln(x)-ln(2)+ln(-b\*exp(1/2\*a))-ln(-1/2\*b\*x\*exp(1/2\*a))-Ei(1,-1/2\*b\*x\*exp(1/2\*a)))

**Maxima [A]** time = 1.17256, size = 14, normalized size = 0.52

$$\operatorname{Ei} \left( \frac{1}{2} bx \right) e^{\left(\frac{1}{2} a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(b\*x+a)^(1/2)/x,x, algorithm="maxima")

[Out]  $Ei(1/2*b*x)*e^{(1/2*a)}$

---

**Fricas [A]** time = 1.49306, size = 31, normalized size = 1.15

$$Ei\left(\frac{1}{2}bx\right)e^{\left(\frac{1}{2}a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(b*x+a)^(1/2)/x,x, algorithm="fricas")`

[Out]  $Ei(1/2*b*x)*e^{(1/2*a)}$

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{e^a e^{bx}}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(b*x+a)**(1/2)/x,x)`

[Out] `Integral(sqrt(exp(a)*exp(b*x))/x, x)`

---

**Giac [A]** time = 1.36466, size = 14, normalized size = 0.52

$$Ei\left(\frac{1}{2}bx\right)e^{\left(\frac{1}{2}a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(b*x+a)^(1/2)/x,x, algorithm="giac")`

[Out]  $Ei(1/2*b*x)*e^{(1/2*a)}$

$$3.96 \quad \int \frac{\sqrt{e^{a+bx}}}{x^2} dx$$

**Optimal.** Leaf size=48

$$\frac{1}{2} b e^{-\frac{bx}{2}} \sqrt{e^{a+bx}} \text{Ei}\left(\frac{bx}{2}\right) - \frac{\sqrt{e^{a+bx}}}{x}$$

[Out]  $-(\text{Sqrt}[E^{(a + b*x)}]/x) + (b*\text{Sqrt}[E^{(a + b*x)}]*\text{ExpIntegralEi}[(b*x)/2])/(2*E^{(b*x)/2})$

**Rubi [A]** time = 0.0864004, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$ , Rules used = {2177, 2182, 2178}

$$\frac{1}{2} b e^{-\frac{bx}{2}} \sqrt{e^{a+bx}} \text{Ei}\left(\frac{bx}{2}\right) - \frac{\sqrt{e^{a+bx}}}{x}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Sqrt}[E^{(a + b*x)}]/x^2, x]$

[Out]  $-(\text{Sqrt}[E^{(a + b*x)}]/x) + (b*\text{Sqrt}[E^{(a + b*x)}]*\text{ExpIntegralEi}[(b*x)/2])/(2*E^{(b*x)/2})$

#### Rule 2177

$\text{Int}[(b_.)*(F_)^{((g_.)*((e_.) + (f_.)*(x_)))^{(n_.)*((c_.) + (d_.)*(x_))^{(m_.)}, x\_Symbol] :> \text{Simp}[(c + d*x)^{(m + 1)}*(b*F^{(g*(e + f*x)))^n})/(d*(m + 1)), x] - \text{Dist}[(f*g*n*\text{Log}[F])/(d*(m + 1)), \text{Int}[(c + d*x)^{(m + 1)}*(b*F^{(g*(e + f*x)))^n}, x], x] /;$  FreeQ[{F, b, c, d, e, f, g, n}, x] && LtQ[m, -1] && IntegerQ[2\*m] && !\$UseGamma == True

#### Rule 2182

$\text{Int}[(b_.)*(F_)^{((g_.)*((e_.) + (f_.)*(x_)))^{(n_.)*((c_.) + (d_.)*(x_))^{(m_.)}, x\_Symbol] :> \text{Dist}[(b*F^{(g*(e + f*x)))^n}/F^{(g*n*(e + f*x))}, \text{Int}[(c + d*x)^m * F^{(g*n*(e + f*x))}, x], x] /;$  FreeQ[{F, b, c, d, e, f, g, m, n}, x]

#### Rule 2178

```
Int[(F_)^((g_)*(e_) + (f_)*(x_))/((c_) + (d_)*(x_)), x_Symbol] := Si
mp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; F
reeQ[{F, c, d, e, f, g}, x] && !$UseGamma === True
```

### Rubi steps

$$\begin{aligned}\int \frac{\sqrt{e^{a+bx}}}{x^2} dx &= -\frac{\sqrt{e^{a+bx}}}{x} + \frac{1}{2}b \int \frac{\sqrt{e^{a+bx}}}{x} dx \\ &= -\frac{\sqrt{e^{a+bx}}}{x} + \frac{1}{2} \left( b e^{\frac{1}{2}(-a-bx)} \sqrt{e^{a+bx}} \right) \int \frac{e^{\frac{1}{2}(a+bx)}}{x} dx \\ &= -\frac{\sqrt{e^{a+bx}}}{x} + \frac{1}{2} b e^{-\frac{bx}{2}} \sqrt{e^{a+bx}} \operatorname{Ei}\left(\frac{bx}{2}\right)\end{aligned}$$

**Mathematica [A]** time = 0.0269809, size = 47, normalized size = 0.98

$$\frac{e^{-\frac{bx}{2}} \sqrt{e^{a+bx}} \left( bx \operatorname{Ei}\left(\frac{bx}{2}\right) - 2e^{\frac{bx}{2}} \right)}{2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[E^(a + b*x)]/x^2, x]
```

```
[Out] (Sqrt[E^(a + b*x)]*(-2*E^((b*x)/2) + b*x*ExpIntegralEi[(b*x)/2]))/(2*E^((b*
x)/2)*x)
```

**Maple [B]** time = 0.021, size = 116, normalized size = 2.4

$$-\frac{b}{2} \sqrt{e^{bx+a}} e^{\frac{a}{2}} e^{-\frac{bx}{2}} e^{\frac{a}{2}} \left( 2 \frac{e^{-a/2}}{bx} + 1 - \ln(x) + \ln(2) - \ln\left(-be^{\frac{a}{2}}\right) - \frac{1}{bx} e^{-\frac{a}{2}} \left( 2 + bxe^{\frac{a}{2}} \right) + 2 \frac{e^{-a/2+1/2 bxe^{a/2}}}{bx} + \ln\left(-\frac{bx}{2} e^{\frac{a}{2}}\right) + \operatorname{Ei}\left(\frac{bx}{2}\right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b*x+a)^(1/2)/x^2, x)
```

```
[Out] -1/2*exp(b*x+a)^(1/2)*exp(1/2*a-1/2*b*x*exp(1/2*a))*b*(2/x/b*exp(-1/2*a)+1-
ln(x)+ln(2)-ln(-b*exp(1/2*a))-1/b/x*exp(-1/2*a)*(2+b*x*exp(1/2*a))+2/b/x*ex
p(-1/2*a+1/2*b*x*exp(1/2*a))+ln(-1/2*b*x*exp(1/2*a))+Ei(1,-1/2*b*x*exp(1/2*
```

a)))

**Maxima [A]** time = 1.17219, size = 18, normalized size = 0.38

$$\frac{1}{2} b e^{\left(\frac{1}{2} a\right)} \Gamma\left(-1, -\frac{1}{2} b x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(b\*x+a)^(1/2)/x^2,x, algorithm="maxima")

[Out] 1/2\*b\*e^(1/2\*a)\*gamma(-1, -1/2\*b\*x)

**Fricas [A]** time = 1.43129, size = 80, normalized size = 1.67

$$\frac{b x \operatorname{Ei}\left(\frac{1}{2} b x\right) e^{\left(\frac{1}{2} a\right)} - 2 e^{\left(\frac{1}{2} b x + \frac{1}{2} a\right)}}{2 x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(b\*x+a)^(1/2)/x^2,x, algorithm="fricas")

[Out] 1/2\*(b\*x\*Ei(1/2\*b\*x)\*e^(1/2\*a) - 2\*e^(1/2\*b\*x + 1/2\*a))/x

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{e^a e^{bx}}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(b\*x+a)\*\*(1/2)/x\*\*2,x)

[Out] Integral(sqrt(exp(a)\*exp(b\*x))/x\*\*2, x)



**Giac [A]** time = 1.25625, size = 39, normalized size = 0.81

$$\frac{bx\text{Ei}\left(\frac{1}{2}bx\right)e^{\left(\frac{1}{2}a\right)} - 2e^{\left(\frac{1}{2}bx + \frac{1}{2}a\right)}}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(b\*x+a)^(1/2)/x^2,x, algorithm="giac")

[Out] 1/2\*(b\*x\*Ei(1/2\*b\*x)\*e^(1/2\*a) - 2\*e^(1/2\*b\*x + 1/2\*a))/x

$$3.97 \quad \int \frac{\sqrt{e^{a+bx}}}{x^3} dx$$

**Optimal.** Leaf size=71

$$\frac{1}{8}b^2e^{-\frac{bx}{2}}\sqrt{e^{a+bx}}\text{Ei}\left(\frac{bx}{2}\right) - \frac{\sqrt{e^{a+bx}}}{2x^2} - \frac{b\sqrt{e^{a+bx}}}{4x}$$

[Out]  $-\text{Sqrt}[E^{(a + b*x)}]/(2*x^2) - (b*\text{Sqrt}[E^{(a + b*x)}])/(4*x) + (b^2*\text{Sqrt}[E^{(a + b*x)}]*\text{ExpIntegralEi}[(b*x)/2])/(8*E^{((b*x)/2)})$

**Rubi [A]** time = 0.119316, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$ , Rules used = {2177, 2182, 2178}

$$\frac{1}{8}b^2e^{-\frac{bx}{2}}\sqrt{e^{a+bx}}\text{Ei}\left(\frac{bx}{2}\right) - \frac{\sqrt{e^{a+bx}}}{2x^2} - \frac{b\sqrt{e^{a+bx}}}{4x}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Sqrt}[E^{(a + b*x)}]/x^3, x]$

[Out]  $-\text{Sqrt}[E^{(a + b*x)}]/(2*x^2) - (b*\text{Sqrt}[E^{(a + b*x)}])/(4*x) + (b^2*\text{Sqrt}[E^{(a + b*x)}]*\text{ExpIntegralEi}[(b*x)/2])/(8*E^{((b*x)/2)})$

#### Rule 2177

$\text{Int}[(b_*)*(F_*)^{((g_*)*((e_*) + (f_*)*(x_*)))^{(n_*)*((c_*) + (d_*)*(x_*))^{(m_*)}, x\_Symbol] :> \text{Simp}[(c + d*x)^{(m + 1)}*(b*F^{(g*(e + f*x)))^n}/(d*(m + 1)), x] - \text{Dist}[(f*g*n*\text{Log}[F])/(d*(m + 1)), \text{Int}[(c + d*x)^{(m + 1)}*(b*F^{(g*(e + f*x)))^n}, x], x] /; \text{FreeQ}\{F, b, c, d, e, f, g, n\}, x] \&\& \text{LtQ}[m, -1] \&\& \text{IntegerQ}[2*m] \&\& !\$UseGamma == True$

#### Rule 2182

$\text{Int}[(b_*)*(F_*)^{((g_*)*((e_*) + (f_*)*(x_*)))^{(n_*)*((c_*) + (d_*)*(x_*))^{(m_*)}, x\_Symbol] :> \text{Dist}[(b*F^{(g*(e + f*x)))^n}/F^{(g*n*(e + f*x))}, \text{Int}[(c + d*x)^m * F^{(g*n*(e + f*x))}, x], x] /; \text{FreeQ}\{F, b, c, d, e, f, g, m, n\}, x]$

#### Rule 2178

```
Int[(F_)^((g_)*((e_) + (f_)*(x_)))/((c_) + (d_)*(x_)), x_Symbol] := Si
mp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; F
reeQ[{F, c, d, e, f, g}, x] && !$UseGamma === True
```

### Rubi steps

$$\begin{aligned}
 \int \frac{\sqrt{e^{a+bx}}}{x^3} dx &= -\frac{\sqrt{e^{a+bx}}}{2x^2} + \frac{1}{4}b \int \frac{\sqrt{e^{a+bx}}}{x^2} dx \\
 &= -\frac{\sqrt{e^{a+bx}}}{2x^2} - \frac{b\sqrt{e^{a+bx}}}{4x} + \frac{1}{8}b^2 \int \frac{\sqrt{e^{a+bx}}}{x} dx \\
 &= -\frac{\sqrt{e^{a+bx}}}{2x^2} - \frac{b\sqrt{e^{a+bx}}}{4x} + \frac{1}{8} \left( b^2 e^{\frac{1}{2}(a-bx)} \sqrt{e^{a+bx}} \right) \int \frac{e^{\frac{1}{2}(a+bx)}}{x} dx \\
 &= -\frac{\sqrt{e^{a+bx}}}{2x^2} - \frac{b\sqrt{e^{a+bx}}}{4x} + \frac{1}{8} b^2 e^{-\frac{bx}{2}} \sqrt{e^{a+bx}} \text{Ei}\left(\frac{bx}{2}\right)
 \end{aligned}$$

**Mathematica [A]** time = 0.0423023, size = 56, normalized size = 0.79

$$\frac{e^{-\frac{bx}{2}} \sqrt{e^{a+bx}} \left( b^2 x^2 \text{Ei}\left(\frac{bx}{2}\right) - 2e^{\frac{bx}{2}} (bx + 2) \right)}{8x^2}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[E^(a + b\*x)]/x^3, x]

[Out] (Sqrt[E^(a + b\*x)]\*(-2\*E^((b\*x)/2)\*(2 + b\*x) + b^2\*x^2\*ExpIntegralEi[(b\*x)/2]))/(8\*E^((b\*x)/2)\*x^2)

**Maple [B]** time = 0.028, size = 155, normalized size = 2.2

$$\frac{b^2}{4} \sqrt{e^{bx+a}} e^{a-\frac{bx}{2}} e^{\frac{a}{2}} \left( -2 \frac{e^{-a}}{x^2 b^2} - 2 \frac{e^{-a/2}}{bx} - \frac{3}{4} + \frac{\ln(x)}{2} - \frac{\ln(2)}{2} + \frac{1}{2} \ln\left(-be^{\frac{a}{2}}\right) + \frac{e^{-a}}{3x^2 b^2} \left( \frac{9b^2 x^2 e^a}{4} + 6bx e^{a/2} + 6 \right) - \frac{2}{3x^2 b^2} e^{\dots} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(b\*x+a)^(1/2)/x^3, x)

[Out]  $\frac{1}{4} \exp(bx+a)^{1/2} \exp(a-1/2bx \exp(1/2a)) b^2 (-2/x^2/b^2 \exp(-a) - 2/x/b \exp(-1/2a) - 3/4 + 1/2 \ln(x) - 1/2 \ln(2) + 1/2 \ln(-b \exp(1/2a))) + 1/3/b^2/x^2 \exp(-a) * (9/4*b^2*x^2 \exp(a) + 6*b*x \exp(1/2a) + 6) - 2/3/b^2/x^2 \exp(-a + 1/2bx \exp(1/2a)) * (3/2*b*x \exp(1/2a) + 3) - 1/2 \ln(-1/2bx \exp(1/2a)) - 1/2 \text{Ei}(1, -1/2bx \exp(1/2a))$

**Maxima [A]** time = 1.14584, size = 20, normalized size = 0.28

$$-\frac{1}{4} b^2 e^{\left(\frac{1}{2} a\right)} \Gamma\left(-2, -\frac{1}{2} bx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(b*x+a)^(1/2)/x^3,x, algorithm="maxima")`

[Out]  $-1/4*b^2*e^{(1/2*a)}*\text{gamma}(-2, -1/2*b*x)$

**Fricas [A]** time = 1.50765, size = 101, normalized size = 1.42

$$\frac{b^2 x^2 \text{Ei}\left(\frac{1}{2} bx\right) e^{\left(\frac{1}{2} a\right)} - 2 (bx + 2) e^{\left(\frac{1}{2} bx + \frac{1}{2} a\right)}}{8 x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(b*x+a)^(1/2)/x^3,x, algorithm="fricas")`

[Out]  $1/8*(b^2*x^2*\text{Ei}(1/2*b*x)*e^{(1/2*a)} - 2*(b*x + 2)*e^{(1/2*b*x + 1/2*a)})/x^2$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{e^a e^{bx}}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(b*x+a)**(1/2)/x**3,x)`

[Out] Integral(sqrt(exp(a)\*exp(b\*x))/x\*\*3, x)

---

**Giac [A]** time = 1.28556, size = 62, normalized size = 0.87

$$\frac{b^2 x^2 \operatorname{Ei}\left(\frac{1}{2} b x\right) e^{\left(\frac{1}{2} a\right)} - 2 b x e^{\left(\frac{1}{2} b x + \frac{1}{2} a\right)} - 4 e^{\left(\frac{1}{2} b x + \frac{1}{2} a\right)}}{8 x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(b\*x+a)^(1/2)/x^3,x, algorithm="giac")

[Out] 1/8\*(b^2\*x^2\*Ei(1/2\*b\*x)\*e^(1/2\*a) - 2\*b\*x\*e^(1/2\*b\*x + 1/2\*a) - 4\*e^(1/2\*b\*x + 1/2\*a))/x^2

### 3.98

$$\int \frac{\sqrt{e^{a+bx}}}{x^4} dx$$

**Optimal.** Leaf size=92

$$\frac{1}{48} b^3 e^{-\frac{bx}{2}} \sqrt{e^{a+bx}} \text{Ei}\left(\frac{bx}{2}\right) - \frac{b^2 \sqrt{e^{a+bx}}}{24x} - \frac{b \sqrt{e^{a+bx}}}{12x^2} - \frac{\sqrt{e^{a+bx}}}{3x^3}$$

[Out] -Sqrt[E^(a + b\*x)]/(3\*x^3) - (b\*Sqrt[E^(a + b\*x)])/(12\*x^2) - (b^2\*Sqrt[E^(a + b\*x)])/(24\*x) + (b^3\*Sqrt[E^(a + b\*x)]\*ExpIntegralEi[(b\*x)/2])/(48\*E^((b\*x)/2))

**Rubi [A]** time = 0.154713, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$ , Rules used = {2177, 2182, 2178}

$$\frac{1}{48} b^3 e^{-\frac{bx}{2}} \sqrt{e^{a+bx}} \text{Ei}\left(\frac{bx}{2}\right) - \frac{b^2 \sqrt{e^{a+bx}}}{24x} - \frac{b \sqrt{e^{a+bx}}}{12x^2} - \frac{\sqrt{e^{a+bx}}}{3x^3}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[E^(a + b\*x)]/x^4, x]

[Out] -Sqrt[E^(a + b\*x)]/(3\*x^3) - (b\*Sqrt[E^(a + b\*x)])/(12\*x^2) - (b^2\*Sqrt[E^(a + b\*x)])/(24\*x) + (b^3\*Sqrt[E^(a + b\*x)]\*ExpIntegralEi[(b\*x)/2])/(48\*E^((b\*x)/2))

#### Rule 2177

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_), x_Symbol] := Simp[((c + d*x)^(m + 1)*(b*F^(g*(e + f*x)))^n)/(d*(m + 1)), x] - Dist[(f*g*n*Log[F])/(d*(m + 1)), Int[(c + d*x)^(m + 1)*(b*F^(g*(e + f*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && LtQ[m, -1] && IntegerQ[2*m] && !$UseGamma == True
```

#### Rule 2182

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Dist[(b*F^(g*(e + f*x)))^n/F^(g*n*(e + f*x)), Int[(c + d*x)^m*F^(g*n*(e + f*x)), x], x] /; FreeQ[{F, b, c, d, e, f, g, m, n}, x]
```

Rule 2178

Int[(F\_)^((g\_)\*(e\_)+(f\_)\*(x\_)))/((c\_)+(d\_)\*(x\_)), x\_Symbol] :> Simp[(F^(g\*(e-(c\*f)/d))\*ExpIntegralEi[(f\*g\*(c+d\*x)\*Log[F])/d])/d, x] /; FreeQ[{F, c, d, e, f, g}, x] && !UseGamma == True

Rubi steps

$$\begin{aligned}
 \int \frac{\sqrt{e^{a+bx}}}{x^4} dx &= -\frac{\sqrt{e^{a+bx}}}{3x^3} + \frac{1}{6}b \int \frac{\sqrt{e^{a+bx}}}{x^3} dx \\
 &= -\frac{\sqrt{e^{a+bx}}}{3x^3} - \frac{b\sqrt{e^{a+bx}}}{12x^2} + \frac{1}{24}b^2 \int \frac{\sqrt{e^{a+bx}}}{x^2} dx \\
 &= -\frac{\sqrt{e^{a+bx}}}{3x^3} - \frac{b\sqrt{e^{a+bx}}}{12x^2} - \frac{b^2\sqrt{e^{a+bx}}}{24x} + \frac{1}{48}b^3 \int \frac{\sqrt{e^{a+bx}}}{x} dx \\
 &= -\frac{\sqrt{e^{a+bx}}}{3x^3} - \frac{b\sqrt{e^{a+bx}}}{12x^2} - \frac{b^2\sqrt{e^{a+bx}}}{24x} + \frac{1}{48} \left( b^3 e^{\frac{1}{2}(-a-bx)} \sqrt{e^{a+bx}} \right) \int \frac{e^{\frac{1}{2}(a+bx)}}{x} dx \\
 &= -\frac{\sqrt{e^{a+bx}}}{3x^3} - \frac{b\sqrt{e^{a+bx}}}{12x^2} - \frac{b^2\sqrt{e^{a+bx}}}{24x} + \frac{1}{48} b^3 e^{-\frac{bx}{2}} \sqrt{e^{a+bx}} \text{Ei}\left(\frac{bx}{2}\right)
 \end{aligned}$$

**Mathematica [A]** time = 0.0529866, size = 64, normalized size = 0.7

$$\frac{e^{-\frac{bx}{2}} \sqrt{e^{a+bx}} \left( b^3 x^3 \text{Ei}\left(\frac{bx}{2}\right) - 2e^{\frac{bx}{2}} (b^2 x^2 + 2bx + 8) \right)}{48x^3}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[E^(a + b\*x)]/x^4, x]

[Out] (Sqrt[E^(a + b\*x)]\*(-2\*E^((b\*x)/2)\*(8 + 2\*b\*x + b^2\*x^2) + b^3\*x^3\*ExpIntegralEi[(b\*x)/2]))/(48\*E^((b\*x)/2)\*x^3)

**Maple [B]** time = 0.03, size = 189, normalized size = 2.1

$$-\frac{b^3}{8} \sqrt{e^{bx+a}} e^{\frac{3a}{2} - \frac{bx}{2}} e^{\frac{a}{2}} \left( \frac{8}{3x^3 b^3} e^{-\frac{3a}{2}} + 2 \frac{e^{-a}}{x^2 b^2} + \frac{1}{bx} e^{-\frac{a}{2}} + \frac{11}{36} - \frac{\ln(x)}{6} + \frac{\ln(2)}{6} - \frac{1}{6} \ln\left(-be^{\frac{a}{2}}\right) - \frac{1}{9x^3 b^3} e^{-\frac{3a}{2}} \left( \frac{11x^3 b^3}{4} e^{\frac{3a}{2}} + \dots \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(exp(b*x+a)^(1/2)/x^4,x)`

[Out]  $-1/8*\exp(b*x+a)^{(1/2)}*\exp(3/2*a-1/2*b*x*\exp(1/2*a))*b^3*(8/3/x^3/b^3*\exp(-3/2*a)+2/x^2/b^2*\exp(-a)+1/x/b*\exp(-1/2*a)+11/36-1/6*\ln(x)+1/6*\ln(2)-1/6*\ln(-b*\exp(1/2*a))-1/9/b^3/x^3*\exp(-3/2*a)*(11/4*b^3*x^3*\exp(3/2*a)+9*b^2*x^2*\exp(a)+18*b*x*\exp(1/2*a)+24)+1/3/b^3/x^3*\exp(-3/2*a+1/2*b*x*\exp(1/2*a))*(b^2*x^2*\exp(a)+2*b*x*\exp(1/2*a)+8)+1/6*\ln(-1/2*b*x*\exp(1/2*a))+1/6*Ei(1,-1/2*b*x*\exp(1/2*a)))$

**Maxima [A]** time = 1.13943, size = 20, normalized size = 0.22

$$\frac{1}{8}b^3e^{\left(\frac{1}{2}a\right)}\Gamma\left(-3,-\frac{1}{2}bx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(b*x+a)^(1/2)/x^4,x, algorithm="maxima")`

[Out]  $1/8*b^3*e^{(1/2*a)}*\gamma(-3, -1/2*b*x)$

**Fricas [A]** time = 1.46683, size = 119, normalized size = 1.29

$$\frac{b^3x^3Ei\left(\frac{1}{2}bx\right)e^{\left(\frac{1}{2}a\right)} - 2\left(b^2x^2 + 2bx + 8\right)e^{\left(\frac{1}{2}bx + \frac{1}{2}a\right)}}{48x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(b*x+a)^(1/2)/x^4,x, algorithm="fricas")`

[Out]  $1/48*(b^3*x^3*Ei(1/2*b*x)*e^{(1/2*a)} - 2*(b^2*x^2 + 2*b*x + 8)*e^{(1/2*b*x + 1/2*a)})/x^3$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{e^a e^{bx}}}{x^4} dx$$



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(b\*x+a)\*\*(1/2)/x\*\*4,x)

[Out] Integral(sqrt(exp(a)\*exp(b\*x))/x\*\*4, x)

**Giac [A]** time = 1.30918, size = 85, normalized size = 0.92

$$\frac{b^3 x^3 \operatorname{Ei}\left(\frac{1}{2} b x\right) e^{\left(\frac{1}{2} a\right)} - 2 b^2 x^2 e^{\left(\frac{1}{2} b x + \frac{1}{2} a\right)} - 4 b x e^{\left(\frac{1}{2} b x + \frac{1}{2} a\right)} - 16 e^{\left(\frac{1}{2} b x + \frac{1}{2} a\right)}}{48 x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(b\*x+a)^(1/2)/x^4,x, algorithm="giac")

[Out] 1/48\*(b^3\*x^3\*Ei(1/2\*b\*x)\*e^(1/2\*a) - 2\*b^2\*x^2\*e^(1/2\*b\*x + 1/2\*a) - 4\*b\*x\*e^(1/2\*b\*x + 1/2\*a) - 16\*e^(1/2\*b\*x + 1/2\*a))/x^3



# Chapter 4

## Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

### 4.0.1 Mathematica and Rubi grading function

```
1 (* Original version thanks to Albert Rich emailed on 03/21/2017 *)
2 (* ::Package:: *)
3
4 (* ::Subsection:: *)
5 (*GradeAntiderivative[result,optimal]*)
6
7
8 (* ::Text:: *)
9 (*If result and optimal are mathematical expressions, *)
10 (*      GradeAntiderivative[result,optimal] returns*)
11 (* "F" if the result fails to integrate an expression that*)
12 (*      is integrable*)
13 (* "C" if result involves higher level functions than necessary*)
14 (* "B" if result is more than twice the size of the optimal*)
15 (*      antiderivative*)
16 (* "A" if result can be considered optimal*)
17
18
19 GradeAntiderivative[result_,optimal_] :=
20   If[ExpnType[result]<=ExpnType[optimal],
21     If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
```

```

22     If[LeafCount[result]<=2*LeafCount[optimal],
23         "A",
24         "B"],
25     "C"],
26 If[FreeQ[result,Integrate] && FreeQ[result,Int],
27     "C",
28     "F"]]
29
30
31 (* ::Text:: *)
32 (*The following summarizes the type number assigned an *)
33 (*expression based on the functions it involves*)
34 (*1 = rational function*)
35 (*2 = algebraic function*)
36 (*3 = elementary function*)
37 (*4 = special function*)
38 (*5 = hyperpergeometric function*)
39 (*6 = appell function*)
40 (*7 = rootsum function*)
41 (*8 = integrate function*)
42 (*9 = unknown function*)
43
44
45 ExpnType[expn_] :=
46     If[AtomQ[expn],
47         1,
48     If[ListQ[expn],
49         Max[Map[ExpnType,expn]],
50     If[Head[expn]===Power,
51         If[IntegerQ[expn[[2]]],
52             ExpnType[expn[[1]]],
53         If[Head[expn[[2]]]===Rational,
54             If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
55                 1,
56                 Max[ExpnType[expn[[1]],2]],
57             Max[ExpnType[expn[[1]],ExpnType[expn[[2]],3]],
58     If[Head[expn]===Plus || Head[expn]===Times,
59         Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
60     If[ElementaryFunctionQ[Head[expn]],
61         Max[3,ExpnType[expn[[1]]],
62     If[SpecialFunctionQ[Head[expn]],
63         Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
64     If[HypergeometricFunctionQ[Head[expn]],
65         Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
66     If[AppellFunctionQ[Head[expn]],
67         Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
68     If[Head[expn]===RootSum,

```

```

69   Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 7]],
70   If[Head[expn]===Integrate || Head[expn]===Int,
71     Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 8]],
72   9]]]]]]]]]]
73
74
75 ElementaryFunctionQ[func_] :=
76   MemberQ[{
77     Exp, Log,
78     Sin, Cos, Tan, Cot, Sec, Csc,
79     ArcSin, ArcCos, ArcTan, ArcCot, ArcSec, ArcCsc,
80     Sinh, Cosh, Tanh, Coth, Sech, Csch,
81     ArcSinh, ArcCosh, ArcTanh, ArcCoth, ArcSech, ArcCsch
82   }, func]
83
84
85 SpecialFunctionQ[func_] :=
86   MemberQ[{
87     Erf, Erfc, Erfi,
88     FresnelS, FresnelC,
89     ExpIntegralE, ExpIntegralEi, LogIntegral,
90     SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
91     Gamma, LogGamma, PolyGamma,
92     Zeta, PolyLog, ProductLog,
93     EllipticF, EllipticE, EllipticPi
94   }, func]
95
96
97 HypergeometricFunctionQ[func_] :=
98   MemberQ[{Hypergeometric1F1, Hypergeometric2F1, HypergeometricPFQ}, func]
99
100
101 AppellFunctionQ[func_] :=
102   MemberQ[{AppellF1}, func]

```

## 4.0.2 Maple grading function

```

1 # File: GradeAntiderivative.mpl
2 # Original version thanks to Albert Rich emailed on 03/21/2017
3
4 #Nasser 03/22/2017 Use Maple leaf count instead since buildin
5 #Nasser 03/23/2017 missing 'ln' for ElementaryFunctionQ added
6 #Nasser 03/24/2017 corrected the check for complex result
7 #Nasser 10/27/2017 check for leafsize and do not call ExpnType()
8 #
9 #Nasser 12/22/2019 Added debug flag, added 'dilog' to special functions
10 #
    see problem 156, file Apostol_Problems

```

```

11
12 GradeAntiderivative := proc(result,optimal)
13 local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
    debug:=false;
14
15     leaf_count_result:=leafcount(result);
16     #do NOT call ExpnType() if leaf size is too large. Recursion problem
17     if leaf_count_result > 500000 then
18         return "B";
19     fi;
20
21     leaf_count_optimal:=leafcount(optimal);
22
23     ExpnType_result:=ExpnType(result);
24     ExpnType_optimal:=ExpnType(optimal);
25
26     if debug then
27         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
    ExpnType_optimal);
28     fi;
29
30 # If result and optimal are mathematical expressions,
31 # GradeAntiderivative[result,optimal] returns
32 #   "F" if the result fails to integrate an expression that
33 #     is integrable
34 #   "C" if result involves higher level functions than necessary
35 #   "B" if result is more than twice the size of the optimal
36 #     antiderivative
37 #   "A" if result can be considered optimal
38
39 #This check below actually is not needed, since I only
40 #call this grading only for passed integrals. i.e. I check
41 #for "F" before calling this. But no harm of keeping it here.
42 #just in case.
43
44
45 if not type(result,freeof('int')) then
46     return "F";
47 end if;
48
49
50 if ExpnType_result<=ExpnType_optimal then
51     if debug then
52         print("ExpnType_result<=ExpnType_optimal");
53     fi;
54     if is_contains_complex(result) then
55         if is_contains_complex(optimal) then

```

```

56     if debug then
57         print("both result and optimal complex");
58     fi;
59     #both result and optimal complex
60     if leaf_count_result<=2*leaf_count_optimal then
61         return "A";
62     else
63         return "B";
64     end if
65     else #result contains complex but optimal is not
66         if debug then
67             print("result contains complex but optimal is not");
68         fi;
69         return "C";
70     end if
71     else # result do not contain complex
72         # this assumes optimal do not as well
73         if debug then
74             print("result do not contain complex, this assumes optimal do not
as well");
75         fi;
76         if leaf_count_result<=2*leaf_count_optimal then
77             if debug then
78                 print("leaf_count_result<=2*leaf_count_optimal");
79             fi;
80             return "A";
81         else
82             if debug then
83                 print("leaf_count_result>2*leaf_count_optimal");
84             fi;
85             return "B";
86         end if
87     end if
88     else #ExpnType(result) > ExpnType(optimal)
89         if debug then
90             print("ExpnType(result) > ExpnType(optimal)");
91         fi;
92         return "C";
93     end if
94
95 end proc:
96
97 #
98 # is_contains_complex(result)
99 # takes expressions and returns true if it contains "I" else false
100 #
101 #Nasser 032417

```

```

102 is_contains_complex:= proc(expression)
103   return (has(expression,I));
104 end proc:
105
106 # The following summarizes the type number assigned an expression
107 # based on the functions it involves
108 # 1 = rational function
109 # 2 = algebraic function
110 # 3 = elementary function
111 # 4 = special function
112 # 5 = hyperpergeometric function
113 # 6 = appell function
114 # 7 = rootsum function
115 # 8 = integrate function
116 # 9 = unknown function
117
118 ExpnType := proc(expn)
119   if type(expn,'atomic') then
120     1
121   elif type(expn,'list') then
122     apply(max,map(ExpnType,expn))
123   elif type(expn,'sqrt') then
124     if type(op(1,expn),'rational') then
125       1
126     else
127       max(2,ExpnType(op(1,expn)))
128     end if
129   elif type(expn,'^^') then
130     if type(op(2,expn),'integer') then
131       ExpnType(op(1,expn))
132     elif type(op(2,expn),'rational') then
133       if type(op(1,expn),'rational') then
134         1
135       else
136         max(2,ExpnType(op(1,expn)))
137       end if
138     else
139       max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
140     end if
141   elif type(expn,'^+') or type(expn,'^*') then
142     max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
143   elif ElementaryFunctionQ(op(0,expn)) then
144     max(3,ExpnType(op(1,expn)))
145   elif SpecialFunctionQ(op(0,expn)) then
146     max(4,apply(max,map(ExpnType,[op(expn)])))
147   elif HypergeometricFunctionQ(op(0,expn)) then
148     max(5,apply(max,map(ExpnType,[op(expn)])))

```



```

149   elif AppellFunctionQ(op(0,expn)) then
150       max(6,apply(max,map(ExpnType,[op(expn)])))
151   elif op(0,expn)='int' then
152       max(8,apply(max,map(ExpnType,[op(expn)]))) else
153       9
154   end if
155 end proc:
156
157
158 ElementaryFunctionQ := proc(func)
159     member(func,[
160         exp,log,ln,
161         sin,cos,tan,cot,sec,csc,
162         arcsin,arccos,arctan,arccot,arcsec,arccsc,
163         sinh,cosh,tanh,coth,sech,csch,
164         arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
165 end proc:
166
167 SpecialFunctionQ := proc(func)
168     member(func,[
169         erf,erfc,erfi,
170         FresnelS,FresnelC,
171         Ei,Ei,Li,Si,Ci,Shi,Chi,
172         GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
173         EllipticF,EllipticE,EllipticPi])
174 end proc:
175
176 HypergeometricFunctionQ := proc(func)
177     member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
178 end proc:
179
180 AppellFunctionQ := proc(func)
181     member(func,[AppellF1])
182 end proc:
183
184 # u is a sum or product. rest(u) returns all but the
185 # first term or factor of u.
186 rest := proc(u) local v;
187     if nops(u)=2 then
188         op(2,u)
189     else
190         apply(op(0,u),op(2..nops(u),u))
191     end if
192 end proc:
193
194 #leafcount(u) returns the number of nodes in u.
195 #Nasser 3/23/17 Replaced by build-in leafCount from package in Maple

```

```

196 leafcount := proc(u)
197     MmaTranslator[Mma][LeafCount](u);
198 end proc:

```

### 4.0.3 Sympy grading function

```

1 #Dec 24, 2019. Nasser M. Abbasi:
2 #           Port of original Maple grading function by
3 #           Albert Rich to use with Sympy/Python
4 #Dec 27, 2019 Nasser. Added `RootSum`. See problem 177, Timofeev file
5 #           added 'exp_polar'
6 from sympy import *
7
8 def leaf_count(expr):
9     #sympy do not have leaf count function. This is approximation
10    return round(1.7*count_ops(expr))
11
12 def is_sqrt(expr):
13     if isinstance(expr,Pow):
14         if expr.args[1] == Rational(1,2):
15             return True
16         else:
17             return False
18     else:
19         return False
20
21 def is_elementary_function(func):
22     return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
23                    asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
24                    asinh,acosh,atanh,acoth,asech,acsch
25                    ]
26
27 def is_special_function(func):
28     return func in [ erf,erfc,erfi,
29                    fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
30                    gamma,loggamma,digamma,zeta,polylog,LambertW,
31                    elliptic_f,elliptic_e,elliptic_pi,exp_polar
32                    ]
33
34 def is_hypergeometric_function(func):
35     return func in [hyper]
36
37 def is_appell_function(func):
38     return func in [appellf1]
39
40 def is_atom(expn):
41     try:

```

```

42     if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
43         return True
44     else:
45         return False
46
47     except AttributeError as error:
48         return False
49
50 def expnType(expn):
51     debug=False
52     if debug:
53         print("expn=",expn,"type(expn)=",type(expn))
54
55     if is_atom(expn):
56         return 1
57     elif isinstance(expn,list):
58         return max(map(expnType, expn)) #apply(max,map(ExpnType,expn))
59     elif is_sqrt(expn):
60         if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
61             return 1
62         else:
63             return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn)))
64     elif isinstance(expn,Pow): #type(expn,'^^')
65         if isinstance(expn.args[1],Integer): #type(op(2,expn),'integer')
66             return expnType(expn.args[0]) #ExpnType(op(1,expn))
67         elif isinstance(expn.args[1],Rational): #type(op(2,expn),'rational')
68             if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
69                 return 1
70             else:
71                 return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn)
72 ))
73     else:
74         return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
75 ExpnType(op(1,expn)),ExpnType(op(2,expn)))
76     elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'+' or type
77 (expn,'*')
78         m1 = expnType(expn.args[0])
79         m2 = expnType(list(expn.args[1:]))
80         return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
81     elif is_elementary_function(expn.func): #ElementaryFunctionQ(op(0,expn))
82         return max(3,expnType(expn.args[0])) #max(3,ExpnType(op(1,expn)))
83     elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
84         m1 = max(map(expnType, list(expn.args)))
85         return max(4,m1) #max(4,apply(max,map(ExpnType,[op(expn)])))
86     elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
87 expn))
88         m1 = max(map(expnType, list(expn.args)))

```

```

85     return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
86 elif is_appell_function(expn.func):
87     m1 = max(map(expnType, list(expn.args)))
88     return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
89 elif isinstance(expn,RootSum):
90     m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,
Apply[List,expn]],7]],
91     return max(7,m1)
92 elif str(expn).find("Integral") != -1:
93     m1 = max(map(expnType, list(expn.args)))
94     return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
95 else:
96     return 9
97
98 #main function
99 def grade_antiderivative(result,optimal):
100
101     leaf_count_result = leaf_count(result)
102     leaf_count_optimal = leaf_count(optimal)
103
104     expnType_result = expnType(result)
105     expnType_optimal = expnType(optimal)
106
107     if str(result).find("Integral") != -1:
108         return "F"
109
110     if expnType_result <= expnType_optimal:
111         if result.has(I):
112             if optimal.has(I): #both result and optimal complex
113                 if leaf_count_result <= 2*leaf_count_optimal:
114                     return "A"
115                 else:
116                     return "B"
117             else: #result contains complex but optimal is not
118                 return "C"
119         else: # result do not contain complex, this assumes optimal do not as
well
120             if leaf_count_result <= 2*leaf_count_optimal:
121                 return "A"
122             else:
123                 return "B"
124     else:
125         return "C"

```

## 4.0.4 SageMath grading function

1 #Dec 24, 2019. Nasser: Ported original Maple grading function by

```

2 #           Albert Rich to use with Sagemath. This is used to
3 #           grade Fracas, Giac and Maxima results.
4 #Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
5 #           'arctan2','floor','abs','log_integral'
6
7 from sage.all import *
8 from sage.symbolic.operators import add_vararg, mul_vararg
9
10 def tree(expr):
11     debug=False;
12     if debug:
13         print ("Enter tree(expr), expr=",expr)
14         print ("expr.operator()=",expr.operator())
15         print ("expr.operands()=",expr.operands())
16         print ("map(tree, expr.operands()=",map(tree, expr.operands()))
17
18     if expr.operator() is None:
19         return expr
20     else:
21         return [expr.operator()+list(map(tree, expr.operands()))
22
23 def leaf_count(anti):
24     debug=False;
25
26     if debug: print ("Enter leaf_count, anti=", anti, " len(anti)=", len(anti))
27
28     if len(anti) == 0: #special check for optimal being 0 for some test cases.
29         if debug: print ("len(anti) == 0")
30         return 1
31     else:
32         if debug: print ("round(1.35*len(flatten(tree(anti))))=",round(1.35*len(
33         flatten(tree(anti))))
34         return round(1.35*len(flatten(tree(anti)))) #fudge factor
35         #since this estimate of leaf count is bit lower than
36         #what it should be compared to Mathematica's
37
38 def is_sqrt(expr):
39     debug=False;
40     if expr.operator() == operator.pow: #isinstance(expr,Pow):
41         if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
42             if debug: print ("expr is sqrt")
43             return True
44         else:
45             return False
46     else:
47         return False

```

```

48 def is_elementary_function(func):
49     debug = False
50
51     m = func.name() in ['exp','log','ln',
52         'sin','cos','tan','cot','sec','csc',
53         'arcsin','arccos','arctan','arccot','arcsec','arccsc',
54         'sinh','cosh','tanh','coth','sech','csch',
55         'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
56         'arctan2','floor','abs'
57     ]
58     if debug:
59         if m:
60             print ("func ", func , " is elementary_function")
61         else:
62             print ("func ", func , " is NOT elementary_function")
63
64
65     return m
66
67 def is_special_function(func):
68     debug = False
69
70     if debug: print ("type(func)=", type(func))
71
72     m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
73         'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
74     sinh_integral'
75         'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
76         'polylog','lambert_w','elliptic_f','elliptic_e',
77         'elliptic_pi','exp_integral_e','log_integral']
78
79     if debug:
80         print ("m=",m)
81         if m:
82             print ("func ", func ," is special_function")
83         else:
84             print ("func ", func ," is NOT special_function")
85
86     return m
87
88
89 def is_hypergeometric_function(func):
90     return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U
91     ']
92
93 def is_appell_function(func):

```

```

93     return func.name() in ['hypergeometric']    #[appellf1] can't find this in
          sagemath
94
95 def is_atom(expn):
96
97     #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-type-in-maple/
98     try:
99         if expn.parent() is SR:
100             return expn.operator() is None
101         if expn.parent() in (ZZ, QQ, AA, QQbar):
102             return expn in expn.parent() # Should always return True
103         if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
104             return expn in expn.parent().base_ring() or expn in expn.parent().
          gens()
105         return False
106
107     except AttributeError as error:
108         return False
109
110
111 def expnType(expn):
112     debug=False
113
114     if debug:
115         print(">>>>Enter expnType, expn=", expn)
116         print(">>>>is_atom(expn)=", is_atom(expn))
117
118     if is_atom(expn):
119         return 1
120     elif type(expn)==list:    #isinstance(expn,list):
121         return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
122     elif is_sqrt(expn):
123         if type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
          Rational):
124             return 1
125         else:
126             return max(2,expnType(expn.operands()[0]))    #max(2,expnType(expn.
          args[0]))
127     elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
128         if type(expn.operands()[1])==Integer:    #isinstance(expn.args[1],Integer)
129             return expnType(expn.operands()[0])    #expnType(expn.args[0])
130         elif type(expn.operands()[1])==Rational:    #isinstance(expn.args[1],
          Rational)
131             if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
          Rational)
132                 return 1

```

```

133         else:
134             return max(2,expnType(expn.operands()[0])) #max(2,expnType(expn.
args[0]))
135         else:
136             return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
137         elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
isinstance(expn,Add) or isinstance(expn,Mul)
138             m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
139             m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
140             return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
141         elif is_elementary_function(expn.operator()): #is_elementary_function(expn.
func)
142             return max(3,expnType(expn.operands()[0]))
143         elif is_special_function(expn.operator()): #is_special_function(expn.func)
144             m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
145             return max(4,m1) #max(4,m1)
146         elif is_hypergeometric_function(expn.operator()): #
is_hypergeometric_function(expn.func)
147             m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
148             return max(5,m1) #max(5,m1)
149         elif is_appell_function(expn.operator()):
150             m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
151             return max(6,m1) #max(6,m1)
152         elif str(expn).find("Integral") != -1: #this will never happen, since it
153             #is checked before calling the grading function that is passed.
154             #but kept it here.
155             m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
156             return max(8,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
157         else:
158             return 9
159
160 #main function
161 def grade_antiderivative(result,optimal):
162     debug = False;
163
164     if debug: print ("Enter grade_antiderivative for sagemath")
165
166     leaf_count_result = leaf_count(result)
167     leaf_count_optimal = leaf_count(optimal)
168
169     if debug: print ("leaf_count_result=", leaf_count_result, "
leaf_count_optimal=",leaf_count_optimal)

```



```
170
171
172     expnType_result = expnType(result)
173     expnType_optimal = expnType(optimal)
174
175     if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
176                     expnType_optimal)
177
178     if expnType_result <= expnType_optimal:
179         if result.has(I):
180             if optimal.has(I): #both result and optimal complex
181                 if leaf_count_result <= 2*leaf_count_optimal:
182                     return "A"
183                 else:
184                     return "B"
185             else: #result contains complex but optimal is not
186                 return "C"
187         else: # result do not contain complex, this assumes optimal do not as
188             well
189                 if leaf_count_result <= 2*leaf_count_optimal:
190                     return "A"
191                 else:
192                     return "B"
193         else:
194             return "C"
```